Higher Moment Trading Using ARCH

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40789 Nonlinear Time Series

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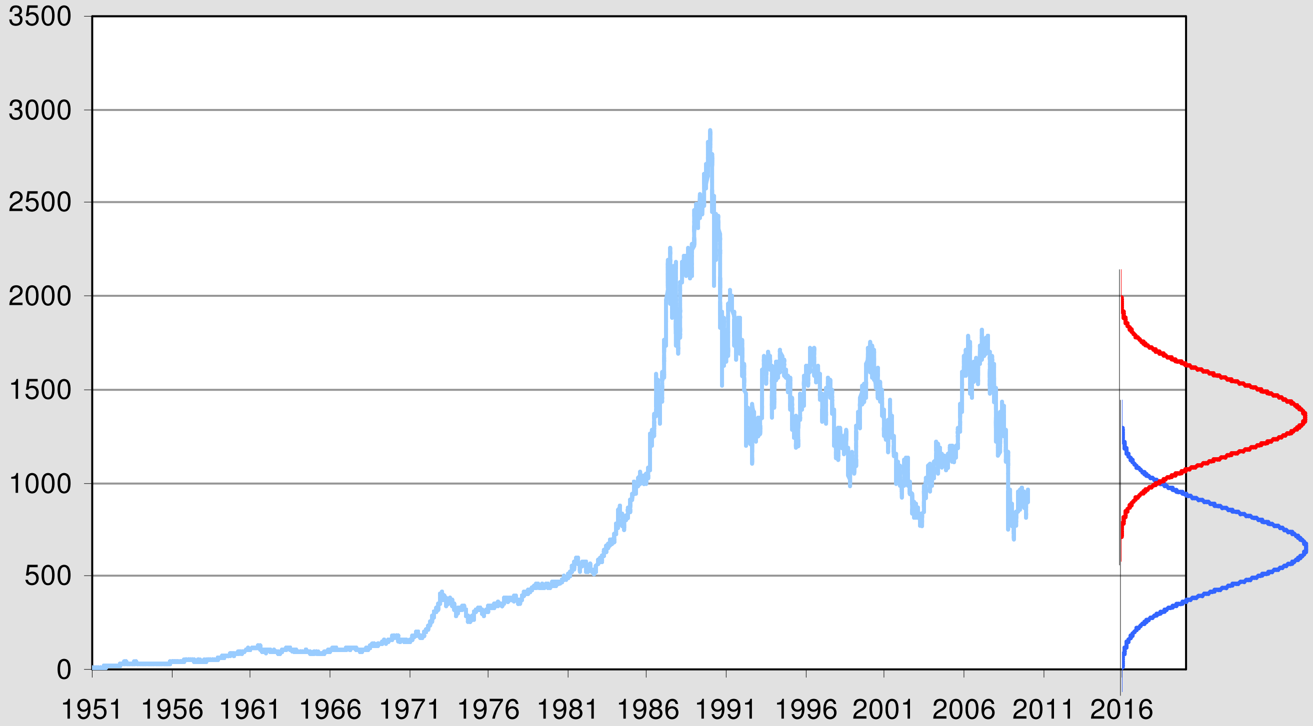
INTRODUCTION
First moment trading

- Common case: trading the mean



Source: Thomson Reuters, own calculations

INTRODUCTION

Black and Scholes (1973)

