Decomposition of Time Series

Econometrics of Seasonality

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Decomposition of Time Series

- macroeconomic time series are subject to two sorts of forces: those that influence the **long-run behavior** of the series and those that influence the **short-run behavior** of the series.
- growth theory focuses on the forces that influence long-run behavior
- business cycle theory focuses on the forces that influence short-run behavior.
- transform a nonstationary series into a stationary series by removing the trend; alleviate a spurious regression problem.

Polynomial Trend Removal

- Assume that y_t is a **trend stationary** process. That is:

  $y_t = \tau_t + c_t$

 τ_t is a deterministic function, typically a low-order polynomial, of t called the trend (or secular or long-run or permanent) component of y_t and c_t is a stationary process, called the cyclical component (or short-run or transitory component) of y_t.

Polynomial Trend Removal

- Assume that the trend is a polynomial in t, so that

  $$y_t = \tau_t + c_t$$
  $$= \beta_0 + \beta_1 t + \ldots + \beta_p t^p + c_t$$

- This is a standard linear regression model with a serially correlated process, c_t. How to efficiently estimate the β's? OLS.

Polynomial Trend Removal

- Then it makes sense to set

$$\hat{\tau}_t = \hat{\beta}_0 + \hat{\beta}_1 t + \ldots + \hat{\beta}_p t^p$$

- and

$$\hat{c}_t = y_t - \hat{\tau}_t$$

- where β–hat denotes the OLS estimator.

Polynomial Trend Removal

- In this case it makes some sense to add the explanatory variable t as a separate regressor in the regression:

(*) $y_t = \beta_0 + \beta_1 x_t + \beta_2 t + \epsilon_t$.

An alternative: Formulate the model as

(**) $y_t^* = \beta_1 x_t^* + \epsilon_t$

- where y_t^* is the residual series from the regression of y_t on a linear time trend and x_t^* is the residual series obtained from the regression of x_t on a linear trend.

Decompositions Based on Differences

- An alternative to the trend stationary assumption to account for trend behavior in a time series is to assume that the series is **difference stationary**, i.e., y_t is stationary in differenced form.

Diffence Stationarity

A time series y_t difference stationary of order d, d a positive integer, if

$\Delta^d y_t$ is stationary
$\Delta^{d-1} y_t$ is not stationary
The MA form of $\Delta^d y_t$ does not have a "unit root"

In practice, d = 1 and, for some rapidly growing nominal time series, d = 2 are the most commonly used values of d.

Suppose y_t is the trend stationary process

$$y_t = \beta_0 + \beta_1 t + \epsilon_t,$$

The Random Walk Process

The simplest case of an I(1) process is the random walk:

$y_t = y_{t-1} + \epsilon_t$, ϵ_t a zero-mean iid process

Note that for the rw –

Δy_t is an iid process: changes in y_t are serially uncorrelated, independent, identically distributed.

$dy_{t+s}/d\epsilon_t = 1$ for all $s > 0$: Innovations have completely permanent effects on the time series!

$y_{t+1} = y_t + \epsilon_{t+1} =$
$y_{t+2} = y_{t+1} + \epsilon_{t+2} = y_{t-1} + \boldsymbol{\epsilon_t} + \epsilon_{t+1} + \epsilon_{t+2}$
…
$y_{t+s} = y_{t-1} + (\boldsymbol{\epsilon_t} + \epsilon_{t+1} + \epsilon_{t+2} + \ldots + \epsilon_{t+s})$

Decomposition Example

- The additive model used is:

Y[t] = T[t] + S[t] + e[t]

- The multiplicative model used is:

Y[t] = T[t] * S[t] + e[t]

```
if (is.null(filter))
        filter <- if (!f%%2)
            c(0.5, rep(1, f - 1), 0.5)/f
        else rep(1, f)/f
    trend <- filter(x, filter)
    season <- if (type == "additive")
        x - trend
    else x/trend
    season <- na.omit(c(as.numeric(window(season, start(x) +
        c(1, 0), end(x))), as.numeric(window(season, start(x),
        start(x) + c(0, f)))))
seasonal <- ts(rep(figure, periods), start = start(x), frequency = f)
random = if (type == "additive") x - seasonal - trend else x/seasonal/trend,
```

