A combined nonparametric test for seasonal unit roots

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Outline

1 Introduction

2 Visualization

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Testing for unit roots and for seasonal unit roots

Unit-root tests assist in allotting time-series variables into the classes I(0) and I(1). For I(0), shocks have no long-run persistence. For I(1), shocks are persistent. The random walk is a basic I(1) process. Many economic variables (GDP etc.) are well modeled by I(1).

Seasonal unit-root tests assist in allotting variables into classes with persistent changes of seasonal patterns and with basically time-constant seasonal patterns. The seasonal random walk is a basic seasonal unit-roots model. Many economic variables do not have exactly pattern-reverting seasonality. Convincing examples for persistent shape changes (seasonal unit roots) are rare, however.

Properties of the two classes

Processes with *deterministic seasonality* rarely change the main qualitative features of their seasonal patterns. Even when they do so, patterns return to their original shapes ('summer remains summer, winter remains winter'): *pattern reversion*;

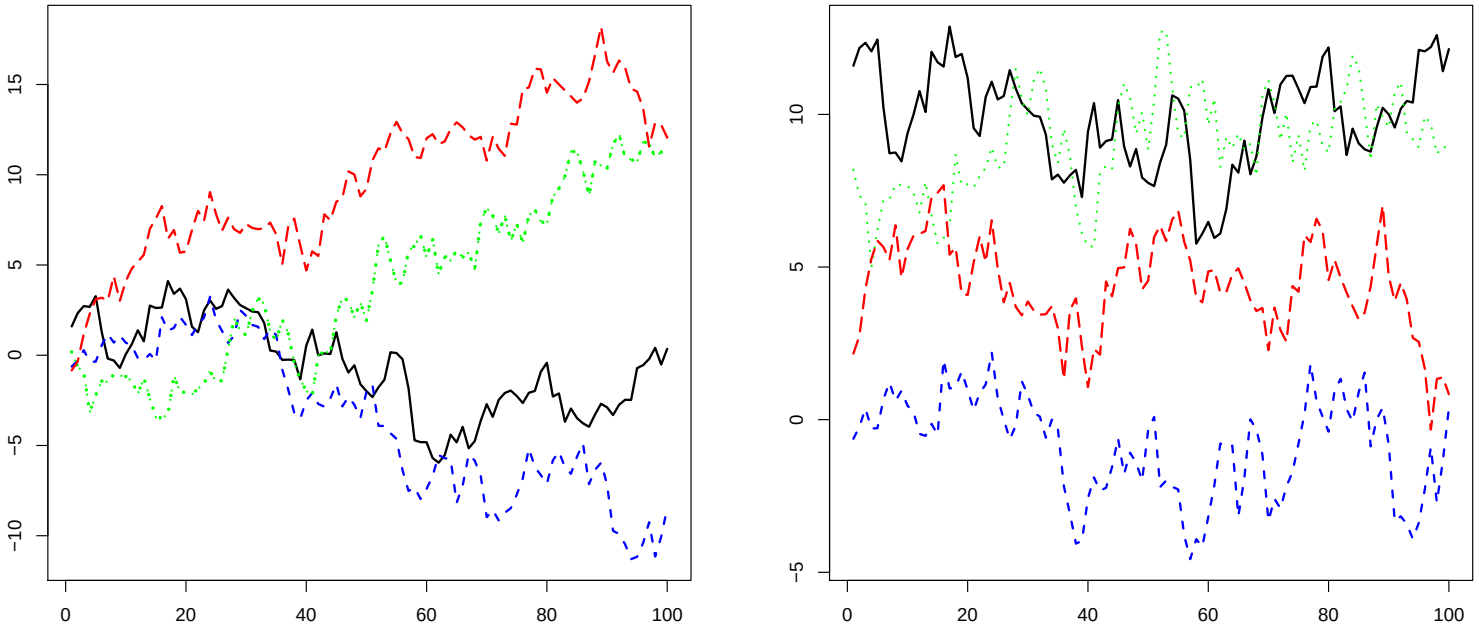
Processes with *seasonal unit roots* rarely change the main qualitative features of their seasonal patterns. If they do so, patterns often will not return to original shapes ('winter becomes summer'): *pattern persistence*;

Processes with weak seasonality frequently change the main qualitative features.