$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

T - t ... time to maturity

S ... spot price of the underlying asset

K ... strike price

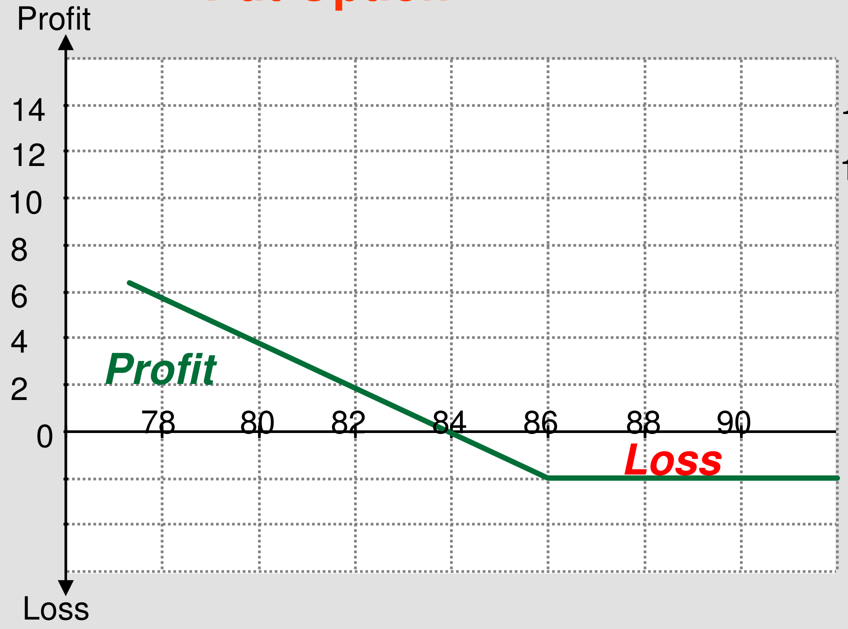
r … risk free interest rate

σ^2 ... volatility in the log-returns of the underlying

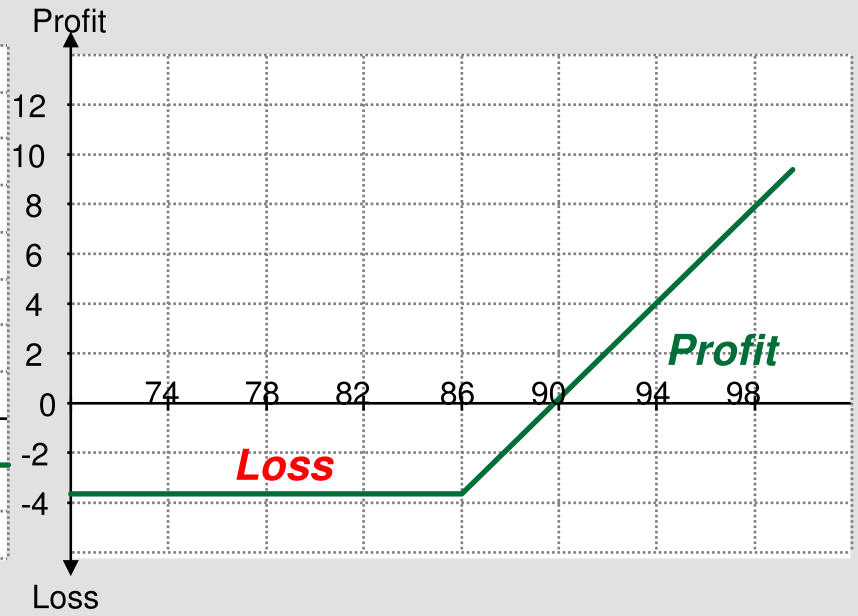
INTRODUCTION
Options

- $P_o = f\,[(S-K), (T-t), \sigma^2]$

Put Option



