# Seasonality in German GDP: Testing for Stationarity and Non-Stationarity

## Term Paper

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**PhD-Course: Econometrics of Seasonality**

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# Contents

# 1 Introduction

This paper investigates the econometric idea of seasonality for typical time series data, namely the German GDP. Most statistical offices provide only seasonal adjusted data, however, the Federal Bureau of Statistics Germany (Statistisches Bundesamt) also provides the original (non-adjusted) data. I use these data to conduct several stationarity and non-stationarity tests. When introducing and testing different model classes I abstract from certain difficulties. In particular, I avoid complicated multi-variate models, but rather try to explain the data generating process by an uni-variate approach.

The two major questions I am concerned with are: a) Is there an overall (zero frequency) unit root in the time series of GDP data, b) Is there stationarity in the seasonal data, or c) Is the time series seasonally integrated? To answer these questions several tests are used, which are the (augmented) Dickey-Fuller test, the Phillips-Perron test, the DF-GLS test, the KPSS test, and the Hylleberg-Engle-Granger-Yoo test.

The remainder is organized as follows. Section 2 describes the German GDP data. In section 3, the relevant model classes are presented. Then, different tests are carried out in section 4 and finally, section 5 concludes.

# 2 Data

I make use of quarterly GDP data from Germany, taken from the Federal Bureau of Statistics Germany (Statistisches Bundesamt). I cover the time from the first quarter of 1991, right after the German re-unification, up to the first quarter of 2012. This leaves me with 85 observations. The currency of consideration is the Euro (values of Deutsche Mark before 1999, are converted into Euro, according to a fixed exchange rate). All numbers are expressed in terms of one billion Euros.

Figure 1 illustrates German GDP data without price adjustment, that is, no deflation index is used. Second, There are three different lines. The yellow line captures the original GDP data, without seasonal adjustments. The red and blue line display seasonally adjusted GDP data, following two different adjustment processes (Census X-12-ARIMA and BV4.1, respectively). The first thing to note is that there is a positive long-term trend for all lines. Second, the original series is strongly fluctuating in the short-term, whereas the adjustedserieses are relatively smooth. Moreover, the adjusted processes look very similar. Hence, choosing among the different seasonal adjustment processes does not make a big difference — at least, for the considered data and time frame. The short-term fluctuations seem to follow a seasonal pattern, since ups and downs happen on a regular basis. A closer look at the data reveals that there is always a sharp drop from the last quarter of one year to the first quarter of the successive year.

