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Seminar Paper Regarding the PhD-Course „Vector Autoregressions":

„Univariate and Multivariate Models to Predict Liechtenstein's GDP "

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1. Introduction

So far, data for Liechtenstein's GDP only existed for the years 1998-2009. This fact has made analysis and prediction of business cycles an even harder task than it already is. Newly calculated GDP-data for the years 1972-1997 (as part of the dissertation of the author), which are comparable and chainable with the official data, finally enable a meaningful application of popular univariate and multivariate time serial methods, such as ARIMA, ARIMAX, "ordinary multiple", VAR, VEC and combined models.

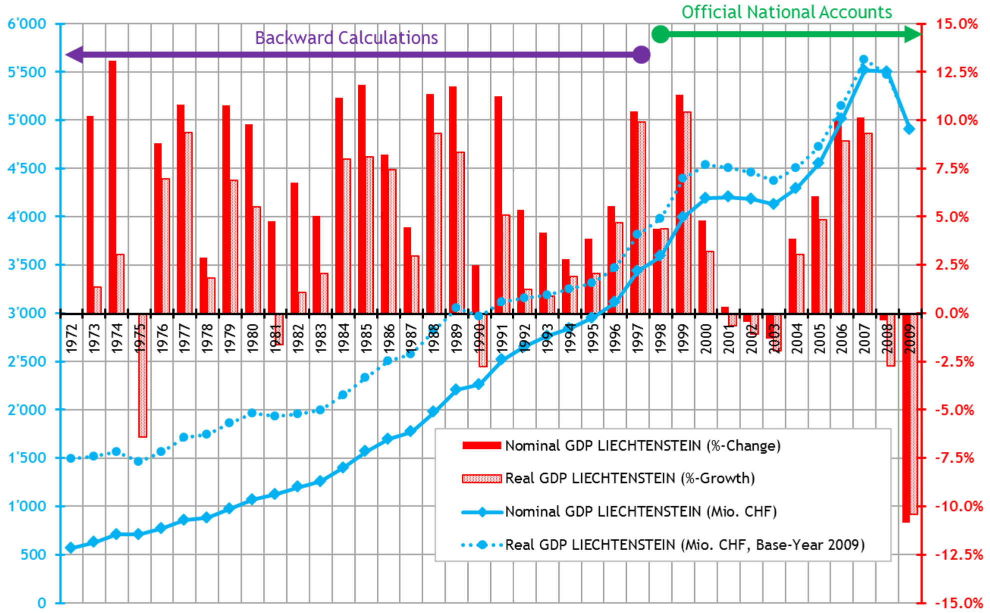
As a first step, different models for the existing sample 1972-2009 are chosen, estimated, evaluated, and compared. Then, a first "prediction" for the year 2010 is made. It turns out, that the Vector Autoregressive model with three lags containing GDP, Liechtenstein's exports, and sales of Liechtenstein's big industrial companies (all in nominal values) features the smallest prediction error among the eight considered model-classes. All the applied models suggest a further decrease of Liechtenstein's nominal GDP in the year 2010. As new GDP-data for the years 2010 and 2011 is going to be available in February, these models will be rerun, evaluated again and used for prediction of GDP 2012.

This text is the written form of the presentation held by the author, which took place on January 24th (2012) at the University of Vienna (Department of Economics) as part of the doctoral course "Vector Autoregressions" hosted by Prof. Robert Kunst.
After this introduction, the second section deals with the descriptive and time serial analysis of the data. The third section will deal with the different considered regression models and their evaluation. In the fourth section, the text will be completed by some concluding remarks.

2. Data Analysis

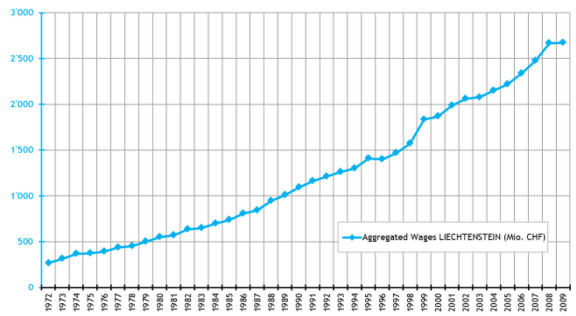
The series that shall be predicted within this empirical project is the nominal GDP of Liechtenstein. There are several reasons, why the nominal GDP is used instead of the real GDP: Along with the official National Accounts, only nominal values are published, since no official price indices exist for Liechtenstein. Also, all the considered indicator series are measured in nominal terms (Swiss Francs), too.