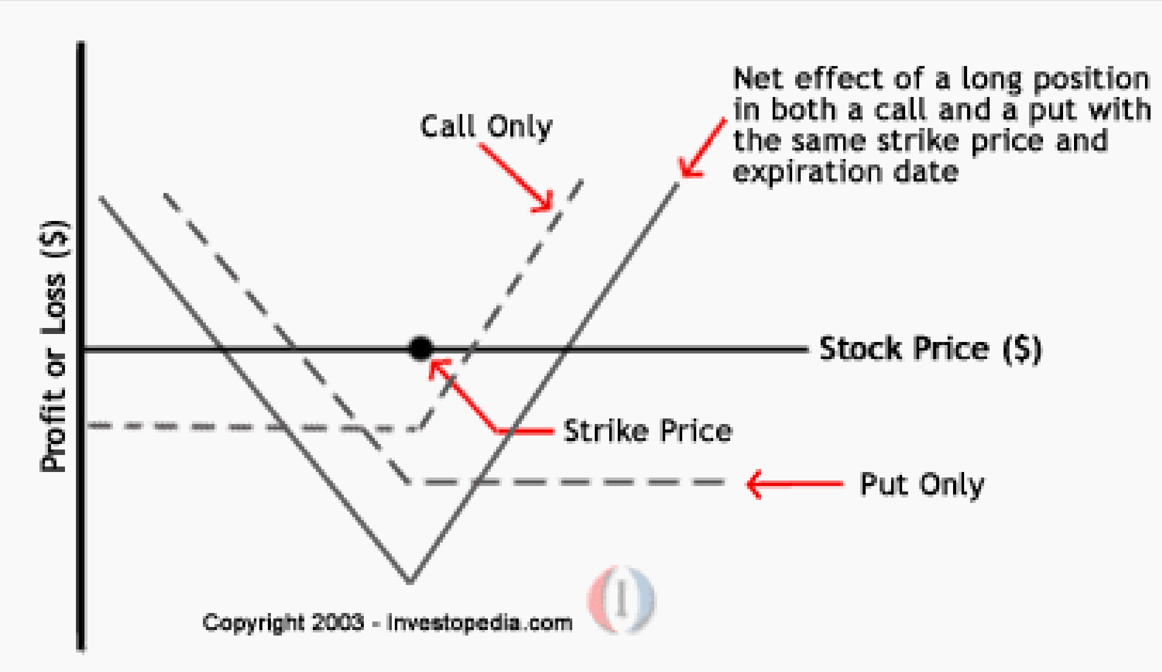
Call Option



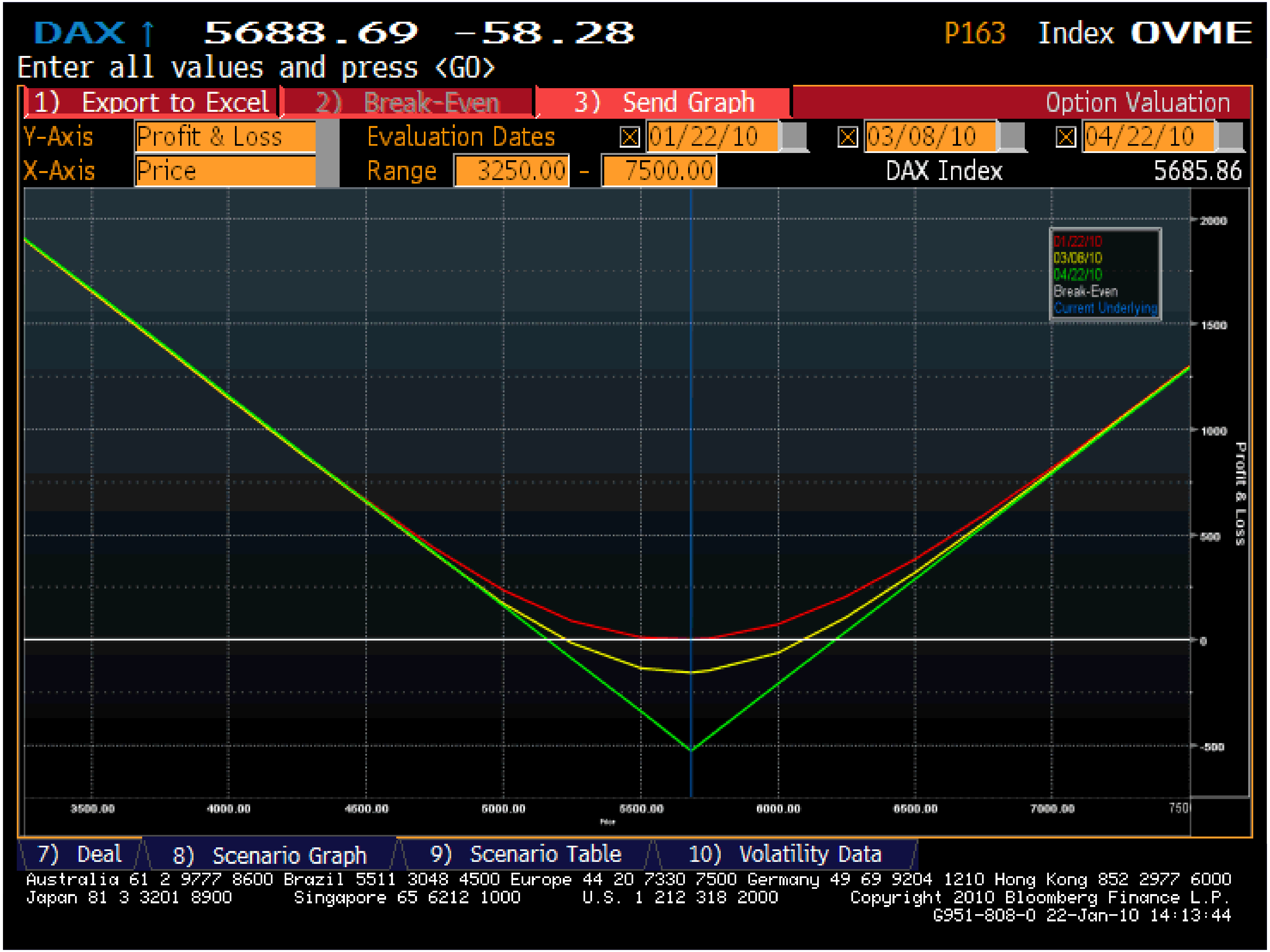
INTRODUCTION

Option-strategy: Straddle

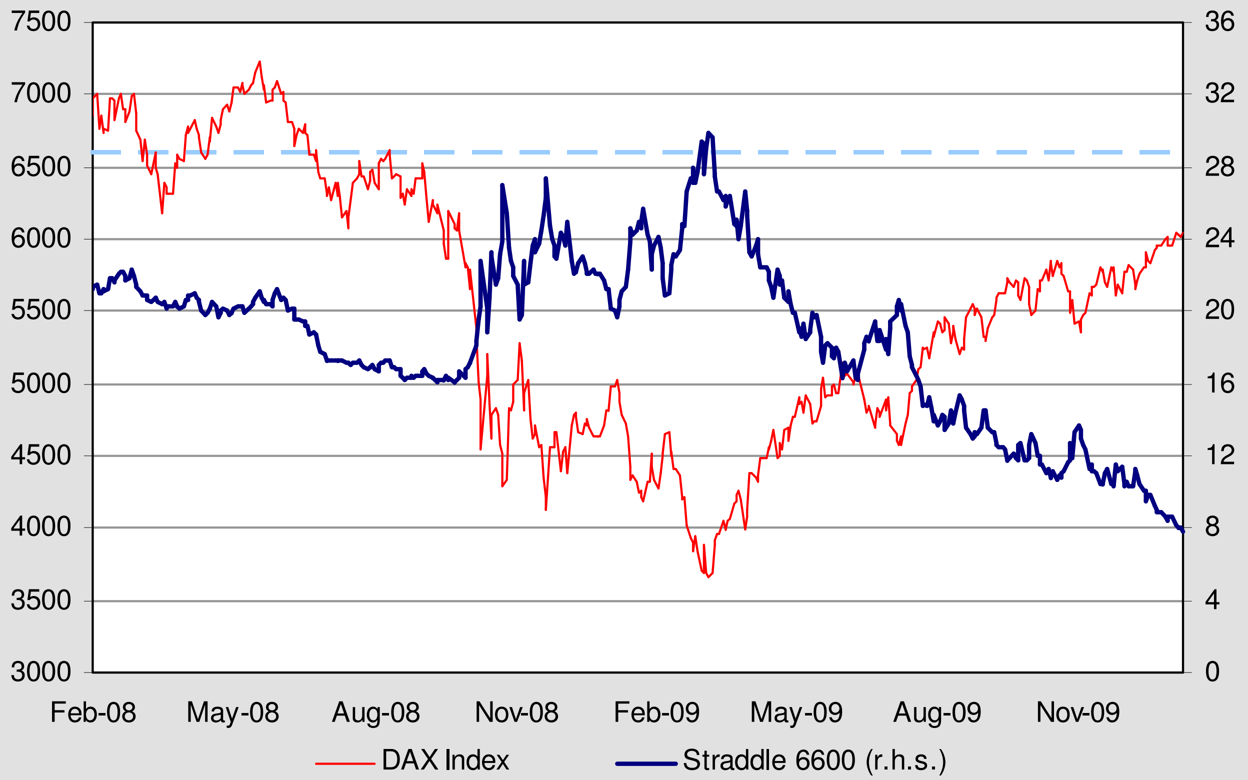
- $P_s = f\,[(S-K), \sigma^2]$



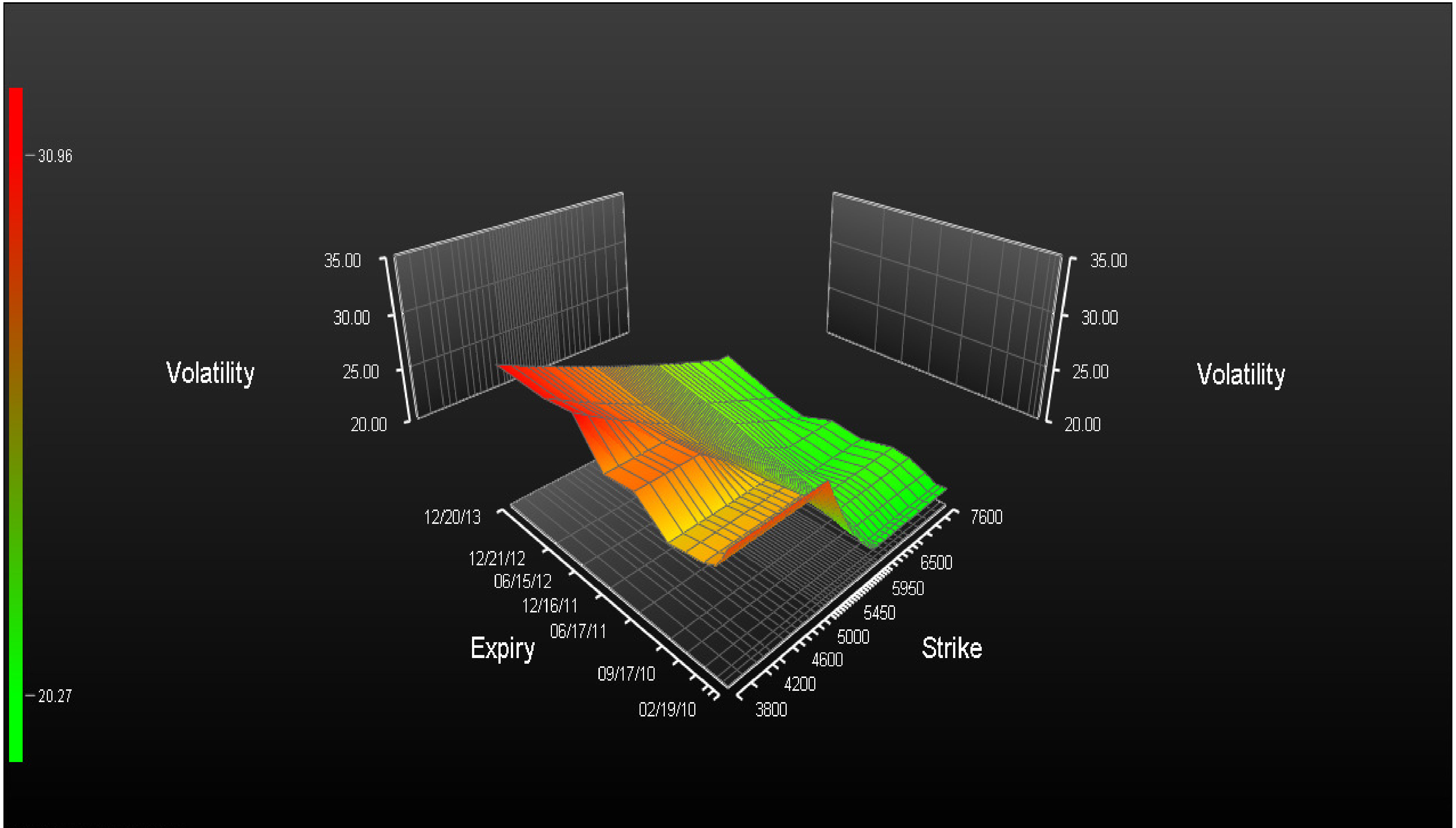
The combination of a straddle and a future contract with the same expiration date and future price = K allows trading a derivative which only depends on the underlying's σ^2


