Testing for Seasonal Integration and Cointegration: The Austrian Consumption Income Relationship

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INTRODUCTION

This paper analyzes the nature of seasonal fluctuations in quarterly observations for Austrian consumption and income data.

-The reliability of empirical studies is often endangered by nonstationarity of the analyzed variables. When data of higher frequency (monthly or quarterly) are analyzed however, the 'spurious regression' problem might reappear as a consequence of seasonality in the series.

-To avoid the problem by simply using seasonally adjusted data has become a well-established practice in much empirical work. On the other hand, seasonal adjustment can have distorting influences on the relationships between economic time series.

The Goal line of the paper is

To Investigate the Seasonal Integration and seasonal Cointegration, in Austrian Consumption and Income data, using both kind of data Seasonally Adjusted and Raw data.

The Data

The analyzed data are quarterly time series from the Austrian National Accounts, 1954 to1994.

The data are:

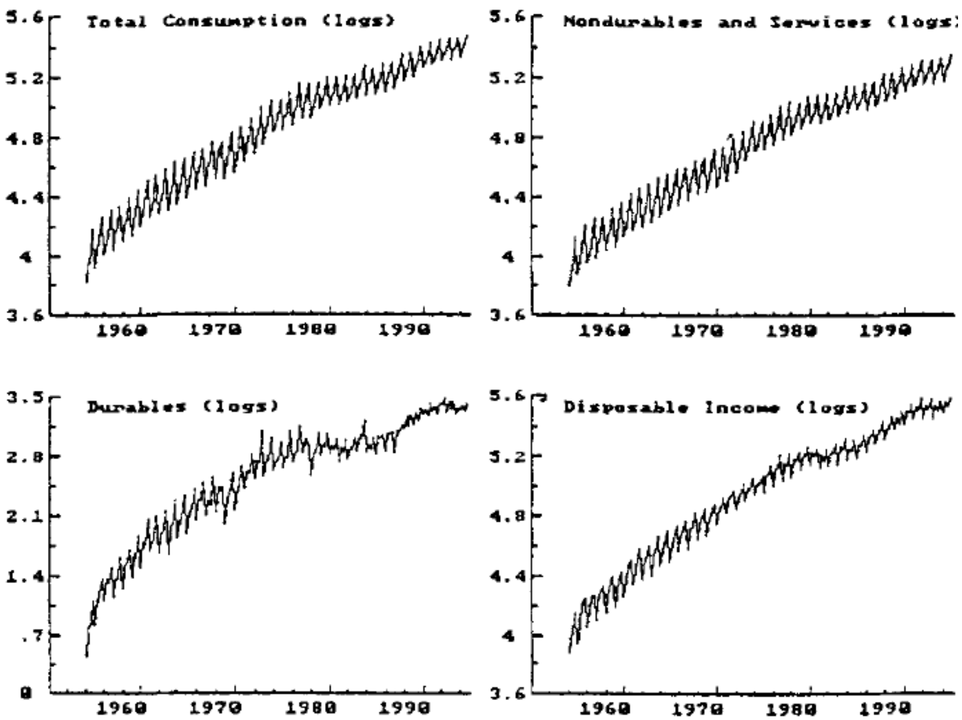
-Total consumption,

-Expenditure on nondurables and services,

-Purchases of durables

- Personal disposable income

Graphs of the natural logarithms of the series



- Expenditure on nondurables and services has seasonal pattern.

- Consumption and Income are trending together.

- Consumption and income are strongly seasonal .