Test developments in unit roots and in seasonal unit roots

	unit roots	seasonal unit roots
parametric	Dickey & Fuller (1979) Phillips & Perron (1988)	Hylleberg, Engle, Granger & Yoo (1990)
multivariate	Engle & Granger (1987) Johansen (1988)	Lee (1992) Johansen & Schaumburg (1999)
inverse design	Kwiatkowski, Phillips, Schmidt & Shin (1992)	Canova & Hansen (1995) Caner (1998)
variance ratios	Breitung (2002)	Taylor (2005)
nonparametric	Aparicio, Escribano & Sipols (2006) Burridge & Guerre (1996)	Kunst & Franses (2011) this paper

Traditional visualization: time plots per quarter



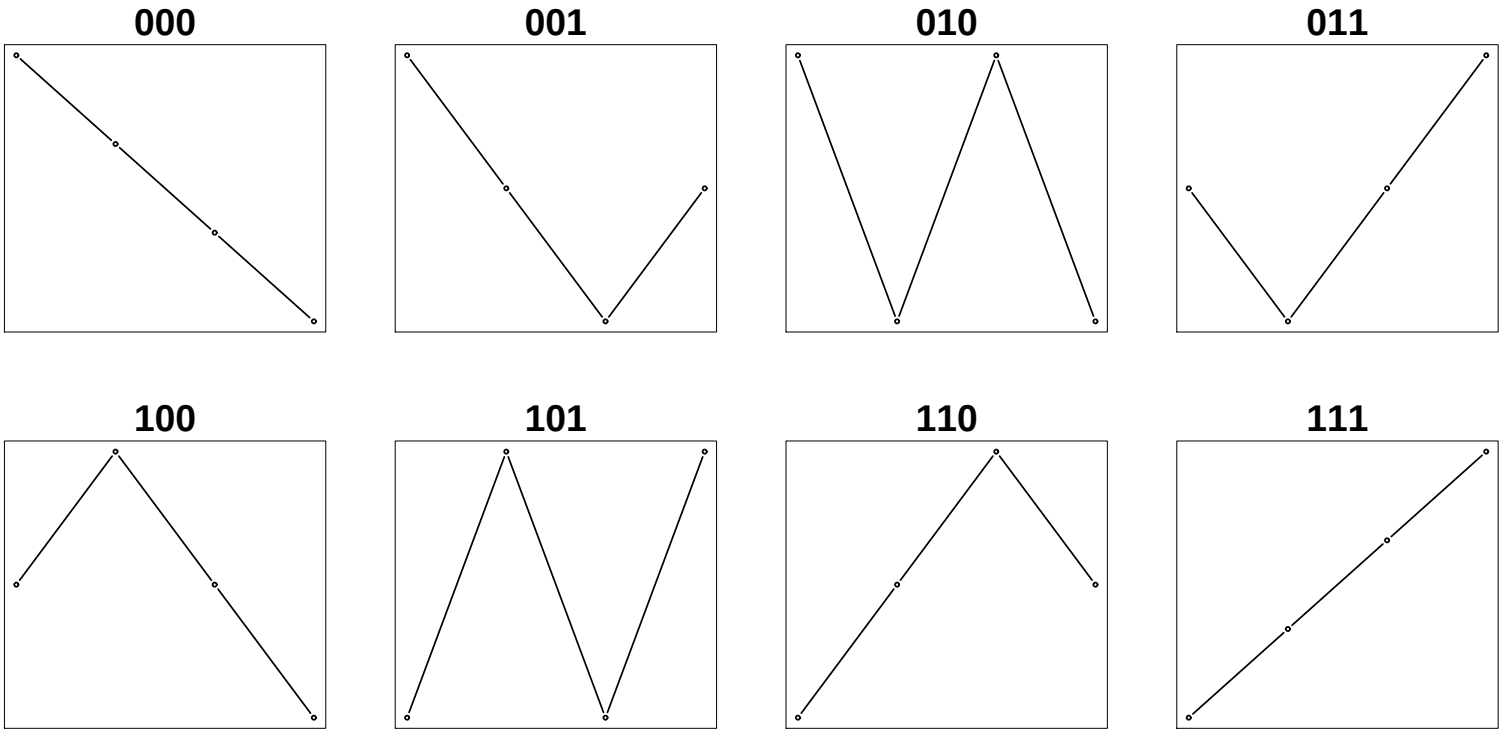
Time plots per quarter for a Gaussian seasonal random walk $x_t = x_{t-4} + \varepsilon_t$ (SRW, left) and a deterministic seasonal process (right).