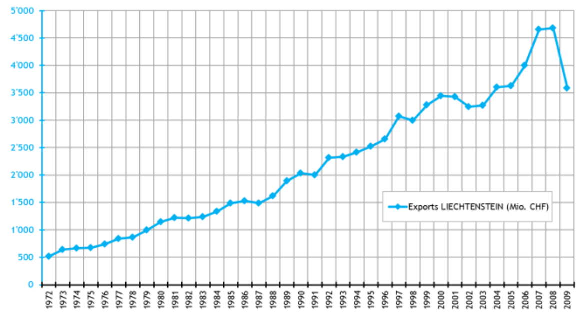
At first, attention should be paid to the dependent series being forecasted: The nominal GDP of Liechtenstein is plotted in the following figure[1]. Several internationally identified recessions are observable, such as the first and the second oil crises, the low economic performance in the first half of the 90s and the two recent recessions. Also evident is the very high range of the growth rates from more than plus 12% to minus 10%.

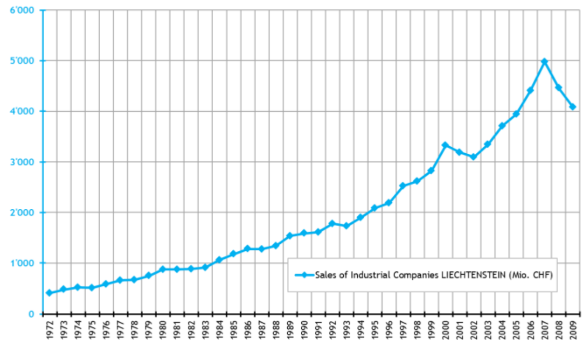


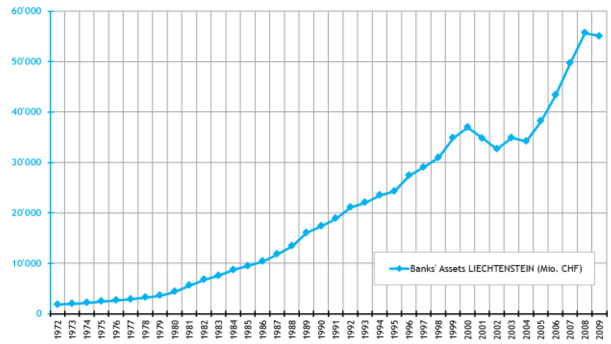
The five chosen indicator series have been selected from about 30 series according to their correlation to Liechtenstein's GDP. The series are obtained from the annual Statistical Yearbook by the National Office of Statistics (AMT FÜR STATISTIK [2011]), except for Swiss GDP which is published by Swiss Statistics (http://www.bfs.admin.ch). Some of the series have been corrected by the author for structural changes or outliers. Also, some data gaps have been filled using other archived data sources. The used indicator series are shown on the following five graphs:

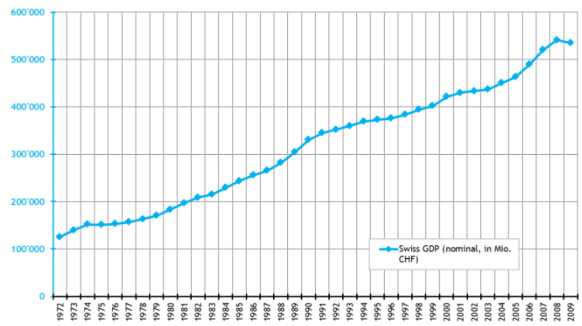
[1] For the calculation of the real GDP, the Swiss Index of Consumer Prices (LIK) has been taken. This approach is supportable as there exists a custom treaty between Switzerland and Liechtenstein since 1923 and both countries use Swiss Francs as their official currency.











The first four displayed series feature a strong growth, while they share a similar business cycle pattern. In nominal terms, the GDP of Liechtenstein is about ten times greater than in 1972, while Swiss GDP (plotted in the fifth graph) has risen about 500% during the same time period. Furthermore, Liechtenstein's GDP can be seen as a leading indicator for the GDP of Switzerland: For both the real/nominal growth rates and the real/nominal business cycle (derived by the filter after HODRICK AND PRESCOTT [1997]), it can be shown that Liechtenstein's GDP is Granger-causal to Swiss GDP.

For the six considered series, unit root tests have been conducted. An augmented Dickey-Fuller-Test (DICKEY AND FULLER [1979]) with one-sided p-values after MACKINNON [1996] was applied using the test strategy proposed by ELDER AND KENNEDY [2001] and the information criterion of AKAIKE [1974] for the determination of the lag order in the estimation setting shown below[2]:

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[2] For the justification of the chosen approach also see NEUSSER [2006, p. 112-115].

$$X_t = \alpha + \delta \cdot t + \phi \cdot X_{t-1} + \gamma_1 \cdot \Delta X_{t-1} + \ldots + \gamma_p \cdot \Delta X_{t-p} + \varepsilon_t$$

The statistical package Eviews seems to estimate an alternative form to the (in the eyes of the author) more intuitive form above. The estimated test equation below can be derived by subtracting $X_{t\text{-}1}$ from both sides of the equation before:

$$\Delta X_t = \alpha + \delta \cdot t + \underbrace{(\phi - 1)}_{\beta} \cdot X_{t-1} + \gamma_1 \cdot \Delta X_{t-1} + \ldots + \gamma_p \cdot \Delta X_{t-p} + \varepsilon_t$$

$$\text{H}_0\text{: } \phi = 1 \text{ (tested via H}_0\text{: } \beta = 0\text{)}$$

The next table shows the results of the conducted unit root tests. The null-hypothesis of the existence of a unit root cannot be rejected for the level of all the series, but for the annual differences of all the series.