DAX ↑ 5688.69 -58.28
P163 Index OVME
Enter all values and press <GO>
1) Export to Excel
2) Break-Even
3) Send Graph
Option Valuation
Y-Axis
Profit & Loss
X-Axis
Price
Evaluation Dates
01/22/10
03/08/10
04/22/10
Range
3250.00
-
7500.00
DAX Index
5685.86
01/22/10
03/08/10
04/22/10
Break-Even
Current Underlying
2000
1500
1000
500
0
-500
Profit & Loss
3500.00
4000.00
4500.00
5000.00
5500.00
6000.00
6500.00
7000.00
750
7) Deal
8) Scenario Graph
9) Scenario Table
10) Volatility Data
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
G951-808-0 22-Jan-10 14:13:44

DATA
Straddle

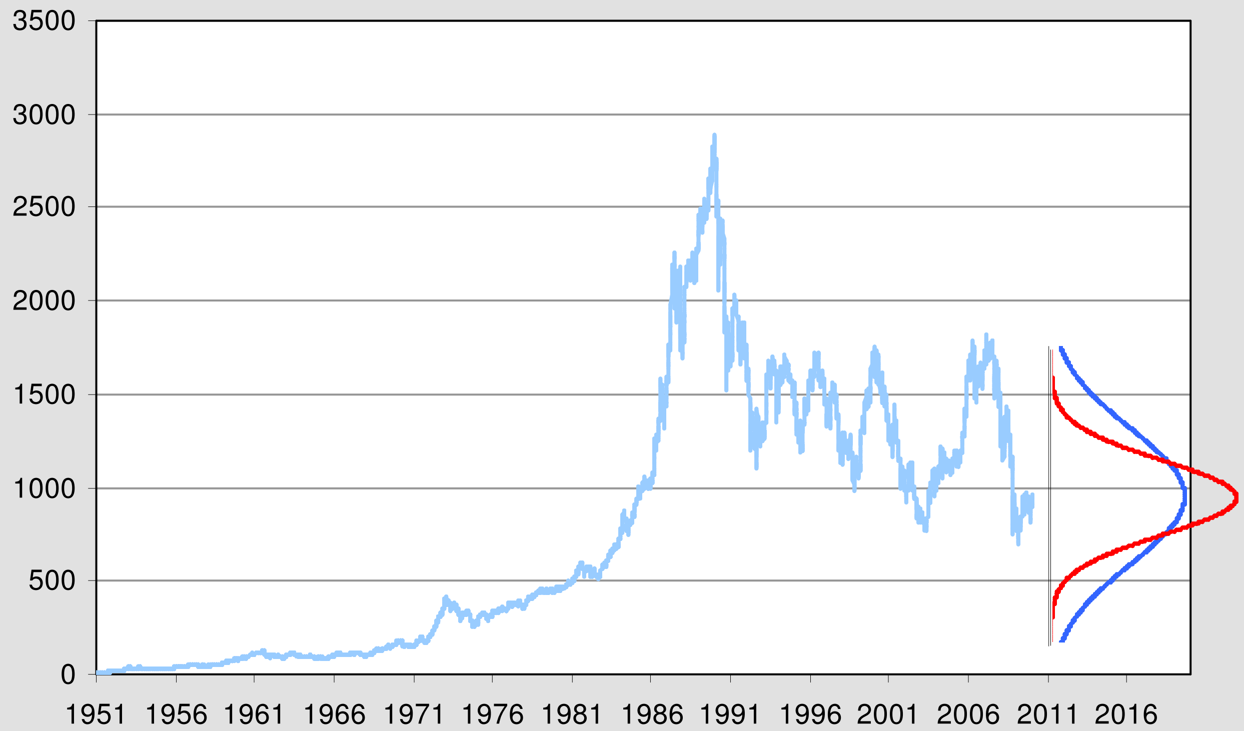


Source: Thomson Reuters, own calculations


30.96
20.27
Volatility
35.00
30.00
25.00
20.00
Volatility
35.00
30.00
25.00
20.00
12/20/13
12/21/12
06/15/12
12/16/11
06/17/11
09/17/10
02/19/10
Expiry
7600
6500
5950
5450
5000
4600
4200
3800
Strike

INTRODUCTION
Higher moment trading

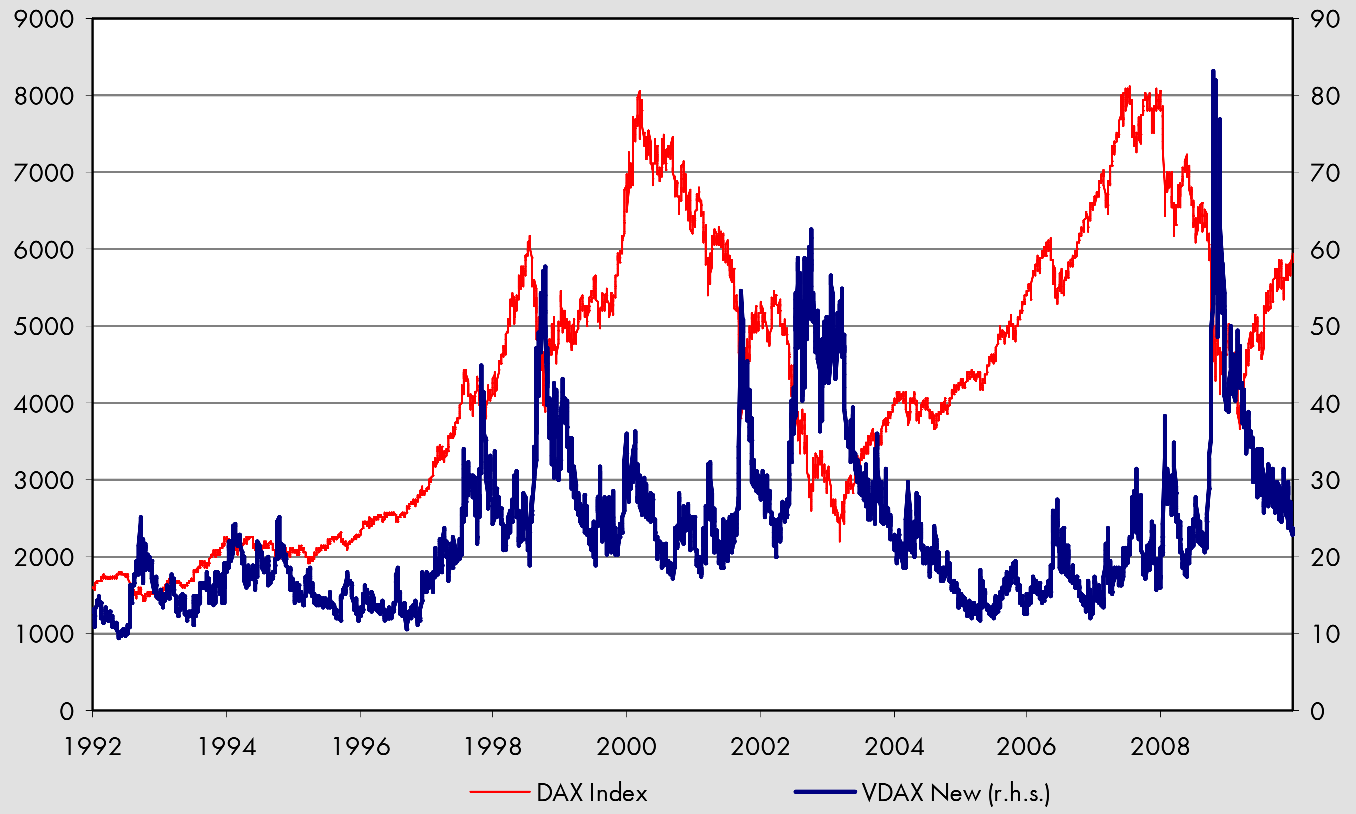
- Trading the variance/kurtosis (higher moment trading)



