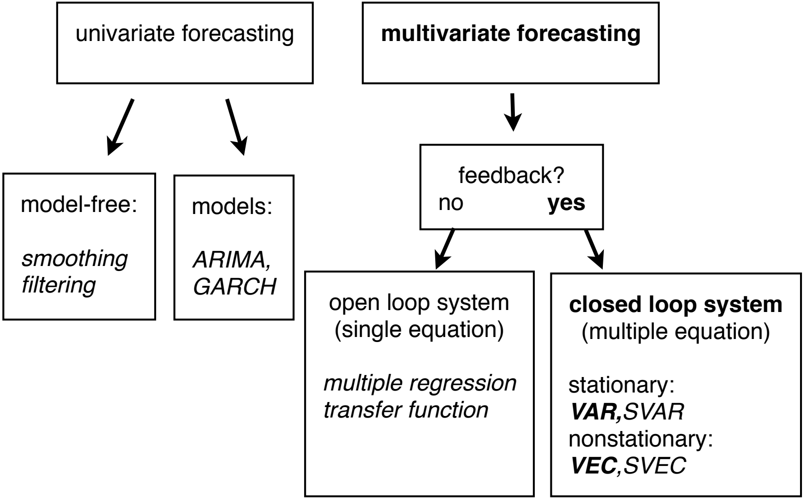
Multivariate forecasting with VAR models

Franz Eigner
University of Vienna

UK Econometric Forecasting
Prof. Robert Kunst

16th June 2009

Overview


univariate forecasting
multivariate forecasting
model-free:
smoothing
filtering
models:
ARIMA,
GARCH
feedback?
no
yes
open loop system
(single equation)
multiple regression
transfer function
closed loop system
(multiple equation)
stationary:
VAR,SVAR
nonstationary:
VEC,SVEC

Definition of VAR(p)

Stationary vector autoregressive process

A VAR consists of a set of K-endogenous variables
$y_t = (y_{1t}, \ldots, y_{kt}, \ldots, y_{Kt})$ for $k = 1, \ldots, K$

A VAR(p) process is defined as

$$y_t = \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t$$

where Φ_i are $(K \times K)$ coefficient matrices for $i = 1, \ldots, p$ and u_t is K-dimensional white noise.

The VAR(p) process is stable (stationary series), if

$$\det(I_K - \Phi_1 z - \ldots - \Phi_p z^p) \neq 0 \quad \text{for } |z| \leq 1 \qquad (1)$$

Definition of bivariate VAR(1)

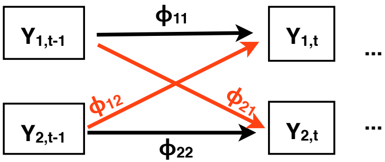
Stationary bivariate vector autoregressive process

Bivariate VAR(1) process $y_t = \Phi_1 y_{t-1} + u_t$

with $u_t^T = (u_{1t}, u_{2t})$ and $\Phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$

consists of two equations:

$$y_{1t} = \phi_{11} y_{1,t-1} + \phi_{12} y_{2,t-1} + u_{1t}$$
$$y_{2t} = \phi_{21} y_{1,t-1} + \phi_{22} y_{2,t-1} + u_{2t}$$



→ Concept of Granger causality

Specification, estimation and structural analysis

Finding optimal time lag p $\longrightarrow$ information criterion

Model specification pitfall

Number of parameters increases tremendously with more lags

Coefficients are estimated by OLS on each equation

Structural analyses

1. Granger causality
2. Impulse response analysis
3. Forecast error variance decomposition

Forecasting

- naive forecast: Minimum mean square error (MMSE)

For a VAR(1) process

$$y_t = \Phi_1 y_{t-1} + u_t$$

one-step-ahead forecast: $\hat{y}_N(1) = \hat{\Phi}_1 y_N$
two-step ahead forecast: $\hat{y}_N(2) = \hat{\Phi}_1 \hat{y}_N(1) = \hat{\Phi}_1^2 y_N$

Forecasts for horizons h are therefore obtained with

$$\hat{y}_N(h) = \hat{\Phi}_1^h y_N$$

Extensions

- VARMA, VMA
- VARX, VARMAX (including exogenous variables)

imposing more restrictions: (model reduction)

- Structural VAR (SVAR)
- Bayesian VAR (BVAR)

VAR model and Cointegration

- before: stationary time series ($\rightarrow$ stability condition)
- now: nonstationary data

one could difference the data, but not adequate in the presence of cointegration.

Cointegration

$y_t \sim I(d)$ is cointegrated, if there exists
a kx1 fixed vector $\beta \neq 0$, so that $\acute{\beta} y_t$ is integrated of order $< d$.

$\rightarrow$ Assume: y_{1t} and y_{2t} are $I(1)$. They are cointegrated when a linear combination of y_{1t} and y_{2t} exists with $(y_{1t} - \beta y_{2t}) \sim I(0)$

Bivariate cointegrated VAR(2)

Consider the bivariate VAR(2)

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + u_t$$

with the matrix polynomial for z=1 ($\rightarrow$ stability condition (1))

$$\Phi(1) = (I - \Phi_1 - \Phi_2) = \Pi$$

$rank(\Pi)$ equals the cointegration rank of the system y_t
0 ... no cointegration ($\rightarrow$ difference VAR)
1 ... one cointegrating vector ($\rightarrow$ VECM)
2 ... process is stable ($\rightarrow$ VAR)

Vector error correction model (VECM)

Implementing cointegration in a VAR(2) using the VECM form:

$$\triangle y_t = y_t - y_{t-1} = \Pi y_{t-1} + \Gamma_1 \triangle y_{t-1} + u_t$$

where $\Gamma_1 = -\Phi_2$ is the transition matrix and $\Pi = \alpha\acute{\beta}$ holds

- α as the »loading matrix« (speed of adjustment)
- $\acute{\beta}$ consisting the independent cointegrating vector
- $\acute{\beta} Y_{t-1}$ as the lagged disequilibrium error
- Πy_{t-1} as the error correction term (long-run part)

(to catch the idea: consider bivariate VAR(1) equation:
$\triangle y_{1t} = \alpha_1(y_{1,t-1} - \beta y_{2,t-1}) + u_{1t}$ with long-run equilibrium $y_{1t} = \beta y_{2t}$)

- estimation by reduced rank regression and forecast as in VAR

Applications of VAR/VEC

VAR

- economics and finance
  growth rates of macroeconomic time series and some asset returns

VEC

- economics
  - Money demand: money, income, prices and interest rates
  - Growth theory: income, consumption and investment
  - Fisher equation: nominal interest rates and inflation
- finance
  cointegration between prices of the same asset trading on different markets due to arbitrage