A NONPARAMETRIC TEST FOR SEASONAL UNIT ROOTS

Robert M. Kunst

Institute for Advanced Studies, Vienna, and University of Vienna

Abstract

We consider a nonparametric test for the null of seasonal unit roots in quarterly and monthly time series that builds on the RUR (range unit root) test by Aparicio, Escribano, and Sipols, so we tentatively use the name RURS (for RUR–seasonal). We find that the test concept is more promising than a formalization of visual aids such as rank changes in plots by period. In order to cope with the sensitivity of the original RUR test to autocorrelation under its null of a unit root, we suggest an augmentation step by autoregression. Whereas the statistics for testing unit roots in quarterly time series have essentially the same limiting distribution as the original RUR statistic, the monthly version uses eight test statistics at the seasonal frequencies with hitherto unexplored limiting distributions. We present some evidence on the size and power of our procedure and we provide illustrations by empirical applications.

The quarterly RURS statistics

The statistics

$$J_1 = n^{-1/2} R_n^{x(1)} \ , \ J_2 = n^{-1/2} R_n^{x[2]},$$
$$J_3 = (n/2)^{-1/2} R_{n/2}^{x[3]} \ , \ J_4 = (n/2)^{-1/2} R_{n/2}^{x[4]}$$

are the RURS (range unit roots seasonal) statistics. For SRW (x_t), they converge to the AES limit distribution. If $\Phi(1) \neq 0$, $J_1 \to 0$. If $\Phi(-1) \neq 0$, $J_2 \to 0$, and if $\Phi(\pm i) \neq 0$, then $J_3 \to 0$ and $J_4 \to 0$.

Correction for autocorrelation works: AR disturbances

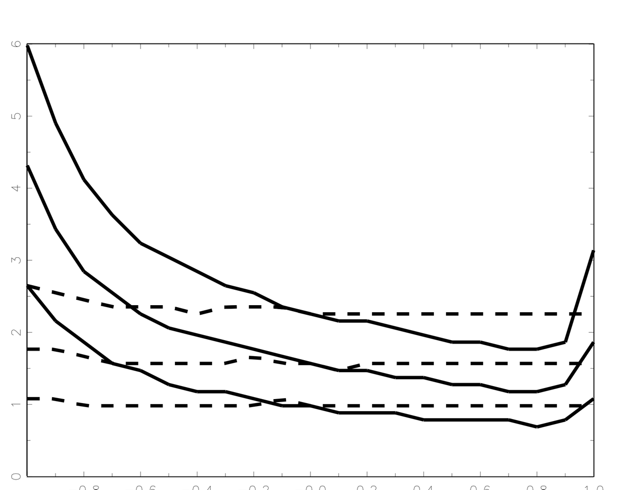

Figure 1: 10%, 50%, and 90% quantiles for the uncorrected (solid) and for the augmentation-corrected (dashed) RURS statistic $J_2^{(n)}$ if it is calculated from trajectories of length $n = 100$ from the data-generation process $\Delta_4 x_t = \phi \Delta_4 x_{t-1} + \varepsilon_t$ and ϕ is varied over the interval $[-1, 1]$. ϕ values on the abscissa.

The RUR test

The RUR test is a nonparametric test for unit roots that was suggested by Aparicio, Escribano, and Sipols (2006 *Journal of Time Series Analysis*, AES).

Essentially, parametric unit-roots tests use test statistics that measure the correlation of levels and differences, which are zero for random walks.

Nonparametric unit-root tests use test statistics that exploit other characteristic properties of random walks, such as the rate of expansion of trajectories or the number of level crossings. The RUR statistic counts the frequency of new extrema within the trajectory.

For a given realization $(x_t, t = 1, \ldots, n)$, define

$$x_{j,j} = \max_{t=1,\ldots,j} x_t, \quad x_{1,j} = \min_{t=1,\ldots,j} x_t.$$

Then, $x_{j,j} - x_{1,j}$ defines the sequence of *ranges* of the series. Any time it increases over $j = 1, \ldots, n$, this is called a *record*. The number of records until n is denoted as $R^{(x)}(n)$ or $R(n)$.

AES show that $R(n) = O(n^{1/2})$ for a random walk with independent increments. One can also show that $R(n) = O(\log n)$ for many stationary processes. This motivates that the RUR (range unit root) statistic

$$J_0^{(n)} = n^{-1/2} R^{(x)}(n)$$

can be the basis for a consistent test, if the null is a random walk and the alternative is stationarity.

Handling RUR non-similarity

AES do not address the issue of the null non-similarity of the RUR test. Assume (x_t) is I(1) but not a random walk. We suggest to eliminate serial correlation under the null by regressing Δx_t on p BIC–selected lags

$$\Delta x_t = \mu + \sum_{k=1}^{p} \gamma_k \Delta x_{t-k} + \varepsilon_t, t \geq p+2.$$

Estimation residuals u_t are accumulated according to

$$\tilde{x}_t = \sum_{j=p+2}^{t} u_j + x_{p+1},$$

such that $\tilde{x}_t$ is ideally a pure random walk without drift. The same can be done for the constructed variables $x^{(1)}$, $x^{[2]}$, $x^{[3]}$, $x^{[4]}$.

Deterministic terms. In the RURS construction, drifts and deterministic seasonal patterns are eliminated. The RURS test is conducted as a one-sided test. The right tail of the AES limit distribution can only materialize in the presence of super-linear trends.