The combination of option strategies and future/foreward contract with the same expiration date allows trading a derivative which only depends on the underlying's σ^2

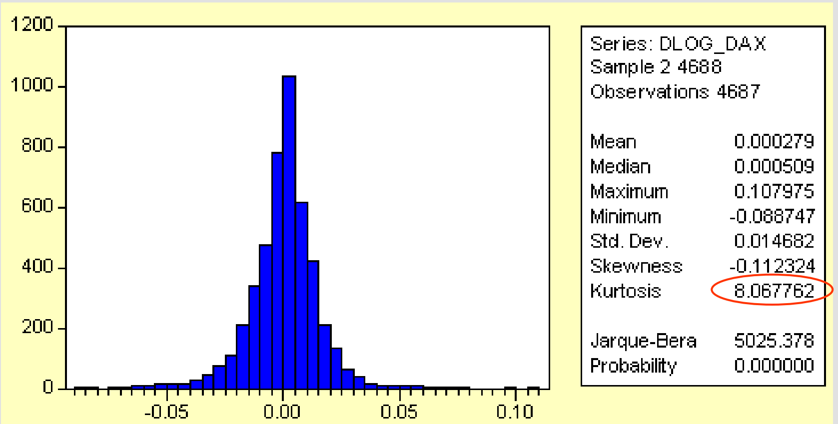
Source: Thomson Reuters, own calculations

