Heteroskedasticity

in the Error Component Model

Baltagi Textbook – Chapter 5

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Introduction

The standard error component model given by equation

$$y_{it} = X_{it}^{'}\beta + u_{it} \qquad i = 1, \dots, N\, ; t = 1, \dots, T$$
$$u_{it} = \mu_i + \nu_{it}$$

assumes that the regression disturbances are homoskedastic with the same variance across time and individuals.

This may be a restrictive assumption for panels, where the cross-sectional units may be of varying size and as a result may exhibit different variation.

The Consequences of Heteroscedasticity

Assuming homoskedastic disturbances when heteroskedasticity is present will still result in consistent estimates of the regression coefficients, but these estimates will not be efficient. Also, the standard errors of these estimates will be biased and one should compute robust standard errors correcting for the possible presence of heteroskedasticity.

Cases of Heteroskedasticity

Case 1: The heteroskedasticity is on the individual specific error component (Mazodier and Trognon (1978))

$$\mu_i \sim \left(0, \omega_i^2\right) \qquad \text{for } i = 1, \dots, N$$

$$\nu_{it} \sim IID(0, \sigma_\nu^2)$$

In vector form,

$$\mu \sim (0, \textstyle\sum_\mu) \qquad \text{where } \textstyle\sum_\mu = diag[\omega_i^2]$$

$$\nu \sim (0, \sigma_\nu^2\, I_{NT})$$

$$\Omega = E(uu') = Z_\mu \Sigma_\mu Z_\mu' + \sigma_\nu^2 \; I_{NT}$$

$$\Omega = diag[\omega^2] \otimes J_T + \; diag \; [\sigma_\nu^2] \otimes I_T$$

Using the Wansbeek and Kapteyn(1982b, 1983) trick, Baltagi and Griffin (1988) derived the transformation as follow:

$$\Omega = diag[T\omega^2 + \sigma_\nu^2] \otimes \bar{J}_T + \; diag \; [\sigma_\nu^2] \otimes E_T$$

$$\Omega^r \; = diag[(\tau_i^2)^r] \otimes \bar{J}_T + \; diag \; [(\sigma_\nu^2)^r] \otimes E_T$$

$$\sigma_\nu \Omega^{-1/2} = diag[\sigma_\nu / \tau_i] \otimes \bar{J}_T + \; (I_N \otimes E_T)$$

Hence,

$$y^* = \sigma_\nu \Omega^{-1/2} y$$

has a typical element

$$y_{it}^* = y_{it} - \theta_i \bar{y}_i$$

where

$$\theta_i = 1 - (\sigma_\nu / \tau_i)$$

Baltagi and Griffin (1988) provided feasible GLS estimator including Rao's (1970,1972) MINQUE estimator for this model.

Phillips (2003) argues that there is no guarantee that feasible GLS and true GLS will have the same asymptotic distributions.

Case 2: The heteroskedasticity is on the remainder error term (Wansbeek (1989))

$$\mu_i \sim IID(0, \sigma_\mu^2)$$

$$\nu_{it} \sim (0, \omega_i^2)$$

In this case,

$$\Omega = E(uu') = diag[\sigma_\mu^2] \otimes J_T + \; diag\,[\omega_i^2] \otimes I_T$$

$$\Omega = diag[T\sigma_\mu^2 + \omega_i^2] \otimes \bar{J}_T + \; diag\,[\omega_i^2] \otimes E_T$$

$$\Omega^r = diag[(\tau_i^2)^r] \otimes \bar{J}_T + \; diag\,[(\sigma_\nu^2)^r] \otimes E_T$$

$$\Omega^{-1/2} = diag[1/\tau_i] \otimes \bar{J}_T + \ diag[1/\omega_i] \otimes E_T$$

$$y^* = \Omega^{-1/2} y$$

$$y_{it}^* = (\bar{y}_i \ / \ \tau_i) + (y_{it} - \bar{y}_i) / \ \omega_i$$

$$y_{it}^* = \frac{1}{\omega_i} (y_{it} - \theta_i \bar{y}_i) \ \text{ where } \ \theta_i = 1 - (\omega_i / \tau_i)$$

Case 3: The heteroskedasticity is both on the individual specific error Component and on the remainder error term (Randolph (1988))

$$\text{Var}\,(\mu_i) = \ \sigma_i^2$$

$$E(\nu\nu') = diag[\sigma_{it}^2] \quad i = 1, \dots, N\,; t = 1, \dots, T$$

Adaptive heteroskedastic estimators (EGLS, GLSAD):