ADF-Tests	Constant α	Trend t	Lags (AIC)	β (t-Value / *p-Value*)	**I(d)**
GDP	Yes	Yes	4	-2.8355/*0.1955*	**I(1)**
ΔGDP	Yes	Yes	3	-4.9479/*0.0018*	
WAGES	Yes	Yes	0	-1.1233/*0.9157*	**I(1)**
ΔWAGES	Yes	Yes	4	-5.3906/*0.0003*	
EXPORTS	Yes	Yes	7	-1.9759/*0.5904*	**I(1)**
ΔEXPORTS	Yes	Yes	4	-4.7718/*0.0030*	
SALES	Yes	Yes	4	1.1356/*0.9999*	**I(1)**
ΔSALES	Yes	Yes	4	-5.9336/*0.0000*	
ASSETS	Yes	Yes	4	-0.4822/*0.9814*	**I(1)**
ΔASSETS	Yes	No	3	-4.4154/*0.0008*	
GDPCH	Yes	Yes	3	-2.6350/*0.2672*	**I(1)**
ΔGDPCH	Yes	Yes	4	-4.0031/*0.0148*	

The unit root tests show that all the used series are integrated of order one. These findings are also supported by other unit root tests such as the tests after PHILLIPS AND PERRON [1988] and KWIATKOWSKI ET AL. [1992]. In order to avoid the danger of spurious regression (see GRANGER AND NEWBOLD [1974]), variables are being differenced.

To check whether the five used indicator series really contain some predictive information to forecast Liechtenstein's GDP, their leading characteristics are investigated. Doing so, causality tests after GRANGER [1969] are carried out. In a first step, the indicator series are tested (each pairwise with the GDP-series) in a univariate setting to explore if the prediction of the GDP can be improved by the inclusion an indicator series compared to the prediction only applying the dynamics of the GDP's own past (only lagged variables of GDP itself). So, it's basically an F-Test for all the $\beta_1, \beta_2, \ldots, \beta_l$ in the following equation:

$$\Delta GDP_t = \alpha_0 + \alpha_1 \cdot \Delta GDP_{t-1} + \cdots + \alpha_l \cdot \Delta GDP_{t-l} + \beta_1 \cdot \Delta INDICATOR_{t-1} + \cdots + \beta_l \cdot \Delta INDICATOR_{t-l} + \varepsilon_t$$

The causality test is also conducted in a multivariate framework, where it has been tested whether all the Phis $\boldsymbol{\Phi}_{12}$, $\boldsymbol{\Phi}_{13}$, $\boldsymbol{\Phi}_{14}$, $\boldsymbol{\Phi}_{15}$ and $\boldsymbol{\Phi}_{16}$ are significant from lag-order one up to order four:

$$\begin{bmatrix} d(GDP) \\ d(WAGES) \\ d(EXPORTS) \\ d(SALES) \\ d(ASSETS) \\ d(GDPCH) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{bmatrix} + \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) & \Phi_{13}(L) & \Phi_{14}(L) & \Phi_{15}(L) & \Phi_{16}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) & \Phi_{23}(L) & \Phi_{24}(L) & \Phi_{25}(L) & \Phi_{26}(L) \\ \Phi_{31}(L) & \Phi_{32}(L) & \Phi_{33}(L) & \Phi_{34}(L) & \Phi_{35}(L) & \Phi_{36}(L) \\ \Phi_{41}(L) & \Phi_{61}(L) & \Phi_{43}(L) & \Phi_{44}(L) & \Phi_{45}(L) & \Phi_{46}(L) \\ \Phi_{51}(L) & \Phi_{52}(L) & \Phi_{53}(L) & \Phi_{54}(L) & \Phi_{55}(L) & \Phi_{56}(L) \\ \Phi_{61}(L) & \Phi_{62}(L) & \Phi_{63}(L) & \Phi_{64}(L) & \Phi_{65}(L) & \Phi_{66}(L) \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \\ u_{6t} \end{bmatrix}$$

The results of both the univariate and the multivariate Granger-tests and the correlation of the absolute annual changes of the variables are shown below:

	Correlation with d(GDP)	Granger-Causality to d(GDP): P-Value of F-test up to Order *l*				Multivariate Granger-Causality (Lag-Order *l*)
		l = 1	*l* = 2	*l* = 3	*l* = 4	
d(WAGES)	0.6074	0.0068	0.1536	0.9359	0.8480	0.0039 (*l* = 1)
d(EXPORTS)	0.7473	0.1131	0.2812	0.0953	0.0308	0.0124 (*l* = 4)
d(SALES)	0.6844	0.0141	0.1107	0.6888	0.7370	0.0096 (*l* = 1)
d(ASSETS)	0.4747	0.0690	0.1563	0.3721	0.2582	0.0601 (*l* = 1)
d(GDPCH)	0.7518	0.0866	0.1788	0.8315	0.9472	0.0774 (*l* = 1)

Considering the results of the causality tests led to the conclusion to include the variables exports[3], industrial sales and wages[4] into the prediction models. The reduction of the number of indicator series makes also sense due to the fact that smaller models might be preferred because of the small number of observations (38 observations in levels, 37 in differences).

[3] Interestingly, exports are granger-causal to GDP when the applied lag-order is four.

[4] Wages have been skipped from the equations later on to save degrees of freedom, since they do not really contribute much to the predictive performance of the applied models.

3. Modelling, Prediction and Evaluation

Due to the small sample size and other considerations, the maximum applied number of lags in the estimated univariate "ordinary multiple" model and the ARIMAX-model was two for the independent variables (indicator series), except for the exports that revealed granger-causality for four lags. Also for the dependent variable (differences of GDP) more than two lags have been tried out. The maximum lag lengths for AR- or MA-terms within the ARIMA-model (without indicator series) was four, like within the applied multivariate VAR- and VEC-models. The following further determinants were considered in the process of the pre-selection within each class of models:

- The goodness-of-fit: Adjusted R^2 and information criteria such as the ones after AKAIKE [1974] or SCHWARZ [1978]. Yet, the main focus was on Akaike's information criteria, since it has good properties when it comes to forecasting (where you might prefer rather more variables/lags) and seems to handle small sample sizes better (see KUNST [2007, p. 22]).
- Significance of coefficients, whereas the insignificance of a coefficient was not always a sufficient reason for excluding this lag/variable.
- No remaining auto-correlation of the residuals.
- The prediction error in-sample (1998-2009), which is of course related to Akaike's information criteria.

Within each model class all the best three/four pre-selected models were then compared by calculating their prediction error out-of-sample (2005-2009). As a last step, the winning models of each model-class were then compared across the different model classes regarding their predictive accuracy in-sample (1998-2009) and more importantly out-of-sample (2005-2009).

3.1. Univariate Models

Three univariate model classes are considered: ARIMA-models (after BOX AND JENKINS [1976]), ARIMAX-models (Combination of ARIMA-terms and additional predictors) and an "ordinary multiple" approach with lagged terms of the variables. The most appropriate model according to the factors mentioned before are displayed in the following tables[5]:

[5] GDP is denoted by BIPFL, exports by EZVKCH and the industrial sales by LIHKK.

Dependent Variable: D(BIPFL)
Method: Least Squares
Date: 01/23/12 Time: 03:49
Sample (adjusted): 1975 2009
Included observations: 35 after adjustments
Convergence achieved after 21 iterations
Backcast: 1973 1974

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	138.9058	7.158848	19.40338	0.0000
AR(1)	1.156441	0.331615	3.487298	0.0015
AR(2)	-0.653286	0.286709	-2.278570	0.0300
MA(1)	-0.652566	0.268470	-2.430682	0.0213
MA(2)	-0.302470	0.270542	-1.118017	0.2724

R-squared	0.469035	Mean dependent var	119.7886
Adjusted R-squared	0.398239	S.D. dependent var	180.1137
S.E. of regression	139.7200	Akaike info criterion	12.84872
Sum squared resid	585650.5	Schwarz criterion	13.07091
Log likelihood	-219.8526	F-statistic	6.625220
Durbin-Watson stat	1.794523	Prob(F-statistic)	0.000604

Dependent Variable: D(BIPFL)
Method: Least Squares
Date: 01/23/12 Time: 18:24
Sample (adjusted): 1977 2007
Included observations: 31 after adjustments
Convergence achieved after 17 iterations
Backcast: 1976

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	53.12575	21.65910	2.452814	0.0218
DBIPFL(-1)	1.018999	0.279831	3.641482	0.0013
DBIPFL(-2)	-0.309340	0.230916	-1.339625	0.1929
DEZVKCH(-1)	-0.138607	0.204620	-0.677389	0.5046
DEZVKCH(-4)	-0.304802	0.157584	-1.934216	0.0650
DLIHKK(-1)	0.337246	0.081155	4.155592	0.0004
MA(1)	-0.997253	0.190879	-5.224528	0.0000