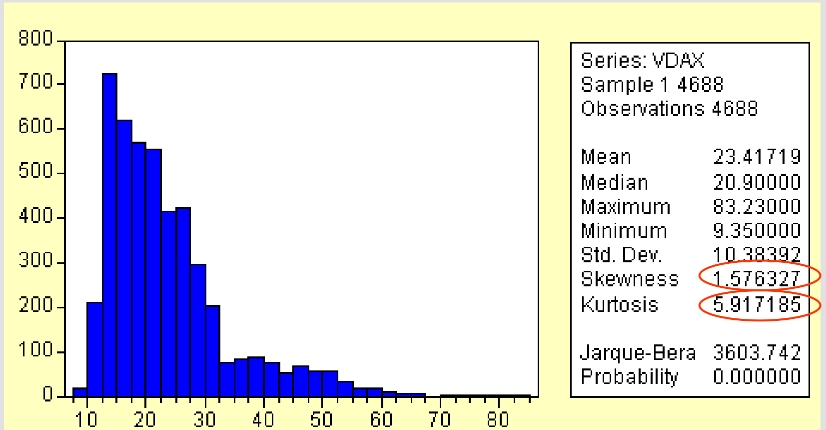
DATA
Volatility Index



Source: Thomson Reuters

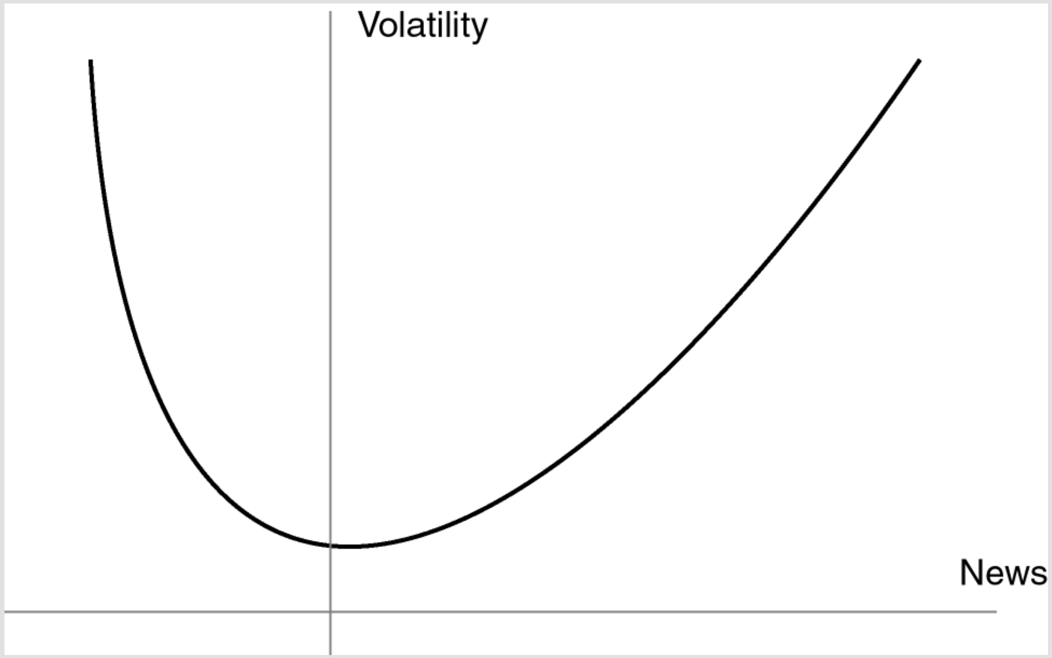
DATA
Substantial leptokurtosis





Source: own calculations

DATA
Skewed volatility

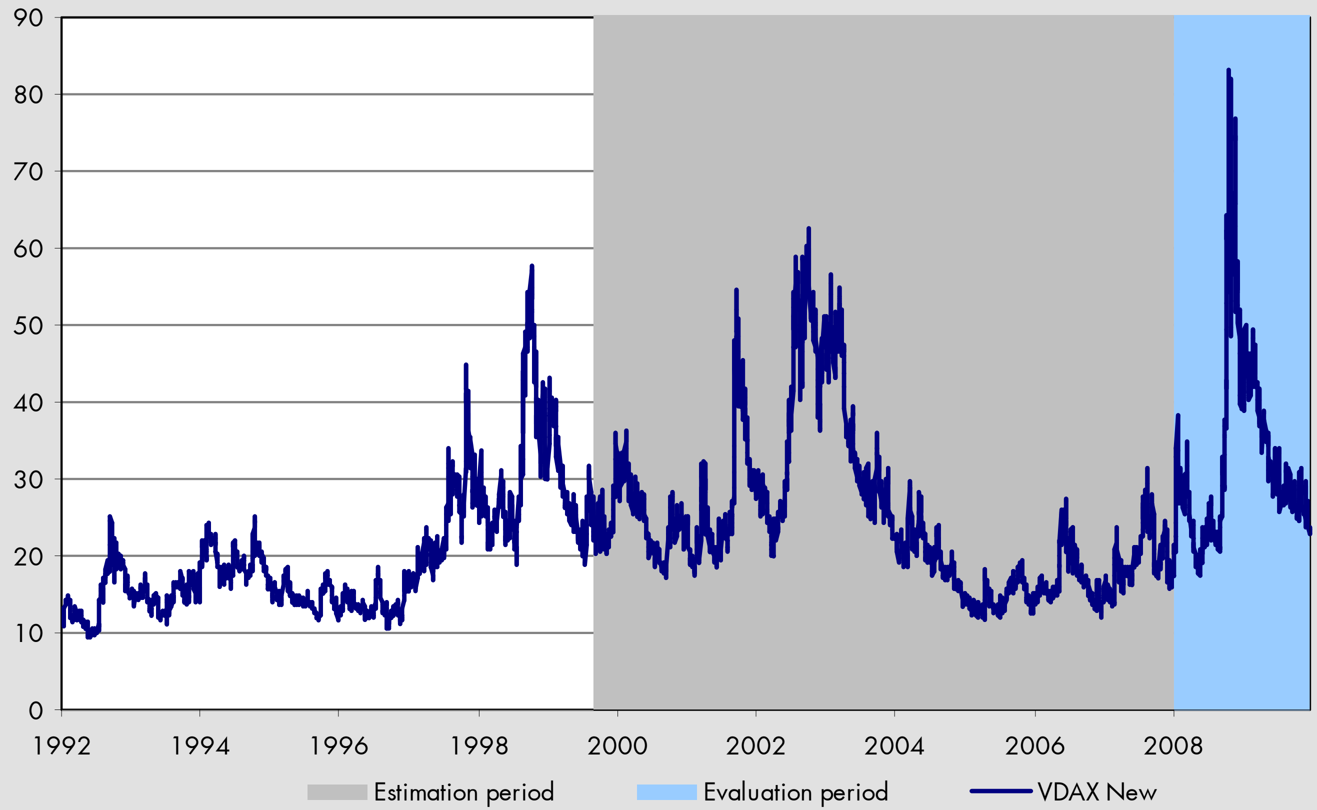


The news impact curve* for equity markets is typically skewed. Negative newsflow increases the volatility stronger than positive one.

* Engle and Ng (1993)

Source: Engle and Ng (1993)

Model
First learning, then doing



Source: Thomson Reuters

Model
ARCH

- ARCH (5)

$$y_t = x_t'\gamma + \varepsilon_t$$

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

DAX=C(1)+C(2)*DAX(-1)

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2 + C(7)*RESID(-4)^2 + C(8)*RESID(-5)^2

Model
Results

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	3.119912	5.147713	0.606077	0.5445
C(2)	1.000033	0.001030	971.1509	0.0000
		Variance Equation		
C	1405.852	141.7636	9.916876	0.0000
ARCH(1)	0.071265	0.025512	2.793369	0.0052
ARCH(2)	0.164289	0.032453	5.062313	0.0000
ARCH(3)	0.179569	0.031037	5.785610	0.0000
ARCH(4)	0.189750	0.033745	5.623094	0.0000
ARCH(5)	0.177770	0.039528	4.497356	0.0000

R-squared	0.997619	Mean dependent var	5284.363
Adjusted R-squared	0.997611	S.D. dependent var	1491.732
S.E. of regression	72.91047	Akaike info criterion	11.21215
Sum squared resid	11519633	Schwarz criterion	11.23306
Log likelihood	-12185.21	Durbin-Watson stat	2.045341

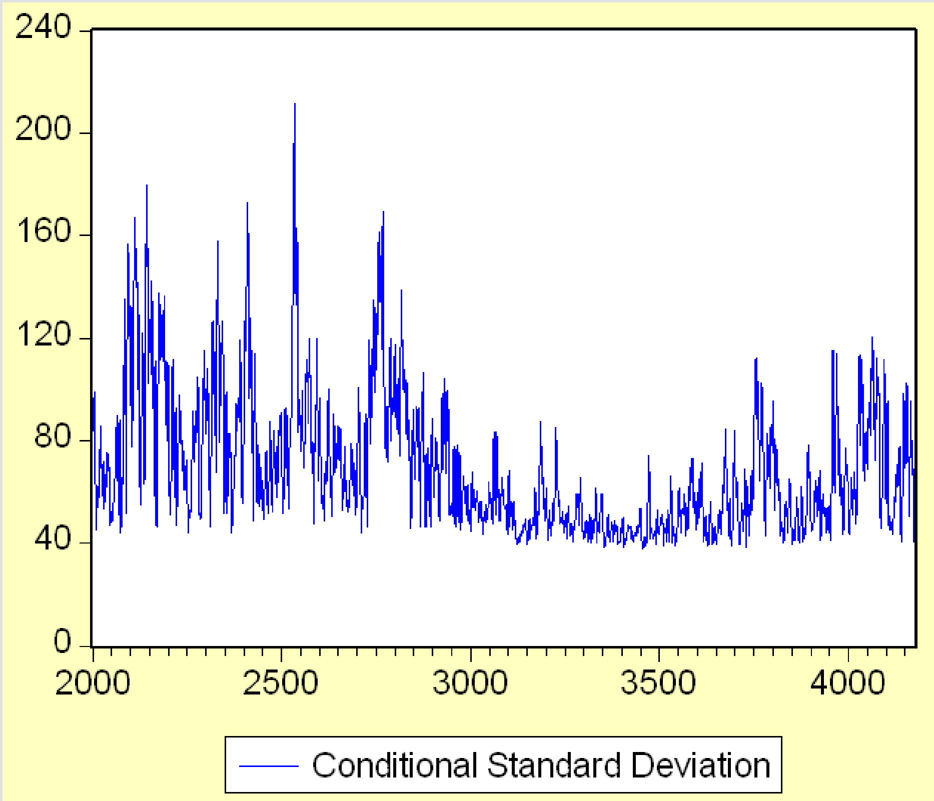
Dependent Variable: DAX
Method: ML - ARCH (BHHH)
Date: 01/20/10 Time: 16:36
Sample: 2000 4174
Included observations: 2175
Convergence achieved after 15 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