Return to the original problem

The feature of concern is not really the existence of seasonal unit roots. Rather, it may be of interest whether a change in the seasonal pattern precedes a permanent shift to a different shape or not.

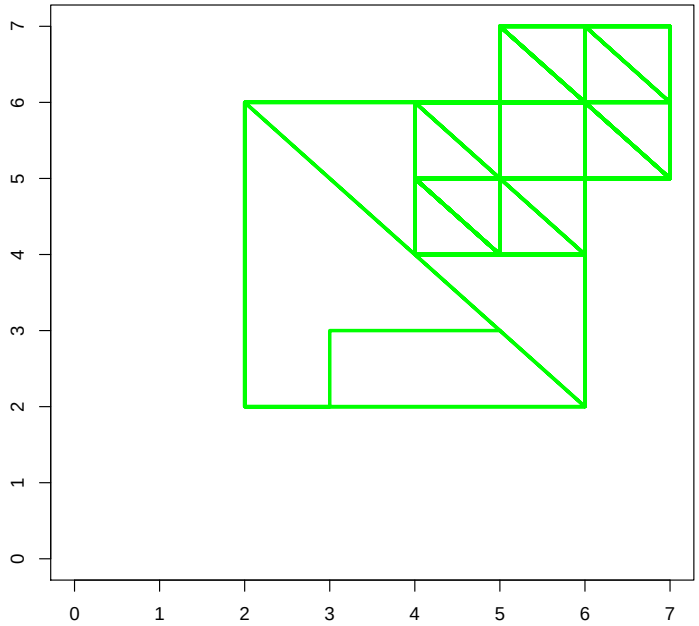
It may thus be convenient to focus on the discretized notion of distinctive seasonal shapes and their transition. The concomitant visualization may assist in the discrimination problem, even in cases where hypothesis tests do not offer a clear conclusion.

The eight seasonal patterns for quarterly data



Rising (1) and falling (0) quarters yield binary representations of numbers 0 to 7. 8 not 16 classes: trend not in focus.

Discrete phase plots



Phase diagram for pattern classes. Generating model is a quarterly Gaussian SRW for 10,000 years. The visualization is unsatisfactory.