R-squared	0.606871	Mean dependent var	153.1387
Adjusted R-squared	0.508589	S.D. dependent var	131.2914
S.E. of regression	92.03620	Akaike info criterion	12.07792
Sum squared resid	203295.9	Schwarz criterion	12.40172
Log likelihood	-180.2078	F-statistic	6.174785
Durbin-Watson stat	1.982920	Prob(F-statistic)	0.000504

Dependent Variable: D(BIPFL)
Method: Least Squares
Date: 01/23/12 Time: 12:49
Sample (adjusted): 1977 2009
Included observations: 33 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	107.9526	34.39236	3.138853	0.0040
DBIPFL(-1)	0.475425	0.252831	1.880405	0.0705
DEZVKCH(-1)	-0.809275	0.206961	-3.910274	0.0005
DEZVKCH(-4)	-0.599394	0.175553	-3.414319	0.0020
DLIHKK(-1)	0.863228	0.160960	5.362983	0.0000

R-squared	0.643842	Mean dependent var	125.1606
Adjusted R-squared	0.592963	S.D. dependent var	184.0790
S.E. of regression	117.4415	Akaike info criterion	12.50848
Sum squared resid	386190.0	Schwarz criterion	12.73523
Log likelihood	-201.3900	F-statistic	12.65421
Durbin-Watson stat	1.975967	Prob(F-statistic)	0.000005

3.2. Multivariate Models

Multivariate models have been estimated as well. The first multivariate model is a Vector Autoregressive Model with the three differenced variables GDP, exports and industrial sales. The estimation output of the VAR(3)-model is displayed in the appendix.

As all the included variables feature a positive trend over time and all are integrated of order one, it is advisable to check if they are cointegrated. Both a trace test and a maximum eigenvalue test indicate that there is one cointegrating relation between the three variables:

Included observations: 33 after adjustments
Trend assumption: Linear deterministic trend
Series: BIPFL LIHKK EZVKCH
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.758326	59.99324	29.79707	0.0000
At most 1	0.282627	13.12772	15.49471	0.1102
At most 2	0.063542	2.166468	3.841466	0.1410

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.758326	46.86552	21.13162	0.0000
At most 1	0.282627	10.96125	14.26460	0.1562
At most 2	0.063542	2.166468	3.841466	0.1410

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

As a consequence of this finding, a Vector Error Correction Model of the following general form has been estimated with one cointegrating relation and constants for the three differenced variables, since they feature a positive trend over time[6]:

$$\begin{bmatrix} \Delta G_t \\ \Delta S_t \\ \Delta E_t \end{bmatrix} = \begin{bmatrix} \mu_G \\ \mu_S \\ \mu_S \end{bmatrix} + \Pi \begin{bmatrix} G_{t-1} \\ S_{t-1} \\ E_{t-1} \end{bmatrix} + \Gamma_1 \begin{bmatrix} \Delta G_{t-1} \\ \Delta S_{t-1} \\ \Delta E_{t-1} \end{bmatrix} + \cdots + \Gamma_4 \begin{bmatrix} \Delta G_{t-4} \\ \Delta S_{t-4} \\ \Delta E_{t-4} \end{bmatrix} + \begin{bmatrix} u_{G,t} \\ u_{S,t} \\ u_{E,t} \end{bmatrix}$$

$$\Pi = \alpha\beta'$$

6 Presumably a random walk with drift, as they have found to be I(1).

Matrix Π includes the cointegrating vector β and the loading vector α, which contains the estimated speed of adjustment to deviations from the long-run equilibrium. Estimating the cointegrating relation and the adjustment parameter, one obtains the following equation for the prediction of GDP:

$$\Delta G_t = \underbrace{338.26}_{\text{trend in level of data}} + \underbrace{\overbrace{0.18}^{\text{speed of adjustment (from loading vector } \alpha)} \cdot \overbrace{(G_{t-1} - 0.85S_{t-1} + 0.09E_{t-1} - 1163.21)}^{\text{cointegrating relation (vector } \beta')}}_{\text{error correction (from matrix } \pi)} + [lagged\ terms\ of\ \Delta G_t, \Delta S_t, \Delta E_t] + e_t$$

The full estimation output of the VEC(1,1)-model can also be found in the appendix.

3.3. Regressive Combination of Different Models

Combining and weighing forecasts of different models can sometimes yield lower prediction errors than the best involved model. Thus, the five estimated models (winning model of each model-calls) are combined:

- In a first step within a *univariate* regression: A multiple model featuring the forecasted values of GDP being used as regressors for the prediction of Liechtenstein's GDP.
- In a second step as a *multivariate* combination: A VAR(3) model incorporating the three models' predictions has also been calculated and used to forecast Liechtenstein's GDP.