DAX=C(1)+C(2)*DAX(-1)

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2 + C(7)*RESID(-4)^2 + C(8)*RESID(-5)^2

Source: own calculations

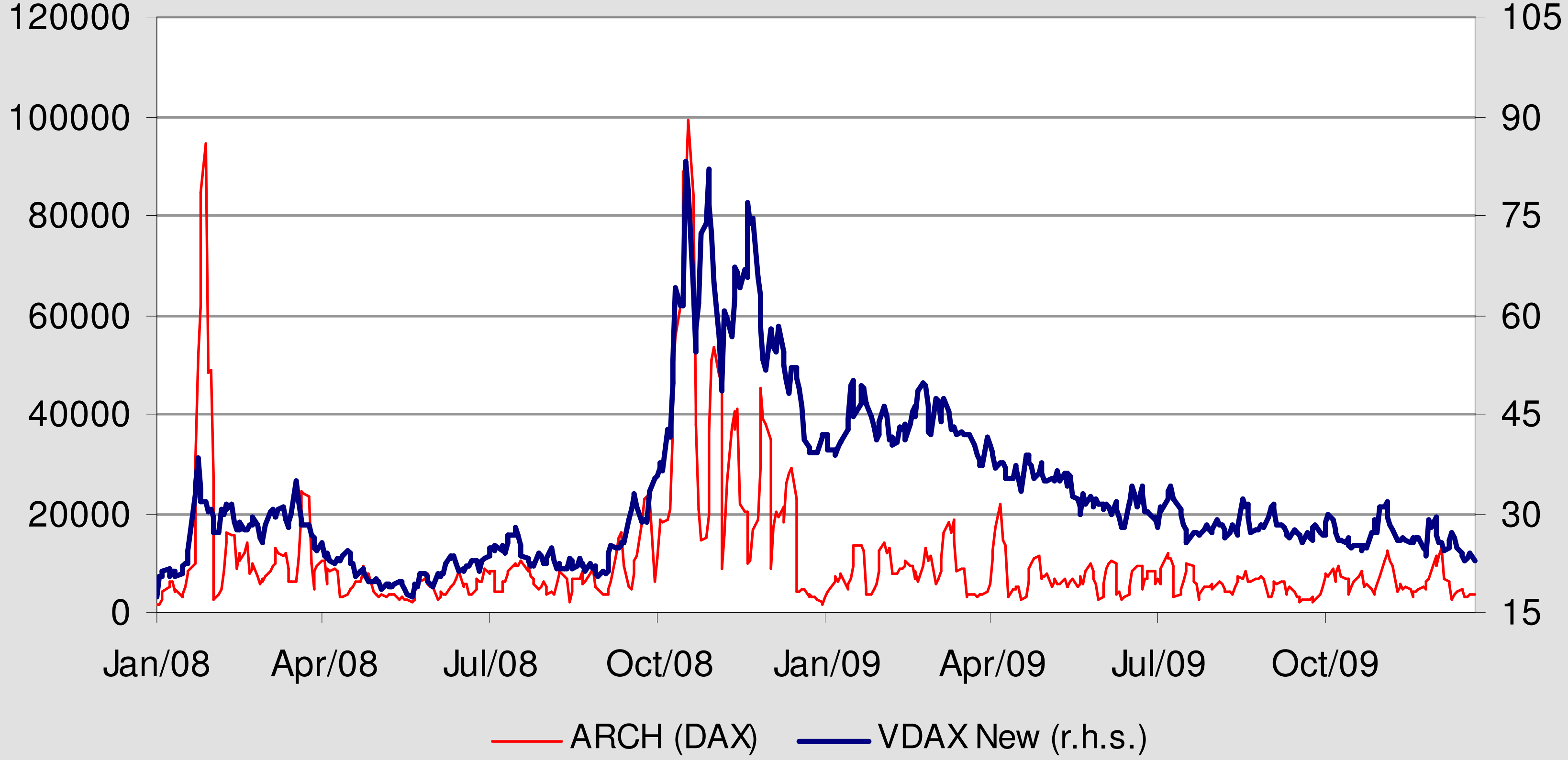
Model
Estimated standard deviation



Source: own calculations

Model

Better in 2008



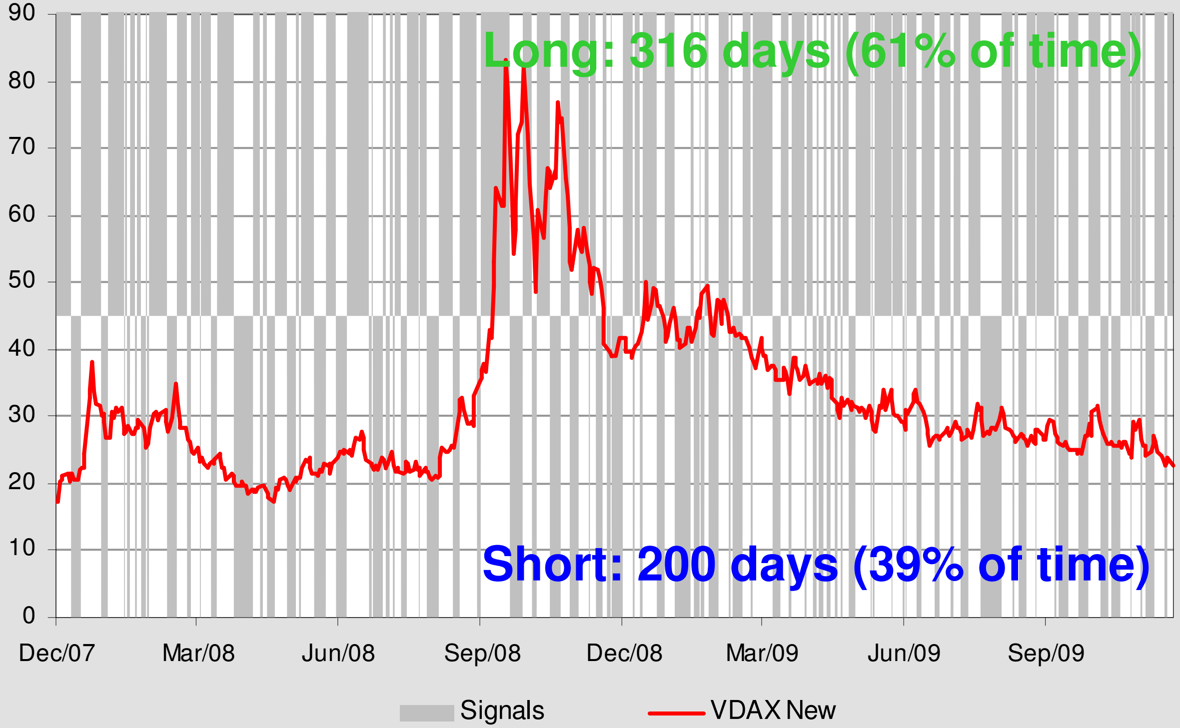
Source: Thomson Reuters, own calculations