Canned AR and MA filters

- Convolution filters have the form

$$y = (a_0 + a_1 L + \ldots + a_p L^p)x$$

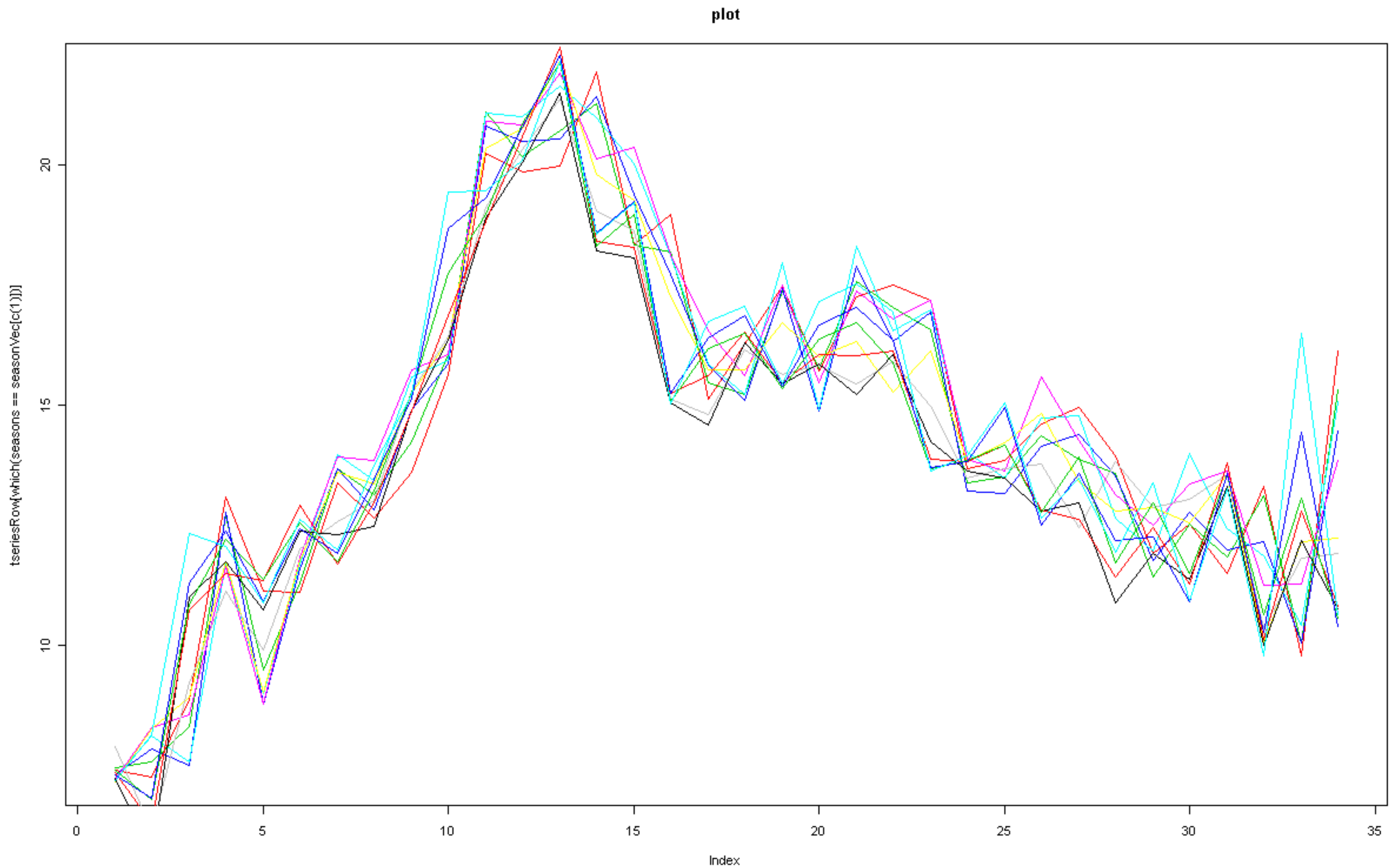
- while recursive filters solve

$$y = (a_1 L + a_2 L^2 + \ldots + a_p L^p)y + x$$

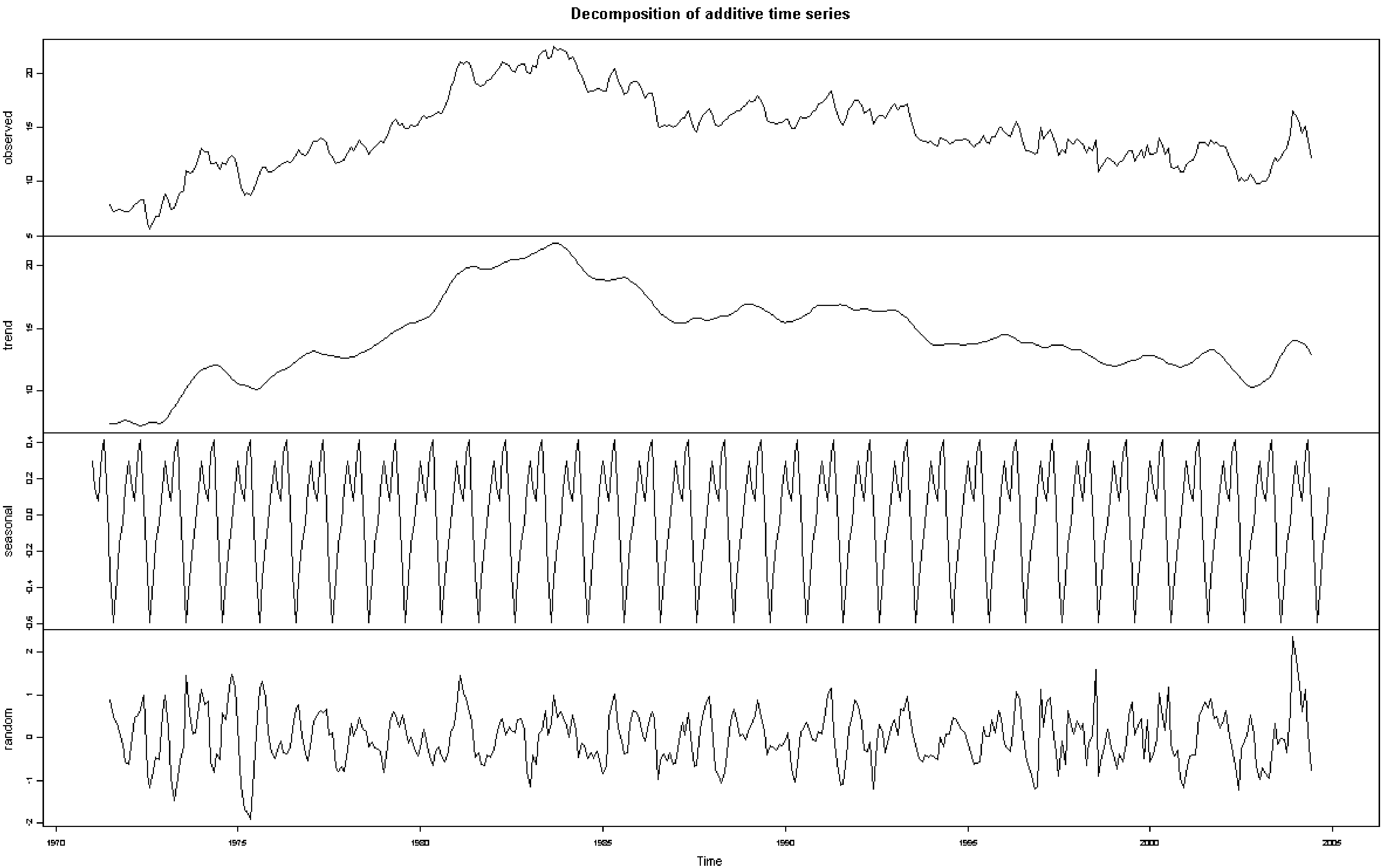
- In both cases, x is the input to the filter and y the output. If x is a vector of innovations, the convolution filter generates a moving average series and the recursive filter generates an autoregressive series. Notice that there is no a_0 term in the recursive filter (it would not make sense theoretically). The recursive filter can equivalently be thought of as solving