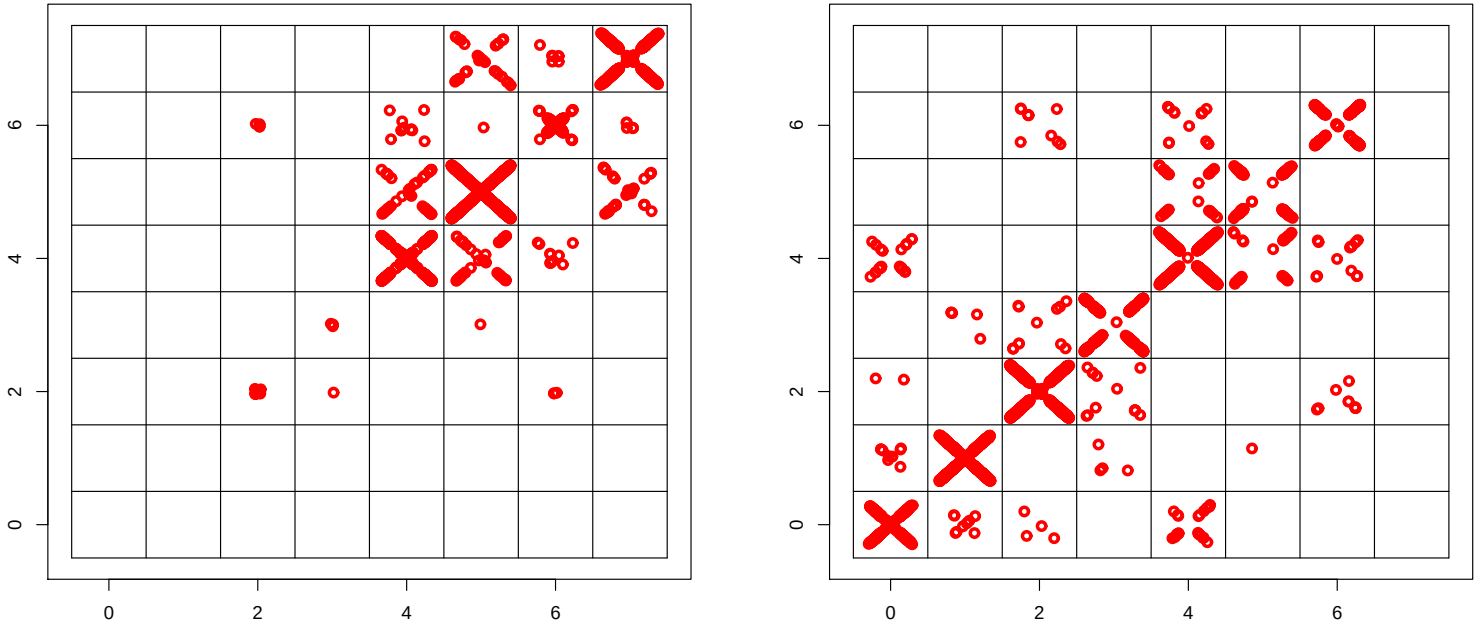
Jittering the phase plots

Observations are not just allotted to the bins $m = 0, \ldots, 7$ but are spread over the intervals $[m - 0.4, m + 0.4]$ according to the following rule:

- Observations are allotted uniformly $(0.5 : 0.5)$ to either $[m - 0.4, m)$ or to $(m, m + 0.4]$;
- Within the intervals, the outer limits $m \pm 0.4$ are set by the maximum 'depth' (largest increase or decrease in any quarter), and points are positioned relative to this maximum.

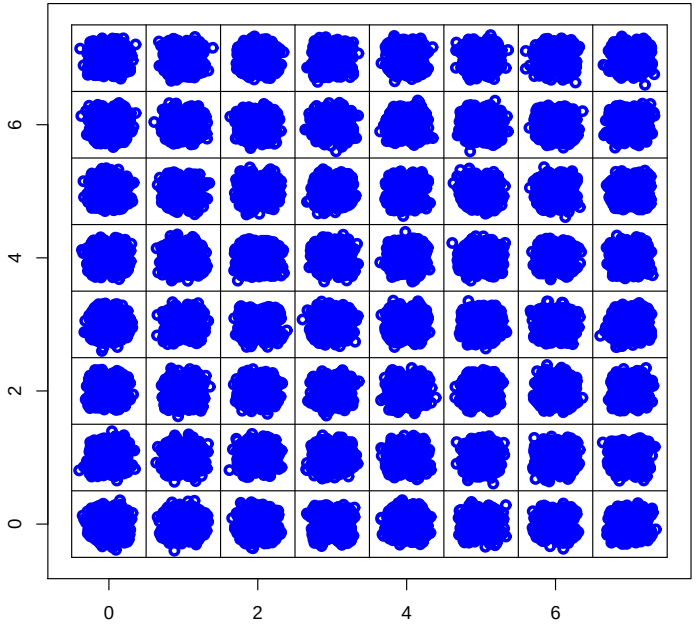
Shallow points are in the centers of the bins, deep points are in the extremes. For seasonal random walks, classes are entered through centers, and corners are approached as the residence in a bin continues.