Roy (2002)-EGLS:

$$E[\,\mu_i \mid \bar{X}_{i.}^{'}\,] = 0$$

$$var[\,\mu_i \mid \bar{X}_{i.}^{'}\,] = \omega(\bar{X}_{i.}^{'}) \equiv \omega_i$$

$$\hat{\sigma}_{\nu}^{2} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\ [(y_{it} - \bar{y}_{i.}) - (X_{it} - \bar{X}_{i.})^{'}\tilde{\beta}]^{2}}{N(T-1) - k}$$

$\tilde{\beta}$ is the fixed effects or within estimator of β.

$$\hat{\omega}_{i} = \frac{\sum_{j=1}^{N}\sum_{t=1}^{T}\hat{u}_{jt}^{2}K_{i.,j.}}{\sum_{j=1}^{N}\sum_{t=1}^{T}K_{i.,j.}} - \hat{\sigma}_{\mu}^{2}$$

where the kernel function is given by $K_{i.,j.} = K\left(\frac{\bar{X}_{i.}^{'} - \bar{X}_{j.}^{'}}{h}\right)$

Monte Carlo results suggest that ignoring the presence of heteroskedasticity on the remainder term has a much more dramatic effect than ignoring the presence of heteroskedasticity on the individual specific error. Hence, the Li and Stengos (1994) estimator should be preferred to the Roy (2002) estimator when a researcher does not know the source of the heteroskedasticity.

Li and Stengos (1994)-GLSAD:

$$\mu_i \sim IID(0, \sigma_\nu^2)$$

$$E[\nu_{it} \mid X_{it}^{'}] = 0 \text{ with } var[\nu_{it} \mid X_{it}^{'}] = \gamma(X_{it}^{'}) \equiv \gamma_{it}$$

$$\sigma_{it}^2 = E\left[u_{it}^2 \mid X_{it}\right] = \sigma_\mu^2 + \gamma_{it}$$

$$\hat{\sigma}_\mu^2 = \frac{\sum_{j=1}^{N} \sum_{t \neq s}^{T} \hat{u}_{it} \hat{u}_{is}}{NT(T-1)}$$

$\hat{u}_{it}$ denote the OLS residual.

$$\hat{\gamma}_{it} = \frac{\sum_{j=1}^{N} \sum_{s=1}^{T} \hat{u}_{js}^2 K_{it,js}}{\sum_{j=1}^{N} \sum_{s=1}^{T} K_{it,js}} - \hat{\sigma}_\mu^2$$

Testing for Heteroskedasticity

Verbon (1980)	Lagrange Multiplier test	H_0: Homoskedasticity H_1: Heteroskedasticity	$\mu_i \sim (0, \sigma^2_{\mu_i})$ $\nu_{it} \sim (0, \sigma^2_{\nu_i})$ $\sigma^2_{\mu_i} = \sigma^2_\mu f(Z_i \theta_2)$ $\sigma^2_{\nu_i} = \sigma^2_\nu f(Z_i \theta_1)$
Lejeune (1996)	ML estimation and LM	Testing general heteroskedastic one way error component $(H_0 = \sigma^2_\mu = 0)$	$\mu_i \sim (0, \sigma^2_{\mu_i})$ $\nu_{it} \sim (0, \sigma^2_{\nu_{it}})$ $\sigma^2_{\mu_i} = \sigma^2_\mu h_\mu(F_i \theta_2)$ $\sigma^2_{\nu_{it}} = \sigma^2_\nu h_\nu(Z_{it} \theta_1)$
Holly & Gardiol (2000)	Score test	Homoskedasticity in a one way error component model $(H_0: \theta_2 = 0)$	H_1: $\mu_i \sim N(0, \sigma^2_{\mu_i})$ $\sigma^2_{\mu_i} = \sigma^2_\mu h_\mu(F_i \theta_2)$

Baltagi, Bresson, Pirotte (2005)	Joint LM test	Homoskedasticity in one way error component ($H_0: \theta_1 = \theta_2 = 0$)
Baltagi et al. (2005)	LM test	H_0: Homoskedasticity of the individual Random Effect assuming homoskedasticity of remainder error

Monte Carlo experiments showed that the joint LM test performed well when both error components were heteroskedastic, and performed second best when one of the components was homoskedastic while the other was not. In contrast, the marginal LM tests performed best when heteroskedasticity was present in the right error component.

References

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Thank you for attention