Figure 1: Time Series of Nominal GDP

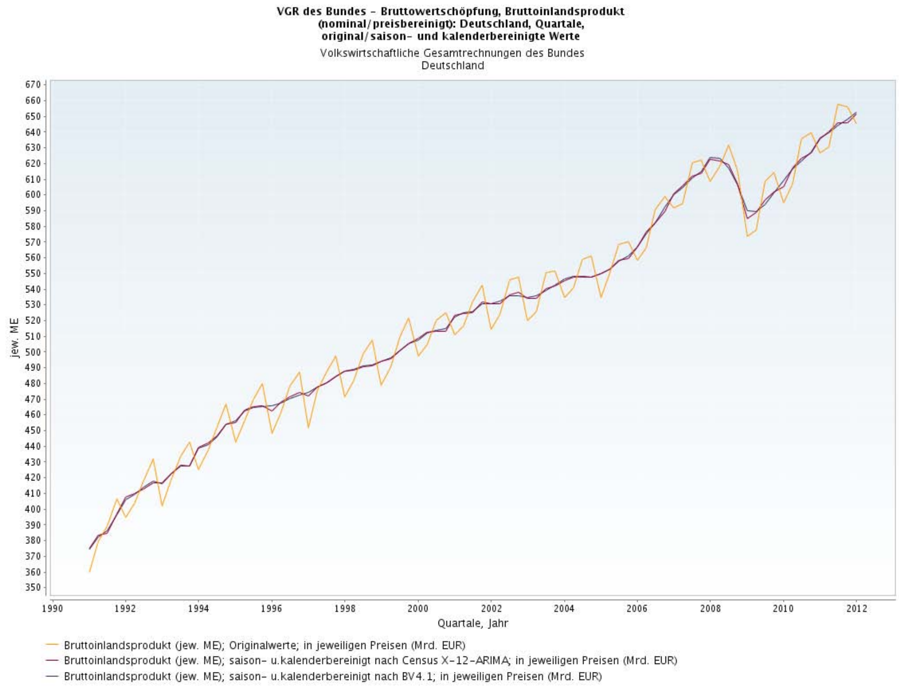
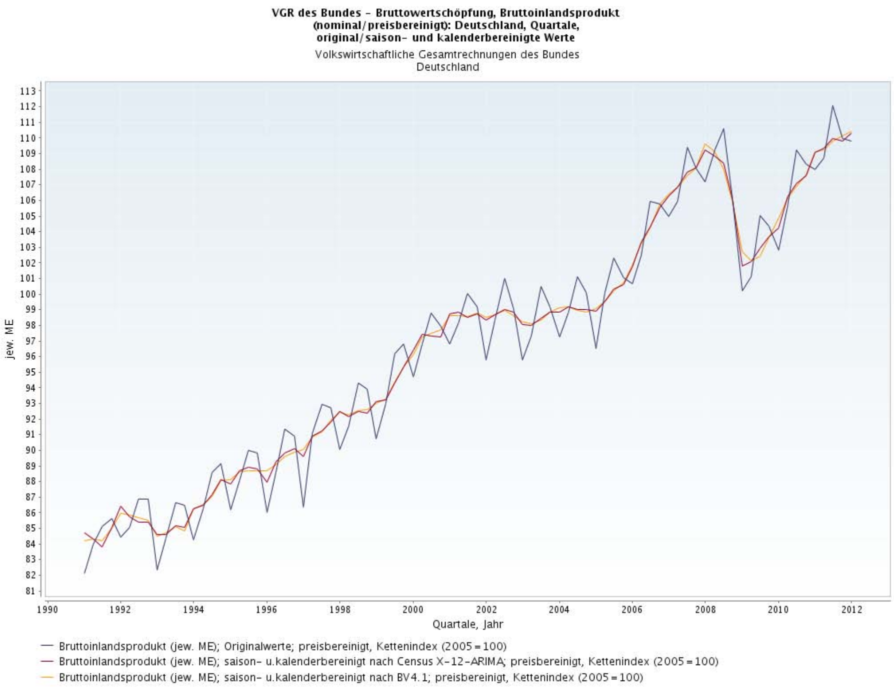



Figure 2 displays the same dataset, but is price-adjusted. To normalize, the average GDP from 2005 is set equal to 100. The overall pattern is the same. One could argue, though, that the slumps (both seasonal and aggregate) look a little more pronounced than the upswings after price-adjustment. This effect is, however, not remarkable and the data from figure 1 and figure 2 can, therefore, be used interchangeably. [1]While the trend is obviously positive, there is an enormous negative spike in 2008. Clearly, this reflects the the impact of the world-wide financial crisis; with a long-lasting effect on the economy. It took about three years, until GDP recovered to its before-crisis level. While it is interesting to investigate trending behavior and depicting real-world events in the graphical representation of the data, the main focus of this paper lies on seasonality.