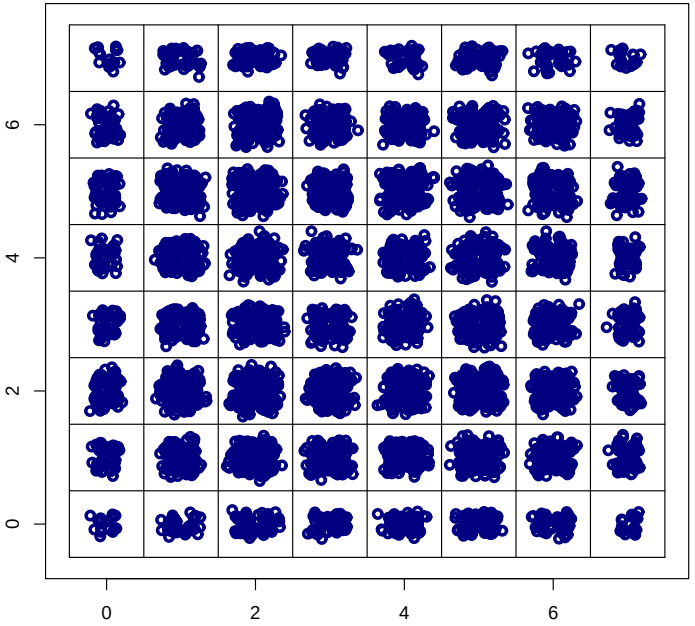
Jittered phase plots of seasonal random walks



10,000 years of $x_t = x_{t-4} + \varepsilon_t$, Gaussian errors.

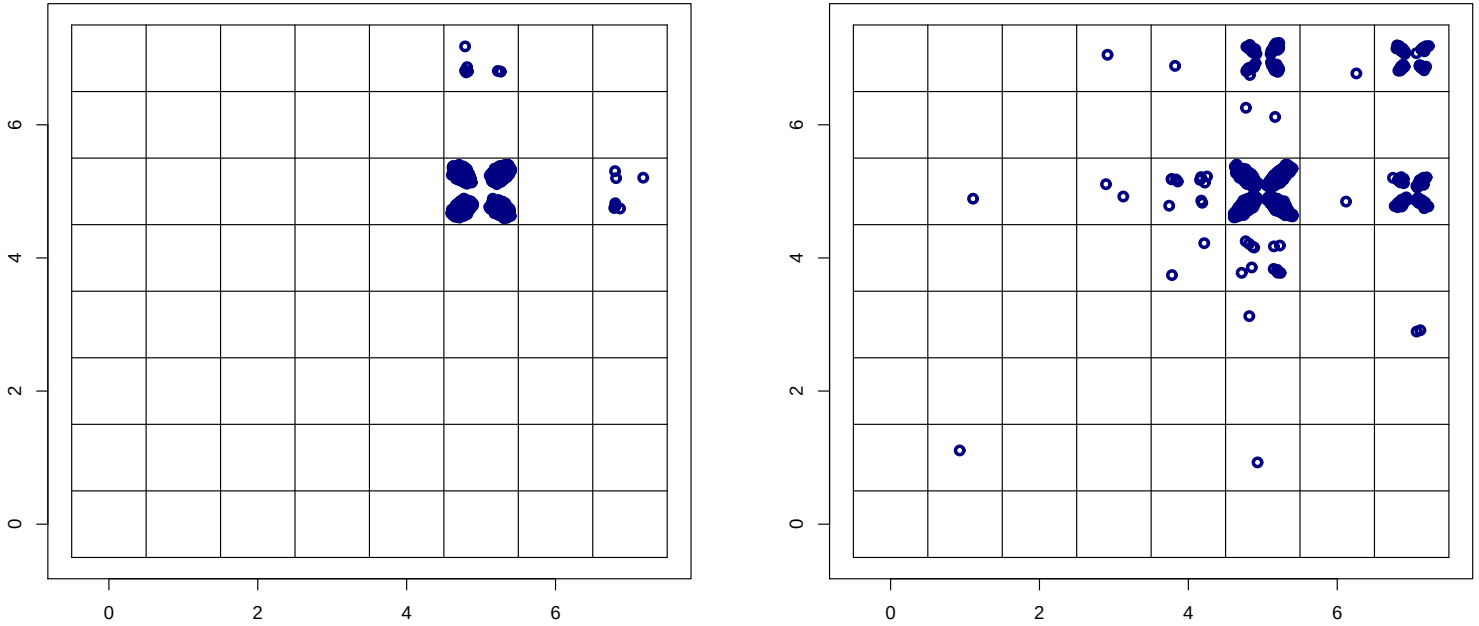
Jittered phase plot of non-seasonal processes





