Some Nonlinear Seasonal Models

Chapter 7 from Ghysels and Osborn: The Econometric

Analysis of Seasonal Time Series

Presented by Mag.Syed Zafar Saeed

OUTLINE

1.INTRODUCTION

2.STOCHASTIC SEASONAL UNIT ROOTS

3. SEASONAL ARCH MODELS

4. THE CLASS OF MODELS

5. THE EFFECTS OF FILTERING ON ARCH MODELS

6. THE GENERAL CASES OF LINEAR FILTER

7.SOME CASES OF LINEAR FILTERS

8. THE CASE OF WEAK GARCH (1,1)

Introduction

- Nonlinear models:
  - Provide tools to model nonlinear relationship between variables (e.g. seasons)
  - Examples of nonlinear dynamics: time-changing (seasonal) variance, asymmetric cycles, higher-moment structures

- (Seasonal) Nonlinearity can be primarily found in high-frequency data like intradaily seasonal patterns
  - S&P's composite stock-price index
  - Exchange rates

- Different seasonal models with different types of nonlinearity:
  - Stochastic seasonal unit roots – *varying impact of seasonal shocks*
  - Seasonal (G)ARCH models – *structure of seasonal variance*
  - Periodic GARCH models – *time-varying seasonal coefficients*
  - Periodic Markov switching models – *seasonal mean shifts*

Stochastic Seasonal Unit Root

- Motivation: <u>not</u> all macroeconomic shocks may have the <u>same</u> impact
- Generalization of linear processes by allowing for random parameters
- First-order seasonal random coefficient autoregressive process:

$$(1-\tilde{\phi}_{s\tau}L^{s})\tilde{y}_{s\tau} = \varepsilon_{s\tau} \qquad s = 1,\ldots,S \qquad \tau = 1,2,\ldots,T_{\tau}$$

where $\tilde{\phi}_{s\tau} = 1 + \tilde{\alpha}_{s\tau}$ and $\tilde{\alpha}_{s\tau} = \rho\tilde{\alpha}_{s,\tau-1} + \xi_{s\tau}$

- $0 \le \rho \le 1$, ε and ξ are i.i.d. and normally distributed with σ^2, ω^2
- The randomized seasonal autoregressive process is then:

$$\Delta_S \tilde{y}_{s\tau} = \tilde{\alpha}_{s\tau}\tilde{y}_{s,\tau-1} + \varepsilon_{s\tau}$$

- It is also called a heteroskedastic seasonally integrated process
  - Conditional on its own normally distributed past $N(\rho\tilde{\alpha}_{s,\tau-1}\tilde{y}_{s,\tau-1}, \sigma^2 + \omega^2\tilde{y}^2_{s,\tau-1})$

heteroskedasticity

- If $\omega^2 = 0$, the process is a regular seasonal random walk with homoskedastic innovations
- Hence, the test hypotheses are $H_0 : \omega^2 = 0$

  $H_A : \omega^2 \rangle 0$

- Taylor-Smith test is used for determination of heteroskedastic seasonal integration

$$S_\rho = \frac{1}{\sigma^4}\sum_{s=1}^{S}\sum_{j}^{T_\tau}\rho^{-2j}\left[\left(\sum_{\tau=j}^{T_\tau}\rho^{\tau}y_{s,\tau-1}\Delta_S y_{s\tau}\right)^2 - \sigma^2\sum_{\tau=j}^{T_\tau}\rho^{2\tau}y_{s,\tau-1}\right]$$

  - $y_{s\tau} = \phi(L)\left[\tilde{y}_{s\tau} - \mu_s^* - \beta_s^*(S\tau + s)\right]$
  - ρ is unidentified under the null hypothesis, hence, the two polar cases S_0 and S_1 are computed
  - the limiting distributions for S_0 and S_1 are nonstandard

- However, this process is <u>not a covariance stationary process</u>

Seasonal (G)ARCH Models

- Application: - financial time series
  - stock-market dividend yields
  - Foreign-exchange volatility
  - Lead-lag relations between two or more simultaneously traded markets

  - volatility of few macroeconomic time series
- The GARCH(p,q) process:

$$X_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \omega + \phi(L)\varepsilon_{t-1}^2 + \theta(L)\sigma_{t-1}^2$$

– if we define $v_t = \varepsilon_t^2 - \sigma_t^2$, one obtains:

$$\Leftrightarrow \varepsilon_t^2 = \omega + [\phi(L) + \theta(L)]\varepsilon_{t-1}^2 - \theta(L)v_{t-1} + v_t$$