Figure 2: Time Series of Real GDP



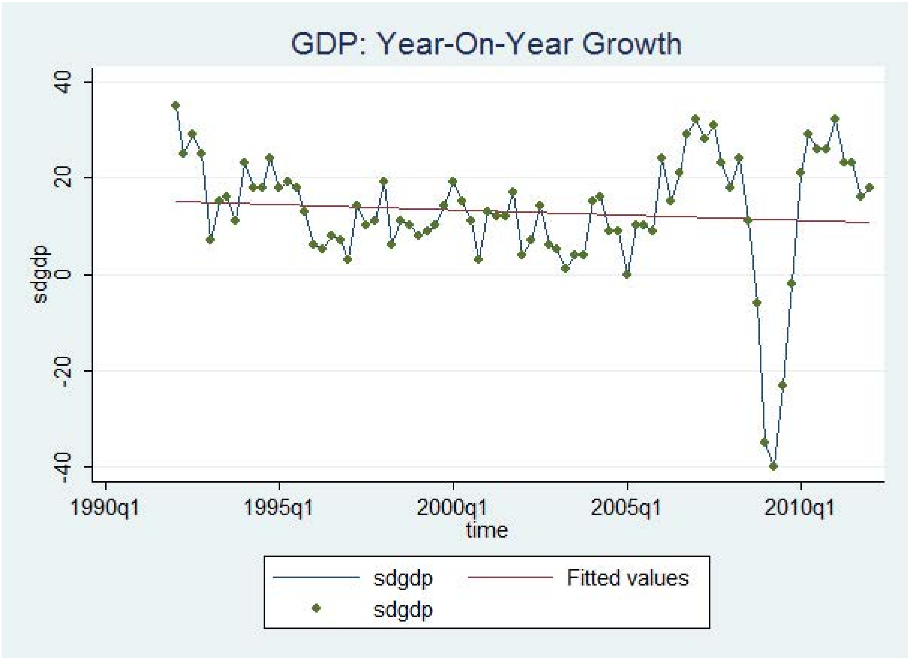

## 2.1 Data Transformation and Visual Inspection

Figure 3 presents the growth rates of GDP from one year to the subsequent year. A first observation is that the yearly growth rates tend to be positive, except for the

[1]The nominal series is in fact smoother than the real series, which is interesting, since several economists often assume that people care about real data and develop their models accordingly. There is, however, also the opposite view that suggests that people rather care about nominal aspects, because they are short-sighted, do not fully understand, or are cognitively incapable of analyzing money devaluation (through inflation).

financial crises period. Second, on a year by year basis, one cannot find a regular pattern.

Figure 3: Year-on-Year Growth Rates of GDP




In contrast, figure 4, illustrates the quarter-by-quarter growth rates, which display a very regular pattern. Ups and downs can be attributed to different quarters within one year. First, the growth rates from one quarter to the subsequent quarter are, on average, positive (red line). Second, there is negative growth from the last quarter (quarter 4) of a year to the first quarter (quarter 1) of the subsequent year. Third, there is a recovery process throughout the rest of the year (quarter 2 - quarter 4). It is clear that there is seasonality in the data and in the remainder, I investigate it in more detail.

Figure 5 shows that seasonal trends do not diverge (overall trend) and there are only a few crossings. Clearly, quarter 4 displays the highest GDP. Note that the winter-GDP is higher than the GDP of quarter 1 and 2 of the subsequent year. Thus, full recovery is only achieved in quarter 3 of the subsequent year.