$$y = (1 - a_1 L - a_2 L^2 - \ldots - a_p L^p)^{-1}x$$

Seasonal Decomposition by Moving Averages

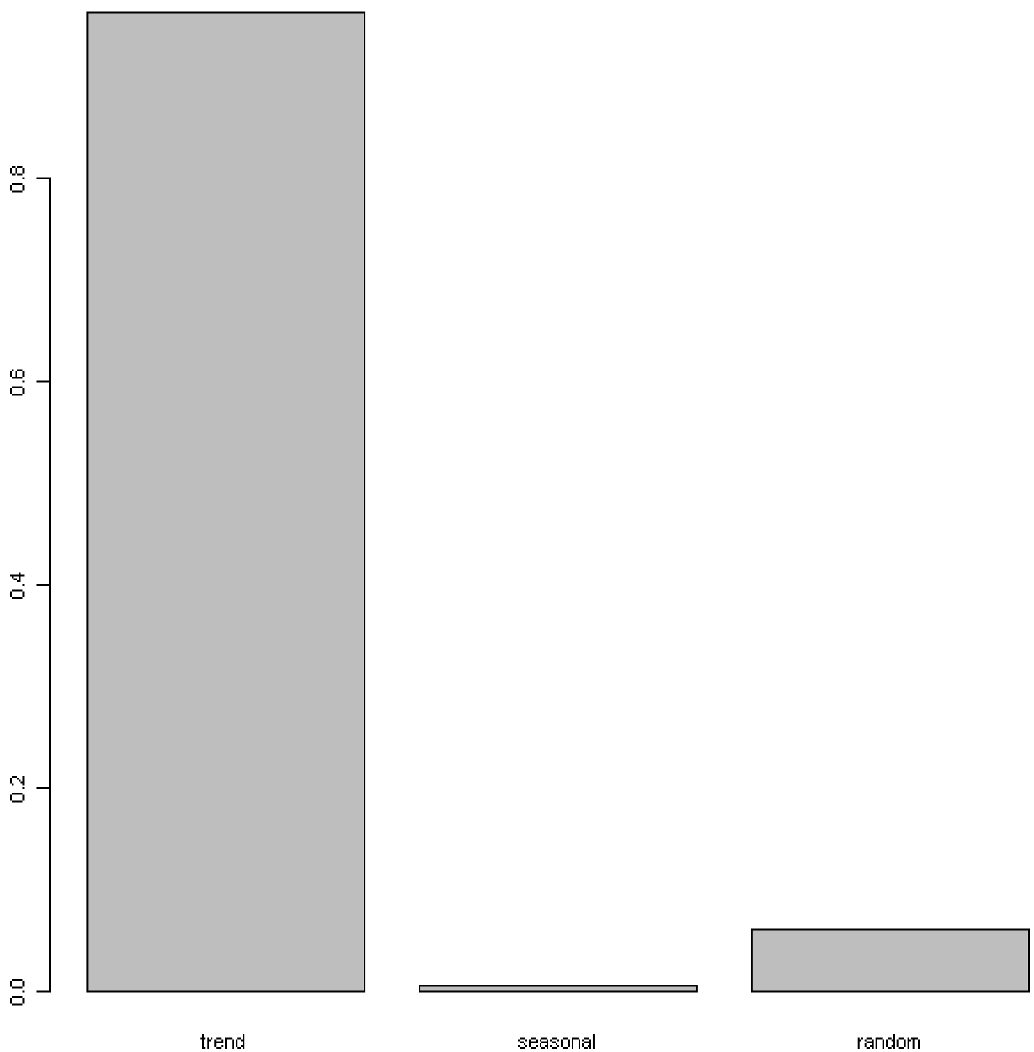


Data – Prices Soft Wheat
1971 – 2004 (monthly)