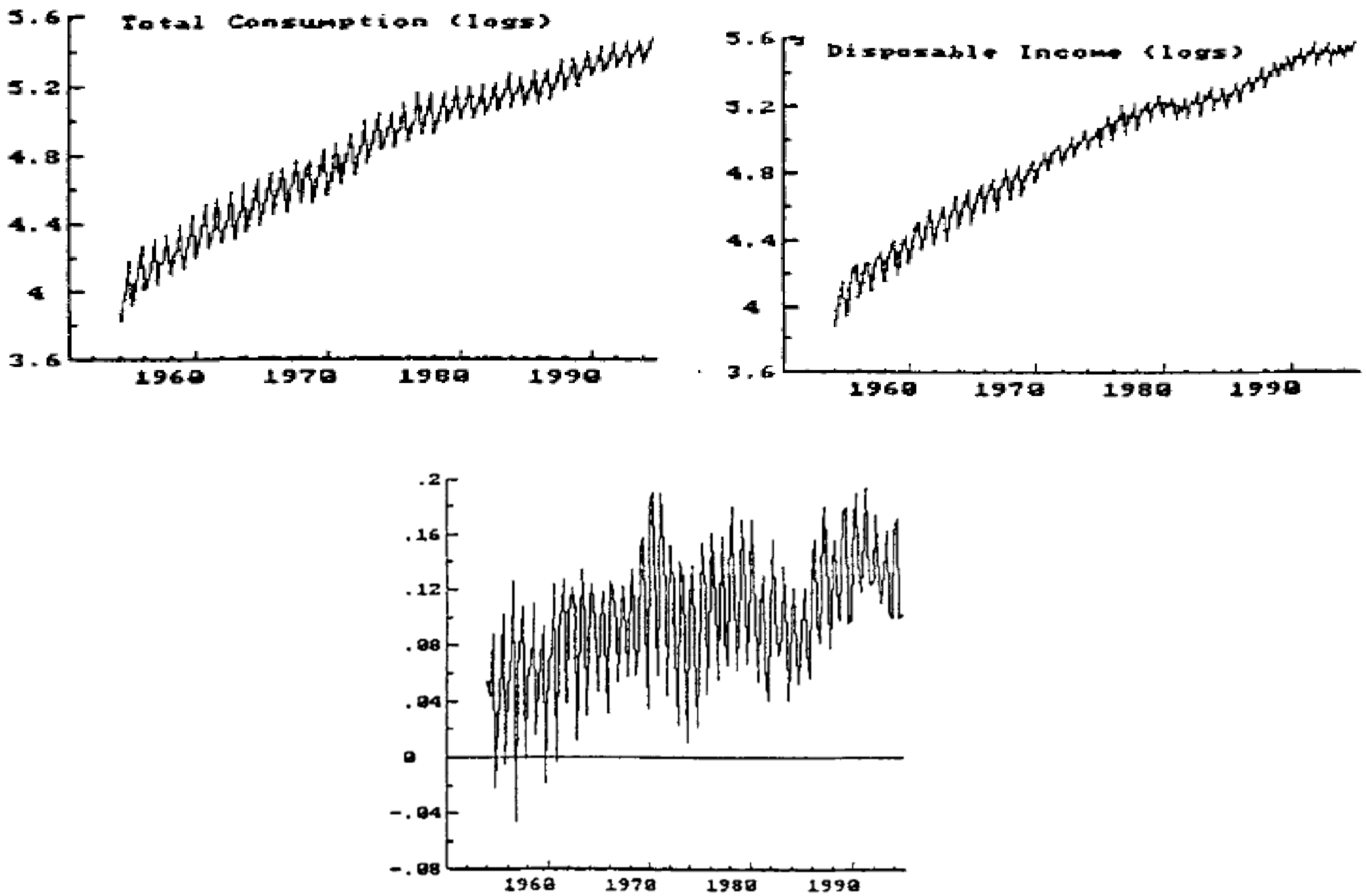


Fig. 2. Saving rates

Although total consumption trends upward during the sample period, it does not do it as strongly as income .This results in a noticeable increase over time in the savings rate.

The estimates for the quarterly seasonal effects

To validate the seasonal fluctuations in the non-trend movement, this regression form has been used.

$$\nabla_1 x_t = \alpha_0 + \alpha_1(Q_{1t} - Q_{4t}) + \alpha_2(Q_{2t} - Q_{4t}) + \alpha_3(Q_{3t} - Q_{4t}) + u_{1t}$$

$$\nabla_1 = (1 - B)$$

- Q_{it} is a zero/one dummy corresponding to quarter i.

Table 1. Percentage seasonal patterns in detrended series

	Quarter							
Variable	1	2	3	4	SEE	R^2	Q(4)	Q(16)
Total Consumption	−22.0	7.2	4.7	10.2	0.031	0.947	148.0	490.5
Nondurables and Services	−21.2	6.1	5.7	9.3	0.027	0.956	142.4	449.3
Durables	−30.5	16.8	−3.6	17.3	0.099	0.798	79.0	261.7
Disposable Income	−15.2	9.4	0.1	5.7	0.032	0.900	178.9	633.2
Savings Rate	6.8	2.2	−4.6	−4.4	0.029	0.736	152.4	472.3

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-The columns (one to forth) report the coefficients of four deterministic seasonal dummies that is the percentage by which each series deviates from its overall mean in each of the four quarters of the year.