Figure 4: Quarter-on-Quarter Growth Rates of GDP

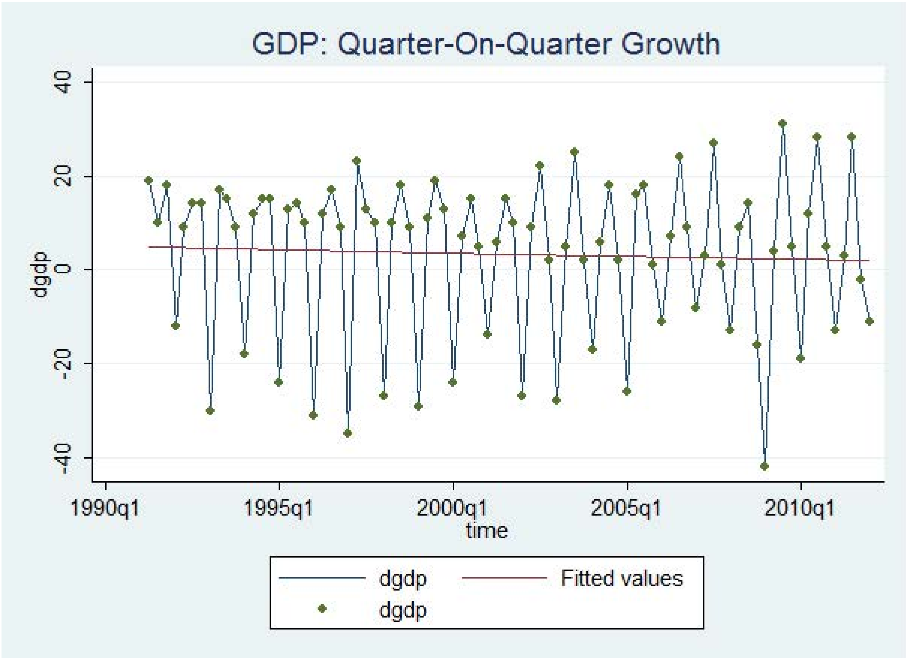
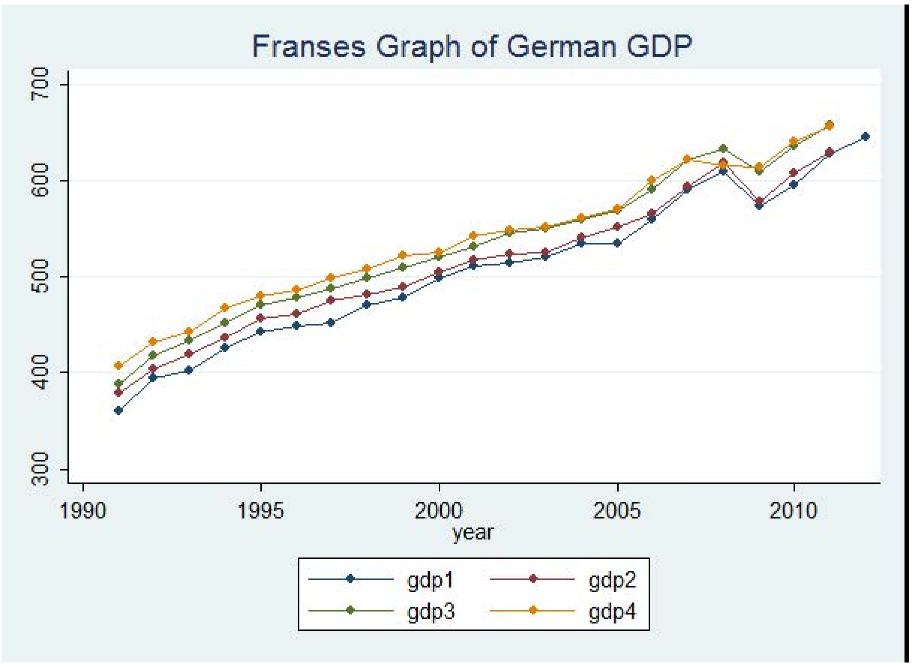



Figure 5: Franses Graph



# 3 Model Classes

In this section I intend to give a short overview. For a self-contained discussion the reader is referred to more advanced texts or the lecture notes *'Econometrics of Seasonality'* by Robert Kunst.

## 3.1 Seasonal Deterministic Models

Seasonal deterministic models are often modeled via the dummy variable representation or its trigonometric counterpart.

The dummy variable representation assumes different means for each season and takes the following form:

$$y_t = \sum_{s=1}^{S} \gamma_s \delta_{st} + z_t \tag{1}$$

S denotes the number of seasons, s the season of consideration (in this work quarterly data, thus S=4), $\delta_{st}$ represents the dummy, $\gamma_s$ is the seasonal mean, and $z_t$ denotes the zero-mean error term. The unconditional mean $\mu$ can be computed by taking the average of the seasonal means. Hence, $m_s = \gamma_s - \mu$ measures the seasonal deviation from the unconditional mean.