An empirical application

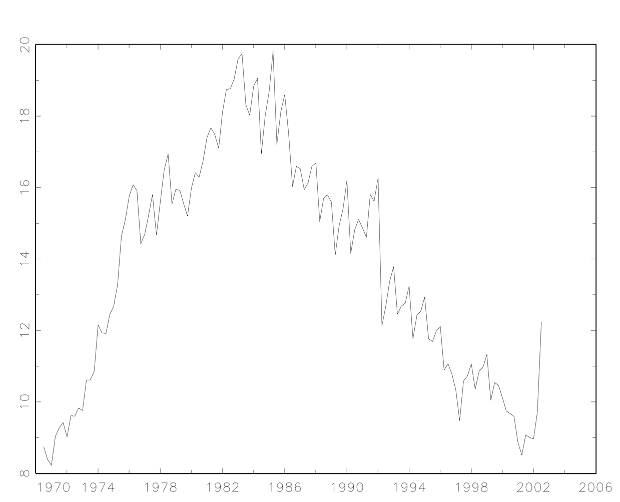

Figure 2: Belgian barley prices, 1971:1–2003:1, quarterly observations.

RURS and HEGY yield different results: Traditional parametric HEGY tests reject unit roots at -1 and at $\pm i$ and indicate purely deterministic seasonal variation. RURS statistics are $J_1 = 2.03$, $J_2 = 1.85$, and $J_3 = J_4 = 1.00$. Unit roots at ± 1 are supported, while $\pm i$ is rejected at the 5% level. The RURS test finds a deterministic annual cycle and a persistently changing semi-annual pattern.

The idea of the RURS test: quarterly version

Assume (x_t) is observed quarterly. If its autoregressive representation $\Phi(B)x_t = \varepsilon_t$ yields a polynomial $\Phi(z)$ with $\Phi(1) = 0$, (x_t) is integrated at frequency zero. If $\Phi(-1) = 0$, it is integrated at the semi-annual frequency π. If $\Phi(\pm i) = 0$, it is integrated at the annual frequency $\pi/2$. The most popular test for seasonal unit roots is the parametric test by Hylleberg, Engle, Granger, Yoo (1990 *Journal of Econometrics*), the HEGY test. It uses auxiliary regressors similar to the following constructed variables.

Assume (x_t) follows a *seasonal random walk* $x_t = x_{t-4} + \varepsilon_t$. Then, the transformed variable

$$x_t^{(1)} = x_t + x_{t-1} + x_{t-2} + x_{t-3}$$

is a random walk. The transformed variable

$$x_t^{(2)} = x_t - x_{t-1} + x_{t-2} - x_{t-3}$$

behaves like $y_t = -y_{t-1} + \varepsilon_t$. Reverting the signs for all observations with t odd yields a random walk $x_t^{[2]}$. Also,

$$x_t^{(3)} = x_t - x_{t-2}$$

behaves like $x_t^{(3)} = -x_{t-2}^{(3)} + \varepsilon_t$. It consists of a modified RW for the odd and another one for even t. Collecting the even t and the odd t separately and reverting signs within the two time series yields two random walks $x_t^{[3]}$ and $x_t^{[4]}$ with sample size $n/2$.

Simulated quantiles

model		1%	5%	10%	median	90%	95%	99%
SRW $n = 1000$	± 1	0.95	1.14	1.23	1.67	2.24	2.40	2.75
	$\pm i$	1.07	1.25	1.38	2.06	2.99	3.35	4.11
SRW $n = 500$	± 1	0.94	1.11	1.20	1.65	2.18	2.41	2.72
	$\pm i$	1.01	1.26	1.39	2.02	2.97	3.28	3.91
SRW $n = 100$	± 1	0.78	0.98	1.08	1.57	2.16	2.26	2.65
	$\pm i$	0.84	1.12	1.26	1.82	2.66	3.08	3.50

Rows ± 1 correspond to the J_1 and J_2 statistics, rows $\pm i$ to J_3 and J_4. Left tails to be used for testing. Values are close to AES values but they are not identical.

A glimpse at the monthly case

The transformation to random walks works only for ± 1 and $\pm i$. At all other frequencies, the asymptotic distribution of the RURS statistic is unknown. Limit trajectories are not Brownian motion. The expansion rate of any RURS statistic will again be $O(n^{1/2})$.

Example: testing at frequency $\pi/6$**.** Assume (x_t) is a monthly SRW $x_t = x_{t-12} + \varepsilon_t$. Then, the dynamic transformation

$$y_t = (1 + \sqrt{3}B + 2B^2 + \sqrt{3}B^3 + B^4 - B^6 - \sqrt{3}B^7 - 2B^8 - \sqrt{3}B^9 - B^{10})x_t$$

will be a pure unit-root process of the form $(1 - \sqrt{3}B + B^2)y_t = \varepsilon_t$ at the angular frequency $\pi/6$. For these processes, a parallel procedure can be used to purge them from serial correlation under the null. Consider the auxiliary regression

$$(1 - \sqrt{3}B + B^2)y_t = \mu + \sum_{j=1}^{p} (1 - \sqrt{3}B + B^2)y_{t-j} + \varepsilon_t, t \geq p+3.$$

Purged trajectories evolve from accumulating estimation residuals u_t

$$\tilde{y}_t = \sqrt{3}\tilde{y}_{t-1} - \tilde{y}_{t-2} + u_t, t \geq p+3.$$

The same can be done for frequencies $\pi/3$, $2\pi/3$, $5\pi/6$. The monthly RURS test uses 8 test statistics, two at $\pm i$ and one at each of the other six seasonal frequencies.