ARMA[max(p,q),q] model

- Analogously, the seasonal GARCH(p,q) process:

$$\varepsilon_t^2 = \omega + [\phi(L) + \theta(L)]\varepsilon_{t-s}^2 - \theta(L)v_{t-s} + v_t$$

- where: $\phi(L)=\sum_{j=1}^{p} b_j L^j$ and $\theta(L)=\sum_{j=1}^{q} a_j L^j$
- If $\sum_{j=1}^{p} b_j + \sum_{j=1}^{q} a_j \langle 1$ then this GARCH(p,q) process has a unique strictly and <u>covariance stationary solution</u>
  - For a GARCH(1,1) process being strictly stationary it is enough to fulfill the following condition: $a_1 + b_1 = 1$
- Maximum-Likelihood-based estimation of coefficients
- The effects of Filtering on ARCH models
  - Seasonal filtering may lead to bias in the autocorrelation function
  - In order to present these biases one may define the weak GARCH(p,q) process, linear filters, and derive the (filtered) autocovariance and autocorrelation functions
  - Then, one can derive conditions to have unbiased autocorrelation function

- The **weak GARCH(p,q) process**:

$$\varepsilon_t^2 = \omega + \sum_{j=1}^{\max(p,q)} (\phi_j + \theta_j)\varepsilon_{t-j}^2 - \sum_{j=-1}^{q} \theta_j v_{t-j} + v_t$$

  - where $\sigma_t^2 = E_{Lt}(\varepsilon_{t+1}^2)$ is the conditional variance with $\varepsilon_{t+1}^2 = \sigma_t^2 + v_{t+1}$ and $E_{Lt}(.)$ is the linear projection on the space spanned by 1, $(\varepsilon_{t-j}, \varepsilon_{t-j}^2) : j \geq 0$
  - v_{t+1} is a Martingale difference sequence with respect to the linear span filtration
- Suppose the following **linear filter** that filters the nonseasonal (ns) components in the data: $z_t^{ns} = z_t^F = v(L)z_t = \sum_{k=-\infty}^{+\infty} v_k L^k z_t$
  - z is a variable with seasonal (s) and nonseasonal components: $z = z^S + z^{NS}$
  - L is the lag operator
- **Autocovariance function**:
  - $\gamma_2(j) = E_L(\varepsilon_t^2 \varepsilon_{t-j}^2)$
  - Applying the linear filter to the residuals one obtains the **filtered autocovariance function**:

$$\gamma_2^F(j) = E_L(\varepsilon_t^F)^2(\varepsilon_{t-j}^F)^2 = E_L(v(L)\varepsilon_t)^2(v(L)\varepsilon_{t-j})^2$$

- In case of weak GARCH(1,1) one can derive the following autocovariance (γ) and **autocorrelation** (ρ) **functions**:

$$\gamma_2(0) = \frac{\left[1+\theta^2 - 2(\phi+\theta)\theta\right]}{\left[1-(\phi+\theta)^2\right]}\sigma_v^2$$

$$\gamma_2(j) = \frac{\phi\left[1-(\phi+\theta)\theta\right]}{\left[1-(\phi+\theta)^2\right]}(\phi+\theta)^{j-1}\sigma_v^2$$

$$\rho_2(j) = \frac{\phi\left[1-\phi\theta-\theta^2\right]}{\left[1-2\phi\theta-\theta^2\right]}(\phi+\theta)^{j-1}$$

- Let define $\lambda \equiv \phi+\theta$ and apply the linear filter v(L) ≡ (1-L^S) to this process, then one obtains the **filtered autocorrelation function**:

$$\rho_2^F(j) \equiv \frac{2+\lambda+\lambda^S}{2+6\lambda\rho_2(S)}\rho_2(j)$$

- As a final step the condition on parameters ϕ and θ is obtained to have an unbiased autocorrelation function

- If the parameters φ and θ solve the following equation:

$$2\lambda^{S}\theta-(1+\theta^{2}-6\phi\theta)\lambda^{S-1}+6\phi\lambda^{S-2}+2\lambda\theta-(1+\theta^{2})=0$$

  the autocorrelation function is **unbiased**

- If the following inequality holds:

$$2\lambda^{S}\theta-(1+\theta^{2}-6\phi\theta)\lambda^{S-1}+6\phi\lambda^{S-2}+2\lambda\theta-(1+\theta^{2})>0$$

  – the autocorrelation function is **upward biased**;

$$2\lambda^{S}\theta-(1+\theta^{2}-6\phi\theta)\lambda^{S-1}+6\phi\lambda^{S-2}+2\lambda\theta-(1+\theta^{2})<0$$

  – the autocorrelation function is **downward biased**