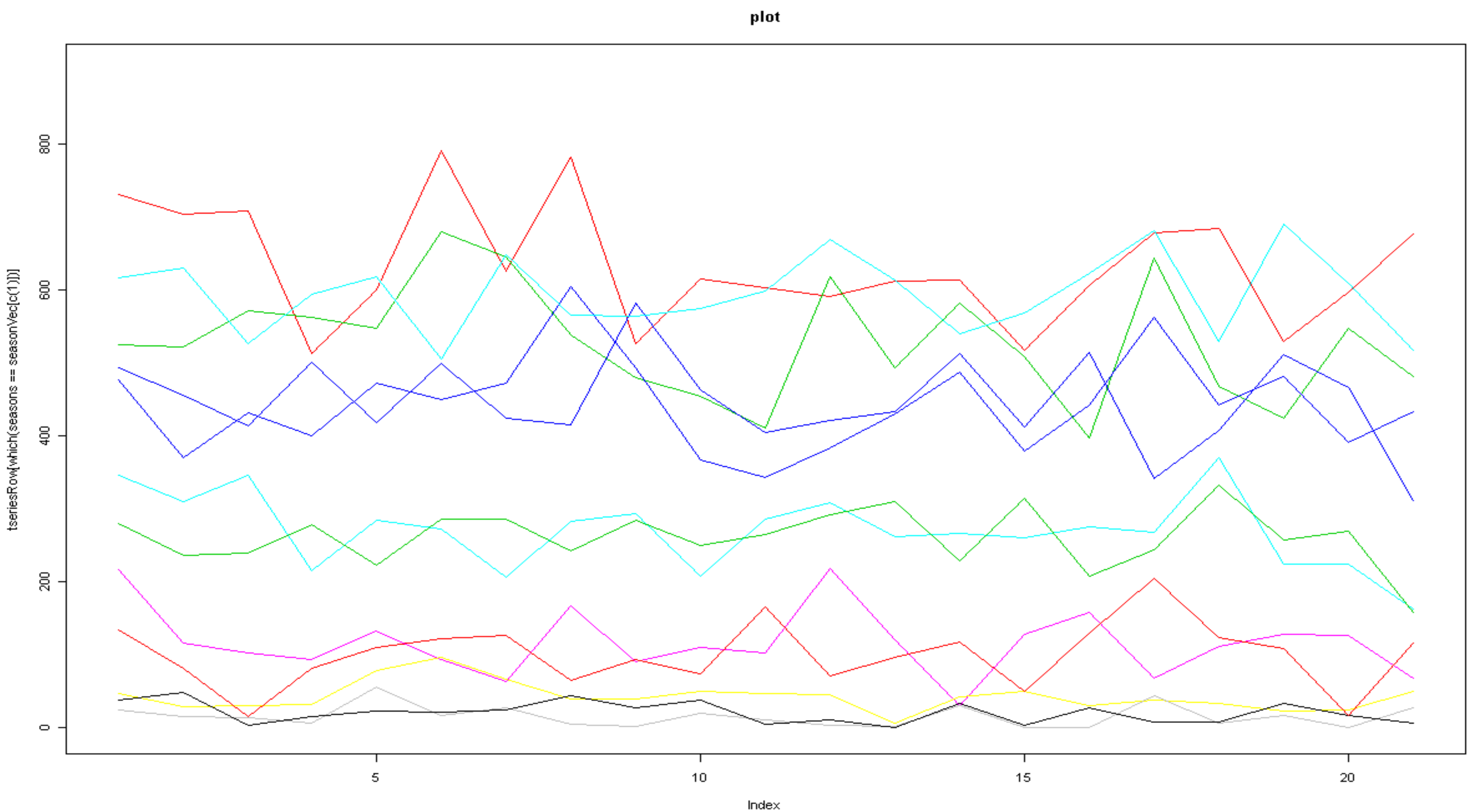


Decomposition of additive time series
observed
trend
seasonal
random
Time
1970
1975
1980
1985
1990
1995
2000
2005