Evaluation
The trading rule

- Evaluation period: 02/01/2008 – 21/12/2009 (515 trading days; daily market close of VDAX New)
- XOR strategy: always invested; either long or short, never neutral
- All in trading: always fully invested
- Long variance if the model forecasts the next day's variance to increase, short variance if the model forecasts a decrease
- Stay long or short as long as the model tells you to

Evaluation

Trading signals

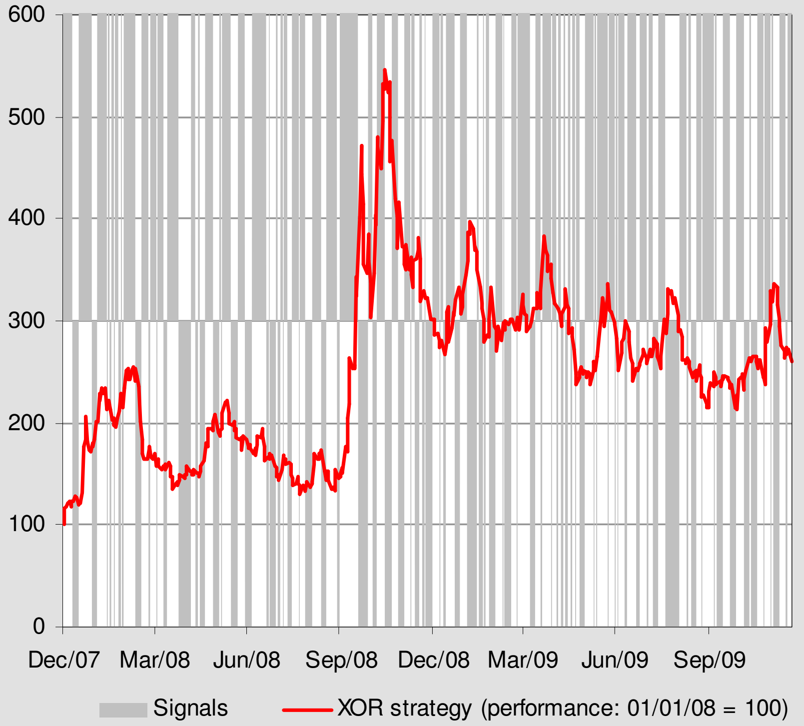


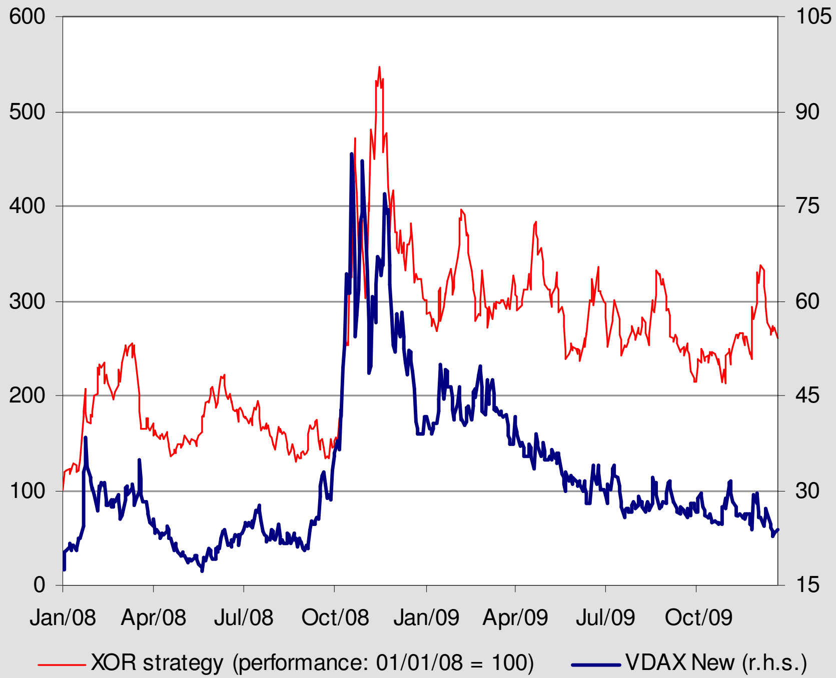
152 trades

1 trade every 3.4 days (≈ one trade a week)

Source: Thomson Reuters, own calculations

Evaluation
Performance: better in 2008





Source: Thomson Reuters, own calculations

Evaluation

Too good to be true?

	2008	***2009***	***Total***
Performance (in % p.a.)	202	-14	61
Maximum (in % p.a.)	447	31	447
Minimum (in % p.a.)	0	-30	0
Sharpe ratio	**30**	**-3**	**10.4**

Sharpe (1966) ratio $$S = \frac{R - R_f}{\sigma} = \frac{E[R - R_f]}{\sqrt{\mathrm{var}[R - R_f]}}$$

Source: own calculations

Conclusion
Too good to neglect

- The ARCH model is able to produce fairly realistic results for implied volatilities of options.
- The applied trading strategy (XOR) based on ARCH(5) generates an impressive performance.
- However, most of this performance is due to the strong increases in volatility during 2008. A simple buy-and-hold would have resulted in a performance of 141% p.a. (XOR: 202% p.a.)
- 2009 XOR fails (-14% p.a.). However, a buy-and-hold would have resulted in a performance of -50% p.a.
- The strategy is biased towards a long-position because the implied volatility of options is heavily skewed.
- There seems to be a structural break in the data.
- More dynamic estimation methods (rolling estimation window) may overcome these shortfalls.
- TARCH and other asymmetric ARCH models promise to be useful. These models adjust for the observed asymmetric shocks to volatility, which could increase the performance during an environment of decreasing volatility.

REFERENCES

Black, Fischer and Myron Scholes (1973). *The Pricing of Options and Corporate Liabilities.* Journal of Political Economy, 81, 637-654.

Engle, Robert and Viktor Ng (1993). *Measuring and Testing the Impact of News on Valatility*. Journal of Finance, 84, 1022-1082.

Sharpe, William (1966). *Mutual Fund Performance*. Journal of Business, 119-138.

APPENDIX

Option strategies