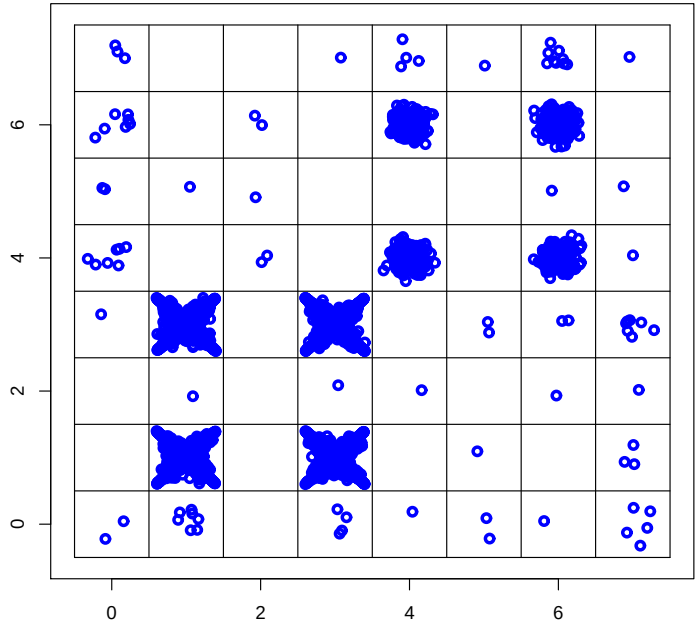
Random walk and white noise.

Jittered phase plot of deterministic seasonality



Generating process is $x_t = 0.4x_{t-4} + \sum_{j=1}^{4} d_t + \varepsilon_t$, with $(d_1, \ldots, d_4) = (0, 8, 3, 10)$.

Jittered phase plot for a periodically integrated process



Generating model is $x_t = \phi_s x_{t-4} + \varepsilon_t$ with $(\phi_1, \ldots, \phi_4) = (1, 0.25, 1, 4)$.

What can be learnt from the jittered plots?

- Processes with seasonal unit roots generate X shapes in the bins (St. Andrew's crosses, saltires), other processes generate blurred crosses or blotches;
- Without jittering, the saltires are replaced by slashes, which is less attractive visually;
- The monthly version has too many classes (2,048), such that the visual impression within the bins is lost.

Two ideas for nonparametric tests

- Count the transitions between pattern classes;
- Consider the average distance from the saltire.

These ideas are reflected in statistics ζ_1 and ζ_2. These are generalizations of non-parametric unit-root test statistics to the seasonal case: the crossings count by Burridge & Guerre and the range test by Aparicio, Escribano & Sipols (2006, AES).

The pros and cons of non-parametric tests

Non-parametric tests do not build upon a likelihood model. If such a likelihood model is valid, then the corresponding LR/LM/Wald tests are more powerful than any non-parametric rival.

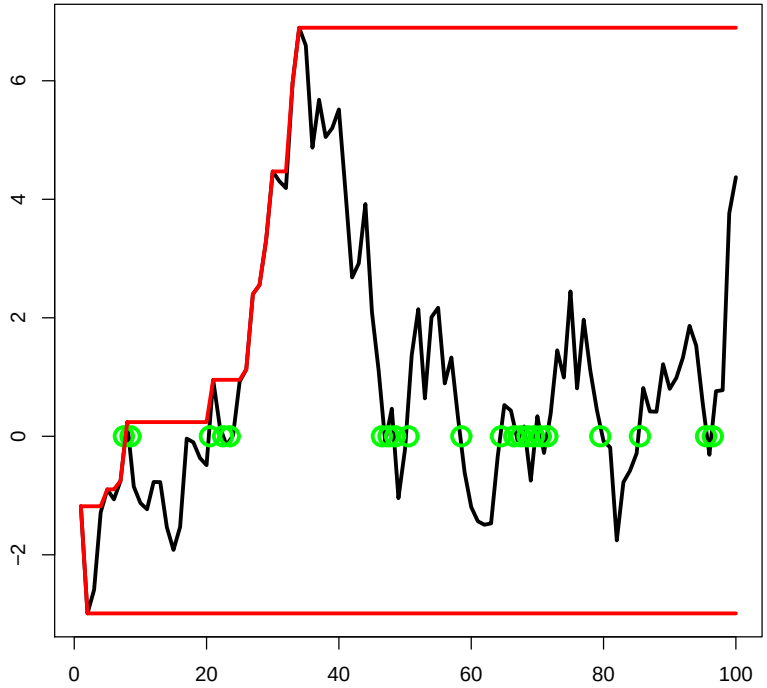
If the likelihood model is invalid, non-parametric tests may still work, while likelihood-based tests may suffer. For example, AES show that their test is robust to structural breaks, to outliers, and to monotonic transformations of variables.