It turns out that the combination of the "ordinary" multiple regression, the VAR(3) and the VECM(1,1) is the best combination. As noted in the next section 3.4., the univariate and multivariate combinations cannot outperform the best single model.

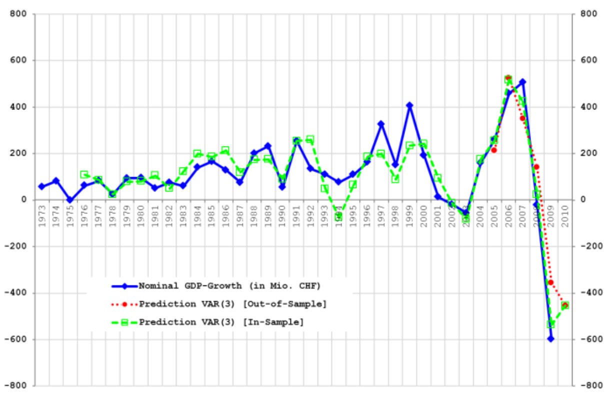
3.4. Evaluation of Predictive Accuracy

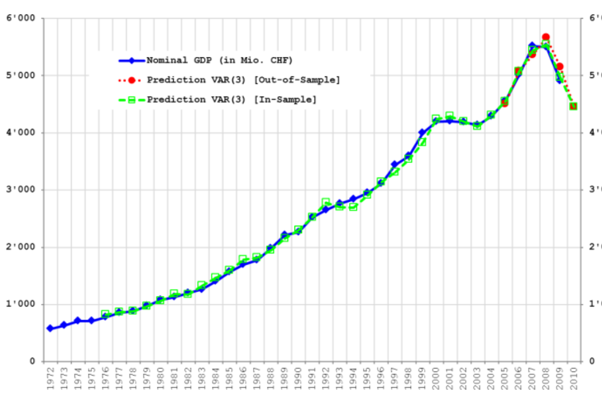
After the estimation of all the relevant models, it now makes sense to proceed to compare the predictive accuracy across the different model-classes. The percentage predictions errors are displayed below:

Annual Nominal GDP of Liechtenstein (1972-2009)	Applied Models	Mean Absolute Prediction Error (%)	
		In-Sample (1998-2009)	Out-of-Sample (2005-2009)
Univariate	[1] Random Walk	5.20%	7.32%
	[2] ARIMA (2,1,2)	3.21%	3.23%
	[3] ARIMAX (1,1,1)	2.06%	4.90%
	[4] Multiple Regression	2.00%	4.29%
Multivariate	[5] VAR (3)	1.27%	2.61%
	[6] VECM (1,1)	2.54%	3.14%
Combination of Model's Predictions	[7] Univariate: [4]+[5]+[6]	1.61%	3.37%
	[8] Multivariate: [4]+[5]+[6]	2.04%	4.25%

Observing the table above with the listing of the models' prediction errors, one can state that all of the models are better than the naive prediction by the benchmark-model (Random Walk), whereas the VAR(3)-model seems to be the best prediction model, both in-sample and out-of-sample. The combinations of different models do not achieve an improvement in forecasting accuracy. The VAR-model seems to outperform the VEC-model, eventhough the series seem to be cointegrated. This might be rather confusing on theoretical grounds, but it can be observed very often in the econometric application that VAR-models are more appropriate in the context of short-term forecasting even though the included series are cointegrated.

Despite of the small sample size (38 annual observations from 1972 to 2009), the multivariate models prove to provide better forecasts compared to the less complex univariate models. Also within the multivariate model setting, lag orders of three or even four appear to have better prediction properties than more parsimonious specifications. The small number of observations, even after including the new backward calculations 1997-1972, was a constantly present challenge and probably a reason for the rather high differences between prediction errors in-sample compared to predictive accuracy out-of-sample. The inspected time period for the evaluation of the prediction errors featured strong volatility and sharp turning points within the evaluated section (1998-2009) of the in-sample-period and especially for the chosen out-of-sample-period (2005-2009).





The figure above exhibits the reference series (the nominal GDP of Liechtenstein) and the in-sample-predictions 1976-2009 and the out-of-sample-predictions of the rolling regression of the best perfoming model, the VAR(3). The graphs show that the model expects for 2010 an additional downturn of economic activities inside the small economy of Liechtenstein. All the applied models failed to forecast the business cycle's turning point (both in-sample and out-of-sample) and they all predict an on-going recession in the year 2010.