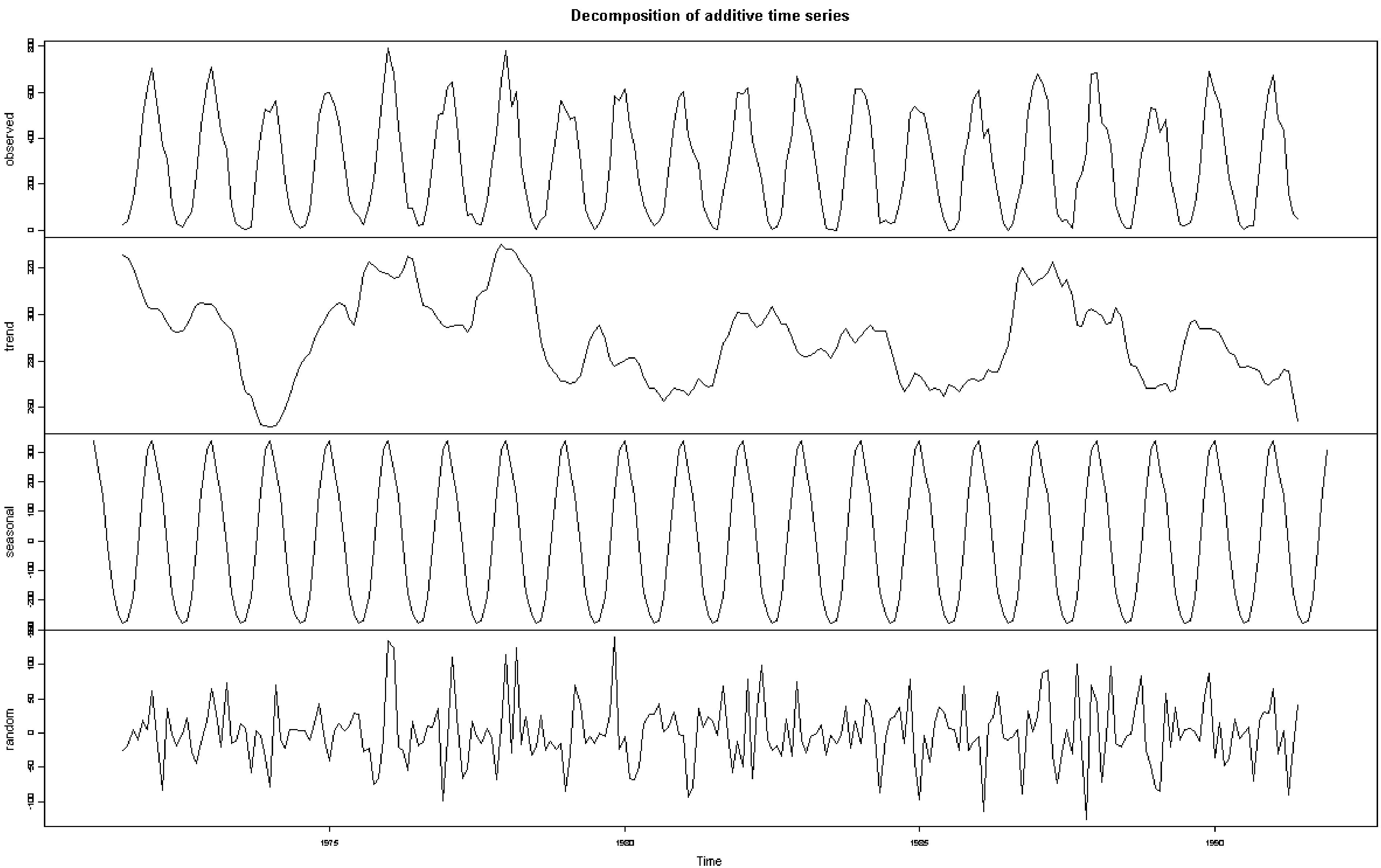
A	B	C
trend	seasonal	random
0.961418137415612	0.00634464451970235	0.061044275260962