-The coefficients approve considerable seasonal fluctuations. And show an extremely large portion of the observed fluctuations in the series is seasonal.

-SEE denotes the standard error of estimate

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-Q(k) is the Box-Ljung test for serial correlation over the first k lags, distributed under the null of no serial correlations as $\chi2(k)$.

-High values of coefficients and high values of Q statistics raise doubts in the validity of these results and a problem with spurious deterministic.

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- Consumption and income experience a large first quarter decline and a substantial fourth quarter increase.

-That the seasonal patterns of total consumption and of expenditure on nondurables and services are almost identical.

-The seasonal pattern of purchases of durables reflects large increases in the second and fourth quarter. (Traditional consumption habits like Christmas).

-The seasonal pattern in the savings rate confirms the above hypothesis that the seasonal movement in consumption and income do not cancel out completely.

Stochastic Seasonality

$$\nabla_1 x_t = \alpha_0 + \alpha_1(Q_{1t} - Q_{4t}) + \alpha_2(Q_{2t} - Q_{4t}) + \alpha_3(Q_{3t} - Q_{4t}) + u_{1t}$$

-This equation which contains no dynamics, only the total long-run seasonal effects can be established. But it is also of interest whether the observed seasonality is stochastic and deterministic.

-Applying a procedure, which is also proposed by Osborn (1990), the writers have found only for the savings rate both forms of seasonality.

-All other variables have no deterministic seasonality
(for the sample period 1954- 1994)

THE ORDER OF INTEGRATION

When the data are seasonal, determining their order of integration is more difficult. OCSB (1998) and HEGY (1990) developed complementary procedures for testing for unit roots at the zero frequency and at the various seasonal frequencies.

In this paper the writers have applied both approaches in order to provide the most complete information.

Definition:

-A quarterly variable x_t is said to be integrated of order (d, D), denoted by *I(d, D)*, if it has a stationary, invertible, nondeterministic ARMA representation after regularly differencing d times and seasonally differencing D times.

Tests of the I(1, 1) hypothesis are derived from the regression

$$\nabla_1\nabla_4 x_t = \alpha_0 + \beta_1\nabla_4 x_{t-1} + \beta_2\nabla_1 x_{t-4} + \sum_{i=1}^{p} \phi_i\nabla_1\nabla_4 x_{t-i} + u_{2t}$$

-Here the do not include deterministic seasonal dummy variables. Because they have assumed, the seasonality of the variables is stochastic, and an attempt to capture it with deterministic variables might give biased results.

- This regression, which contains $\nabla_1 x$ and $\nabla_4 x$ but not the levels of x_t allows to test the null hypothesis I(1, 1) against the I(0, 1) and I(1, 0) alternatives.

-H0: : $\beta_1 = \beta_2 = 0$ (joint F-test)
H1: : $\beta_1 < 0$ or $\beta_2 < 0$

If this test rejected

-Given $\beta_2=0$ that means I(0,1or 0)
H_0: $\beta_1=0$ (one sided t –statistics)

- Given $\beta_1=0$ that means I(1or zero,0)
H_0: $\beta_2=0$

Table 3. OCBS $I(1, 1)$ Seasonal unit root tests

Variable	β_1	β_2	Overall test	p	T	Order of integration
Total Consumption	−5.08**	−2.26	15.46**	8	148	$I(0, 1)$
Nondurables and Services	−3.24**	−1.98	9.85**	8	148	$I(0, 1)$
Durables	−5.10**	−1.61	27.78**	8	148	$I(0, 1)$
Disposable Income	−3.57**	−1.91	12.15**	8	148	$I(0, 1)$
Savings Rate	−7.45**	−1.80	26.89**	8	148	$I(0, 1)$
5% critical value[1]	−2.11	−3.75	3.79			
1% critical value[1]	−2.82	−4.35	4.80			

-Statistics quoted are individual t ratios on β1 and β2, together with an overall 'F' statistic. The table also reports the truncation lag parameter, p, the effective number of observations, T, and the resulting order of integration.

- None of the variables is I(1, 1), neither at the 1 nor at the 5 percent level of significance.

- The I(1, 0) hypothesis is also rejected by the highly significant values of fit for all series.

TESTING FOR SEASONAL COINTEGRATION

Since the outcome of seasonal unit root test does not rule out the possibility of cointegration between the different consumption categories and income, in this part the formal test for seasonal Cointegration has been done.