4. Conclusions

As already outlined in the main part of this paper, the VAR(3)-model clearly outperforms (in terms of its predictive adequacy) the other models, namely the univariate models, the Vector Error Correction model and the combined regression models. The inspected time period for the evaluation of the predictive accuracy in-sample (1998-2009) and out-of-sample (2005-2009) was characterized by high volatility and sharp turning points, which were hard to predict by the estimated models. Furthermore, the high general volatility of the small economy with nominal growth rates between +13% and -10% makes point forecasts more complicating and also enlarges the confidence intervals. Moreover, Liechtenstein's GDP appears to have a leading pattern compared to other nation's GDP, meaning that turning points tend to appear earlier.

Forecasting Liechtenstein's GDP, two additional problems compared to other countries arise: First of all, official figures for nominal GDP only exist for the years from 1998 to 2009. In addition, there is a long publication lag of Liechtenstein's GDP which is almost 14 months. Thus, before forecasting can be started, "nowcasting" has to be carried out first. The National Statistical Office publishes the provisional flash-estimate of GDP for 2010 by the beginning of February 2012. Then, a few days later by the end of February 2012 the Liechtenstein Economic Institute (Konjunkturforschungsstelle Liechtenstein KOFL) "nowcasts" the GDP of 2011 and forecasts the GDP of 2012.

Now, as backwardly calculated GDP-values from 1997 back until 1972 finally exist, ordinary and comparably easy applicable time series models, such as the different models mentioned in the paper here, are useful for:

- *Nowcasting*: The models can serve as complement or improvement of the existing nowcasting procedure applied by the KOFL. In this context, the applied time series are incorporated as coincident indicators for GDP.
- *Forecasting*: These models with a rather low complexity are good benchmark models and can therefore complement the currently applied prediction procedure of KOFL, which also contains iterative-analytical judgment and heuristic proceeding besides econometric modelling of future influences on Liechtenstein's business cycle.

After the nominal GDP-figures for the years 2010 (National Statistical Office) and 2011 (KOFL) are provided by the end of February, this empirical project can be carried out again to obtain a real forecast for the year 2012. The sample will then contain two more observations, therefore exhibiting a total of 40 observations. Future modelling in the course of this empirical prediction project will not deal with absolute differences in levels anymore, but differences of the logarithms will be used instead in the future modelling, evaluation and prediction process. A fruitful extension could be to also incorporate coincident indicators, for example external predictions of variables such as Liechtenstein's exports, which are annually forecasted by KOFL, or the Swiss GDP forecasted by several Swiss institutions dealing with economic research. This would potentially improve the anticipation of turning points within the applied models of this paper.

Appendix

A.1. Estimation Outputs of VAR(3) and VECM(1,1)

The two output tables below show the regression results of the Vector Autoregression and the Vector Error Correction Model. Liechtenstein's GDP is named BIPFL, the industrial sales denoted by LIHKK and the Exports labelled as EZVCHK.

Vector Autoregression Estimates
Date: 01/23/12 Time: 12:47
Sample (adjusted): 1976 2009
Included observations: 34 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	D(BIPFL)	D(LIHKK)	D(EZVKCH)
D(BIPFL(-1))	1.286988	1.118316	1.764383
	(0.25093)	(0.43719)	(0.41195)
	[5.12891]	[2.55799]	[4.28300]
D(BIPFL(-2))	-0.060932	-0.521059	-0.042627
	(0.25053)	(0.43649)	(0.41130)
	[-0.24321]	[-1.19374]	[-0.10364]
D(BIPFL(-3))	-0.960538	-0.818437	-0.737590
	(0.22694)	(0.39538)	(0.37256)
	[-4.23265]	[-2.06998]	[-1.97978]
D(LIHKK(-1))	0.199574	0.098718	0.431088
	(0.17482)	(0.30459)	(0.28701)
	[1.14157]	[0.32410]	[1.50200]
D(LIHKK(-2))	0.621270	0.463685	0.271095
	(0.16739)	(0.29165)	(0.27481)
	[3.71141]	[1.58988]	[0.98647]
D(LIHKK(-3))	-0.290748	0.147430	-0.367098
	(0.17259)	(0.30071)	(0.28335)
	[-1.68457]	[0.49028]	[-1.29557]
D(EZVKCH(-1))	-0.956818	-0.953132	-1.370800
	(0.17124)	(0.29834)	(0.28112)
	[-5.58767]	[-3.19476]	[-4.87619]
D(EZVKCH(-2))	-0.579940	-0.630797	-0.763636
	(0.23736)	(0.41354)	(0.38967)
	[-2.44332]	[-1.52535]	[-1.95969]
D(EZVKCH(-3))	0.270871	0.377844	0.169204
	(0.21543)	(0.37533)	(0.35367)
	[1.25736]	[1.00669]	[0.47842]
C	152.4434	185.2962	128.5172
	(29.7996)	(51.9191)	(48.9221)
	[5.11562]	[3.56894]	[2.62698]
D78	-162.1426	-137.2822	-113.8122
	(87.1943)	(151.916)	(143.147)
	[-1.85956]	[-0.90367]	[-0.79507]
D91	386.2385	215.9106	229.5656
	(92.1434)	(160.539)	(151.272)
	[4.19171]	[1.34491]	[1.51757]
R-squared	0.862515	0.699318	0.824823
Adj. R-squared	0.793772	0.548977	0.737234
Sum sq. resids	149597.3	454105.2	403193.3
S.E. equation	82.46137	143.6703	135.3771
F-statistic	12.54702	4.651546	9.417006
Log likelihood	-190.8627	-209.7392	-207.7177
Akaike AIC	11.93310	13.04348	12.92457
Schwarz SC	12.47182	13.58220	13.46329
Mean dependent	123.3265	104.8427	85.70150
S.D. dependent	181.5837	213.9280	264.0954
Determinant resid covariance (dof adj.)		1.07E+12	
Determinant resid covariance		2.90E+11	
Log likelihood		-593.4195	
Akaike information criterion		37.02468	
Schwarz criterion		38.64082	