Sometimes is is more convenient and insightful to use the trigonometric representation of equation 1. In this way, one can better see the aggregate mean, the yearly cycle, the semi-annual cycle, and which of those are dominating.

## 3.2 Seasonal Stochastic Models

### 3.2.1 Stationary Models

A very simple form is the linear stationary seasonal model, given by the following expression:

$$y_t = \phi_S y_{t-S} + \epsilon_t, \quad |\phi_S| < 1 \tag{2}$$

### 3.2.2 Non-Stationary Models

An important model class is the seasonal unit-root non-stationary model. This is simply a special case of (2), in which $\phi_S$ is set equal to 1.

$$y_t = y_{t-S} + \epsilon_t \tag{3}$$

The seasonal random walk consists of S independent random walks. Since the variance is increasing the process is not stationary.

# 4 Tests

## 4.1 Tests for Non-Seasonal Unit Root

At first, I test whether there is a unit root in the non-seasonal, that is, yearly data. This involves the standard test developed by Dickey and Fuller, with the three standard cases: a) no constant/no trend, b) constant/no trend, and c) constant/trend. In practice the augmented Dickey-Fuller test is often applied such that the imposed assumptions are not violated. This augmented version includes lagged values of the dependent variable in order to whiten the noise term. Then, I check whether the same results are obtained when applying the Phillips-Perron test. Additionally, I use the more advanced method of a DF-GLS test. All these tests have an unit root null hypothesis. In contrast, a KPSS test has a null of trend-stationarity. Finally, I run the KPSS test.

## 4.2 Tests for Seasonal Unit Roots or Seasonal Integration

In addition to testing non-seasonal patterns, it is interesting to investigate the seasonal structure of the data. There are several seasonal stationarity tests (the null hypothesis is stationarity), such as, the non-parametric Canova-Hansen test, the parametric Caner test, and the Tam-Reinsel test. Moreover, there are several tests that directly test for non-stationarity. I focus on the latter group of tests. One candidate is the Dickey-Hasza-Fuller (DHF) test, which is a seasonal extension of the original DF test. It has the drawback that it either rejects or not rejects the null of non-stationarity. In contrast, the Hylleberg-Engle-Granger-Yoo (HEGY) test allows to test for the appearance of specific unit roots.

## 4.3 Conducting the Tests

### 4.3.1 (Augmented) Dickey-Fuller

When investigating the DF-test without constant and without trend (and several different lag length), there seems to be a unit root. We fail to reject the null, because 1.599 is larger than the critical values, as seen in table 1.

Table 1: Dickey Fuller Case A (No Constant/No Trend)

| | Test Statistic | 1% Critical Value | 5% Critical 10% Critical Value Value |
|---|---|---|---|
| Z(t) | 1.599 | -2.606 | -1.950 -1.610 |
| D.gdp | Coef. | Std. Err. t | P>t [95% Conf. Interval] |
| gdp<br>L1. | .0056329 | .0035218 1.60 | 0.114 -.0013718 .0126376 |

A similar picture is obtained for different lag length and also when including a constant. Now, looking at table 2, which performs a DF-test when a trend is included, delivers an entirely different conclusion. Since the test statistic -4.514 is smaller than the critical values (on all significance levels; 1, 5, and 10 percent) we reject the null, and thus infer stationarity. Including a trend seems reasonable, as we have seen in the visual inspection section. This time, I reported the table when including four lag terms. Again, the qualitative results remain unchanged for different lag length.