-The starting point for doing the test for seasonal contegration is the Vector Autoregression:

$$\nabla_4 X_t = \alpha_0 + \delta \cdot t + \Pi_1 Z_{1,t-1} + \Pi_2 Z_{2,t-1} + \Pi_3 Z_{3,t-2} + \Pi_4 Z_{3,t-1} + \sum_{i=1}^{p} \Phi_i \nabla_4 Z_{i-i} + \varepsilon_t$$

X_t is the vector of observed variables

Z_{it}´s are the vectors equivalent to

$$z_{1t} = (1 + B + B^2 + B^3)\nabla_1 x_t \ ,$$

$$z_{2t} = -(1 - B + B^2 - B^3)\nabla_1 x_t \ ,$$

$$z_{3t} = -(1 - B^2)\nabla_1 x_t \ .$$

-π_i and Φ_i are the matrices of the coefficients to be estimated

-The null hypothesis of testing Cointegration at long run frequency is
H_0 : rank(π_1)< n(the number of variables in X_t) or $\pi_1 = \gamma_1 \alpha`_1$
γ_1 and α_1 are m*r mtrices

-The null hypothesis of testing Cointegration at frequencies (ω=0,1/2,1/4) is
$H_0 : \pi_k = \gamma_k \alpha`_k$ for k=1,2,3
Where the column of $\alpha`_k$ are the Cointegrating vectors.
A test of the hypothesis that there are r cointegrating vectors is based on trace statistics:

$$\eta(r) = -T \sum_{i=r+1}^{n} \ln(1-\lambda_i) \qquad \text{for } r = 0, 1, \ldots, n-1$$

Where
λi are the smallest eigenvalue of $\pi_{k..}$

Before presenting the outcome of tests for seasonal cointegration, Table 6 reviews the results of applying the standard Johansen procedure for testing for cointegration at the zero frequency to both seasonally adjusted data and seasonally unadjusted data.

-The results seem to indicate that total consumption as well as its major components and disposable income are cointegrated. That implies a long-run equilibrium relationship between these variables.

Table 6. Testing for cointegration at the long-run frequency

Model	Seasonally Adjusted Data[1] Trace Statistic[3]				Seasonally Unadjusted Data[2] Trace Statistic[3]			
	$r = 0$	$r \leq 1$	p	T	$r = 0$	$r \leq 1$	p	T
Total Consumption = f(Disp Inc)	26.51*	9.01	5	152	48.87**	5.76	4	160
Nondurables and Services = f(Disp Inc)	29.72*	8.46	5	152	51.92**	4.98	4	160
Durables = f(Disp Inc)	28.85*	11.86	5	152	30.48*	8.52	4	160

-The results for raw and adjusted series differ in table7 shows,Cointegration at the long-run frequency is found only between purchases of durables and disposable income (contradiction with table 6)

-The gradual change of behavior from a post-war to a typical post-industrial economy, which has taken place during our sample period, could be a possible explanation for Non-cointegration of consumption and income.

Table 7. Testing for seasonal cointegration

Model	Trace Statistic[1]						p	T
	$r = 0$			$r \leq 1$				
	$\omega = 0$	$\omega = \frac{1}{2}$	$\omega = \frac{1}{4}$	$\omega = 0$	$\omega = \frac{1}{2}$	$\omega = \frac{1}{4}$		
Total Consumption = f(Disp Inc)	10.87	17.51**	14.44*	1.72	1.68	2.99	4	160
Nondurables and Services = f(Disp Inc)	10.84	19.22**	13.61*	1.52	1.49	2.66	4	160
Durables = f(Disp Inc)	29.26*	14.35*	13.16*	3.52	1.51	2.30	4	160

-At the seasonal frequencies all kinds of consumption categories have cointegration with income.

Table 7. Testing for seasonal cointegration

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Conclusion

Model	Seasonally Adjusted Data[1] Trace Statistic[3] $r = 0$	$r \leq 1$	p	T
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The inconsistencies between the results in the Tables (6) and (7) have a serious Implication:

-Conclusions about the long-run relationship between consumption and income are obviously sensitive to whether the data have been seasonally adjusted or not.

-Seasonal adjustment seems to have a distorting effect on the outcome of Johansen-type tests, in general, seasonally adjusted data will have a non-invertible ARIMA representation, while standard tests of integration and cointegration assume this representation to be invertible. Failure to fulfill this assumption might damage the reliability of the obtained results.

THANK YOU