Vector Error Correction Estimates
Date: 01/22/12 Time: 22:54
Sample (adjusted): 1977 2009
Included observations: 33 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
BIPFL(-1)	1.000000
LIHKK(-1)	-0.849273
	(0.29079)
	[-2.92062]
EZVKCH(-1)	0.088225
	(0.28415)
	[0.31049]
C	-1163.212

Error Correction:	D(BIPFL)	D(LIHKK)	D(EZVKCH)
CointEq1	0.176093	0.363381	0.102441
	(0.05880)	(0.05005)	(0.09183)
	[2.99501]	[7.26045]	[1.11553]
D(BIPFL(-1))	0.553835	0.667721	1.353422
	(0.31335)	(0.26673)	(0.48941)
	[1.76748]	[2.50332]	[2.76540]
D(BIPFL(-2))	0.634291	0.353633	0.242679
	(0.36675)	(0.31219)	(0.57282)
	[1.72950]	[1.13274]	[0.42366]
D(BIPFL(-3))	-0.486251	-0.621735	-0.408206
	(0.31504)	(0.26817)	(0.49205)
	[-1.54347]	[-2.31840]	[-0.82960]
D(BIPFL(-4))	-1.113148	-1.456999	-0.880323
	(0.36487)	(0.31059)	(0.56988)
	[-3.05084]	[-4.69107]	[-1.54475]
D(LIHKK(-1))	0.393178	-0.085754	0.510586
	(0.21198)	(0.18045)	(0.33109)
	[1.85476]	[-0.47523]	[1.54212]
D(LIHKK(-2))	-0.115107	-0.612864	-0.215372
	(0.24632)	(0.20967)	(0.38472)
	[-0.46732]	[-2.92293]	[-0.55982]
D(LIHKK(-3))	-0.305693	-0.020911	-0.313470
	(0.21294)	(0.18126)	(0.33259)
	[-1.43559]	[-0.11536]	[-0.94252]
D(LIHKK(-4))	-0.399945	-0.176225	0.041448
	(0.22227)	(0.18920)	(0.34715)
	[-1.79940]	[-0.93141]	[0.11940]
D(EZVKCH(-1))	-0.871510	-0.876472	-1.308533
	(0.18967)	(0.16146)	(0.29625)
	[-4.59482]	[-5.42851]	[-4.41704]
D(EZVKCH(-2))	-0.544306	-0.673574	-0.666090
	(0.26927)	(0.22921)	(0.42057)
	[-2.02142]	[-2.93863]	[-1.58379]
D(EZVKCH(-3))	0.219199	0.107000	0.078387
	(0.27023)	(0.23003)	(0.42207)
	[0.81116]	[0.46515]	[0.18572]
D(EZVKCH(-4))	0.106787	-0.129653	0.001434
	(0.26184)	(0.22289)	(0.40897)
	[0.40783]	[-0.58168]	[0.00351]
C	338.2584	519.2846	247.1678
	(64.8283)	(55.1846)	(101.254)
	[5.21776]	[9.40995]	[2.44106]

R-squared	0.851787	0.922846	0.829648
Adj. R-squared	0.750378	0.870057	0.713091
Sum sq. resids	160710.5	116453.3	392051.2
S.E. equation	91.96983	78.28869	143.6464
F-statistic	8.399543	17.48162	7.117957
Log likelihood	-186.9240	-181.6091	-201.6385
Akaike AIC	12.17721	11.85510	13.06900
Schwarz SC	12.81210	12.48998	13.70388
Mean dependent	125.1606	105.7318	86.14306
S.D. dependent	184.0790	217.1811	268.1774

Determinant resid covariance (dof adj.)	5.32E+11
Determinant resid covariance	1.02E+11
Log likelihood	-558.6412
Akaike information criterion	36.58432
Schwarz criterion	38.62501

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