Table 2: Dickey Fuller Case C (With Trend)

| | Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|---|---|---|---|---|
| Z(t) | -4.514 | -4.084 | -3.470 | -3.162 |

MacKinnon approximate p-value for Z(t) = 0.0014

| D.gdp | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| gdp | | | | | | |
| L1. | -.4227586 | .0936614 | -4.51 | 0.000 | -.6094255 | -.2360916 |
| LD. | .1536495 | .1040528 | 1.48 | 0.144 | -.0537274 | .3610264 |
| L2D. | -.0273002 | .0898536 | -0.30 | 0.762 | -.2063783 | .1517778 |
| L3D. | -.0551457 | .0801727 | -0.69 | 0.494 | -.2149296 | .1046382 |
| L4D. | .7092731 | .0742346 | 9.55 | 0.000 | .5613239 | .8572224 |
| _trend | 1.224826 | .2716355 | 4.51 | 0.000 | .683457 | 1.766194 |
| _cons | 169.124 | 36.97373 | 4.57 | 0.000 | 95.43551 | 242.8126 |

#### 4.3.2 Phillips-Perron

The PP test is an alternative test underlying the same null hypothesis. The result is very much in line with what we obtained from the DF-test. Once, I control for a trend, and investigate different lag length the test rejects the null and hints at a stationary process.

#### 4.3.3 DF-GLS

The Stata dfgls routine includes a trend by default. Running the test delivers results that are in contrast with earlier findings. For lag length from 2 to 11 (maximum lag length is automatically determined by Stata) the DF-GLS test fails to reject the null hypothesis. Therefore, it hints at a unit root.

#### 4.3.4 KPSS

Table 3 presents the KPSS test.

Table 3: KPSS Test

| 10% | 5% | 2.5% | 1% |
|---|---|---|---|
| 0.119 | 0.146 | 0.176 | 0.216 |
| Lag order | Test statistic | | |
| 0 | .166 | | |
| 1 | .122 | | |
| 2 | .12 | | |
| 3 | .113 | | |

For lag length greater than zero, I fail to reject the null hypothesis of trend stationarity. This is because the test statistic is smaller than the critical values at the 5 percent significance level. Thus, the KPSS test is in accordance with the PP and DF test. Only the DF-GLS test suggests that there is a unit root.

#### 4.3.5 HEGY

In the following, I apply the HEGY test to the data. The standard formula looks as follows:

$$\Delta_4 y_t = \pi_1 y_{t-1}^{(1)} - \pi_2 y_{t-1}^{(2)} - \pi_3 y_{t-2}^{(3)} - \pi_4 y_{t-1}^{(3)} + \epsilon_t \tag{4}$$

It can be enriched by deterministic parts (constant, trend, seasonal dummies, seasonal trends) and/or lag augmentation (to get white noise errors). I consider four different specifications for lag length from 1 to 5. Stata automatically selects the best lag length by running sequential tests.

- Specification A: Constant + Trend (Table 4)

Table 4: HEGY Test Specification A

| Stat | Stat | 5% critical | 10% critical |
|---|---|---|---|
| t[Pi1] | -4.503 | -3.512 | -3.183 |
| t[Pi2] | -1.527 | -1.926 | -1.586 |
| t[Pi3] | -0.705 | -1.904 | -1.531 |
| t[Pi4] | -0.455 | -1.673 | -1.303 |
| F[3-4] | 0.346 | 2.966 | 2.273 |
| F[2-4] | 0.981 | 2.718 | 2.145 |
| F[1-4] | 5.914 | 4.269 | 3.589 |

Specification A suggests (in accordance with earlier findings) that there is no overall unit root. But, all seasonal unit roots are present. Moreover, an F-test finds $\pi_2 - \pi_4$ jointly insignificant, which corresponds to the null hypothesis that those values should be zero, in case of unit-root behavior.

- Specification B: Constant + Trend + Seasonal Dummies (Table 5)

Specification B only hints at the semi-annual unit root. There is joint significance $\pi_1 - \pi_4$.

- Specification C: Constant + Seasonal Dummies (Table 6)

Specification C suggests that there are both an overall unit root (zero frequency) and a semi-annual one. This should not come as a surprise, as this is the specification without a trend. The non-seasonal unit root tests also suggest unit-root behavior when the trend was excluded. Therefore, this specification should be watched cautiously. Also note that $\pi_1 - \pi_4$ are jointly significant.