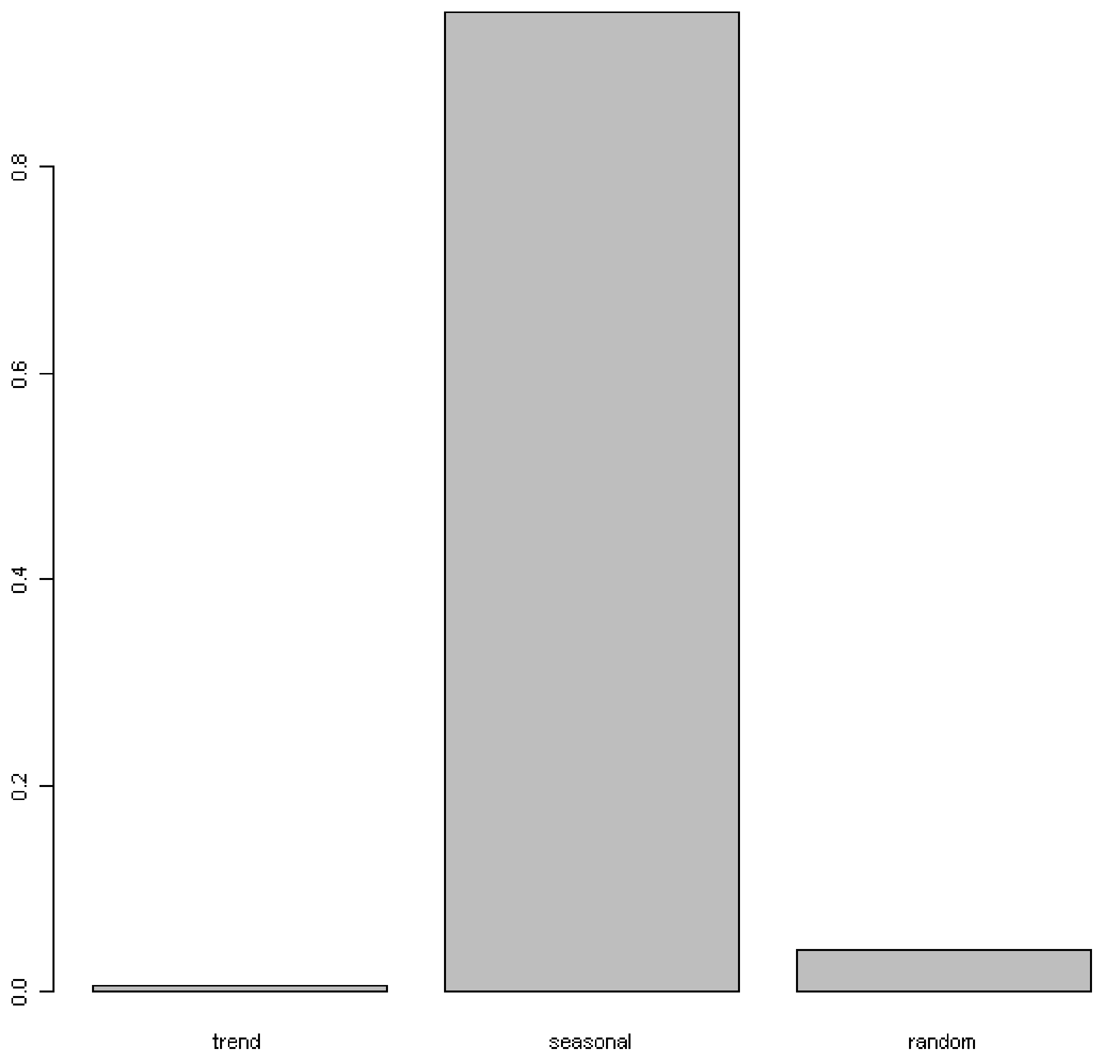
Seasonal Decomposition by Moving Averages



Data – Temperature Index 1980 – 2000 (monthly)


Decomposition of additive time series
observed
trend
seasonal
random
1975
1980
1985
1990
Time

A	B	C
trend	seasonal	random
0.00542119370841522	0.949693166943617	0.0409172496210831



Bibliography

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- Classical Seasonal Decomposition by Moving Averages [http://astrostatistics.psu.edu/datasets/R/html/stats/html/decompose.html]