Table 5: HEGY Test Specification B

| | Stat | 5% critical | 10% critical |
|---|---|---|---|
| t[Pi1] | -4.387 | -3.606 | -3.283 |
| t[Pi2] | -2.033 | -2.999 | -2.672 |
| t[Pi3] | -5.419 | -3.556 | -3.199 |
| t[Pi4] | -6.453 | -1.927 | -1.497 |
| F[3-4] | 72.660 | 6.579 | 5.457 |
| F[2-4] | 73.787 | 6.032 | 5.130 |
| F[1-4] | 66.673 | 6.495 | 5.693 |

Table 6: HEGY Test Specification C

| | Stat | 5% critical | 10% critical |
|---|---|---|---|
| t[Pi1] | -0.411 | -3.005 | -2.668 |
| t[Pi2] | -2.355 | -2.982 | -2.655 |
| t[Pi3] | -4.758 | -3.512 | -3.182 |
| t[Pi4] | -6.015 | -1.968 | -1.530 |
| F[3-4] | 59.956 | 6.583 | 5.535 |
| F[2-4] | 65.385 | 6.054 | 5.172 |
| F[1-4] | 49.093 | 5.833 | 4.980 |

- Specification D: Constant + Seasonal Dummies + Seasonal Trends (Table 7)

Table 7: HEGY Test Specification D

| | Stat | 5% critical | 10% critical |
|---|---|---|---|
| t[Pi1] | -4.503 | -3.390 | -3.081 |
| t[Pi2] | -1.527 | -3.380 | -3.075 |
| t[Pi3] | -0.705 | -4.183 | -3.869 |
| t[Pi4] | -0.455 | -1.852 | -1.435 |
| F[3-4] | 0.346 | 9.867 | 8.536 |
| F[2-4] | 0.981 | 9.333 | 8.140 |
| F[1-4] | 5.914 | 9.035 | 7.978 |

Specification D does not suggest an overall unit root, but all seasonal unit roots (semi-annual + complex). Furthermore, $\pi_2 - \pi_4$ are jointly insignificant. Note that this is the richest specification with constant, seasonal dummies, and seasonal trends.

Now, on which specification should we rely, for a final statement about the 'true' unit-root behavior? As already pointed out, specification c does not contain a trend and should thus be neglected. The three other specifications contain a trend and reject the overall unit root. It is relatively hard to choose one of those three to be the right one, but there are certain differences that can help. As figure 5 has shown, all seasons seem to follow the same trend. Thus, specification d, with seasonal trends is unlikely, since we cannot observe the trends to drift apart. Finally, figure 5 also shows that the seasonal means are different. Thus, I *prefer specification b* over specification a, since it includes seasonal dummies, in addition to a common trend.

In summary, a seasonal stochachastic non-stationary model, with deterministic components describes the data best. In particular, I find that a semi-annual unit-root describes the non-stationary part. A constant, a trend, plus seasonal dummies give a description of the deterministic part.

An alternative Stata routine is the sroot command, developed by Depalo (2009). It has certain advantages, for instance, better small sample properties. Moreover, it comes along with a greater number of options. In general, however, it gives the same predictions as the HEGY test.

Many econometricians prefer log-transformations to level data. Therefore, I took the logarithm of GDP and repeated all tests. Except for minor differences, the test results are robust to log-transformations.

# 5 Conclusion

The previous sections have shown that a zero frequency unit root is rather unlikely. Since the series is trending, trend-stationarity seems the most likely scenario. As soon as I consider seasonal structures as well, the tests hint at some underlying unit root process. In particular, there is very strong evidence for a semi-annual unit root. All HEGY test specifications support such a semi-annual unit root. There is also some evidence for complex unit roots at +i and -i. These results are robust against log-transformations. The most appropriate model to describe the data is a non-stationary stochastic model, that contains lags and deterministic components.

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