For seasonal unit-root tests, these features may be of particular interest. For example, unemployment rates are known not to behave like I(0/1) processes, while their seasonal cycles are very pronounced.

Findings on pros and cons in the literature

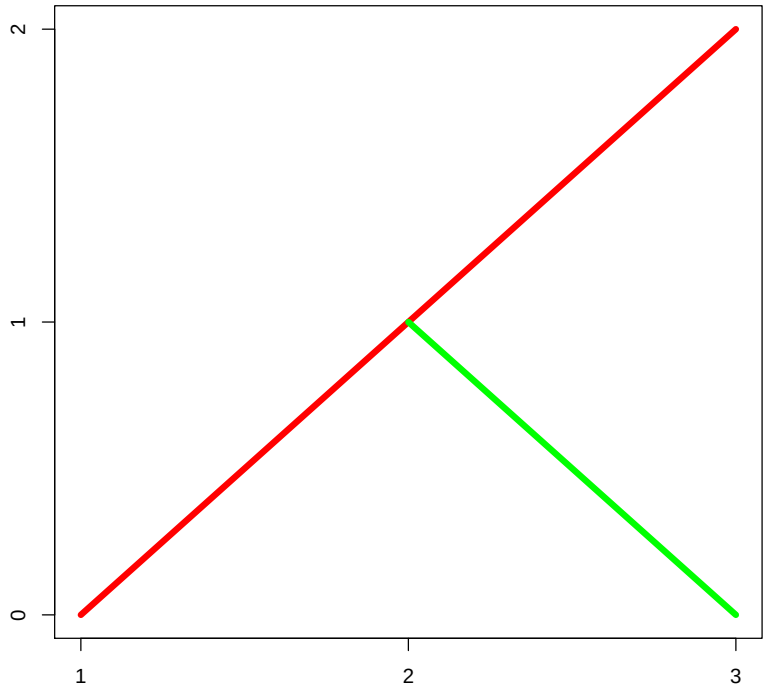
- AES find that their test successfully competes with parametric tests of the Dickey-Fuller type. In many standard (!) designs, it has higher power than the parametric rivals;
- Burridge & Guerre find their level-crossings test to have low power and provide a skeptical viewpoint on non-parametric unit-root tests;
- All such findings concern non-seasonal unit-root tests;
- Kunst & Franses find low power in most designs but succeed in identifying higher power in an empirically relevant design for a non-parametric seasonal unit-root test.

Non-moment characteristics of random walks



The frequency of new extrema (expansion points of the range) and the number of zero passages are typical for random walks. These statistics are invariant under some monotonic transformations.

ζ_1 and ζ_2 appear to be inversely related



For a discrete-valued random walk, if the first increment is positive, there are either two range expansions or one expansion and one zero crossing. For larger T, however, negative correlation between ζ_1 and ζ_2 is small.

The transition count statistic ζ_1

The statistic ζ_1 counts pattern transitions. It is closely related to the level-crossings test by Burridge & Guerre (1996). For seasonal random walks x_t, $x_{4t-j}, j = 0, \ldots, 3$ are random walks, and so are their differences and sums. When they cross levels, the pattern class changes.

The modified crossings count

$$K_T^*(0) = \frac{\hat{\sigma}}{\widehat{\mathrm{MAD}}} T^{-0.5} \sum_{t=1}^{T} I(X_{t-1} \leq x, X_t > x) + I(X_{t-1} > x, X_t \leq x)$$

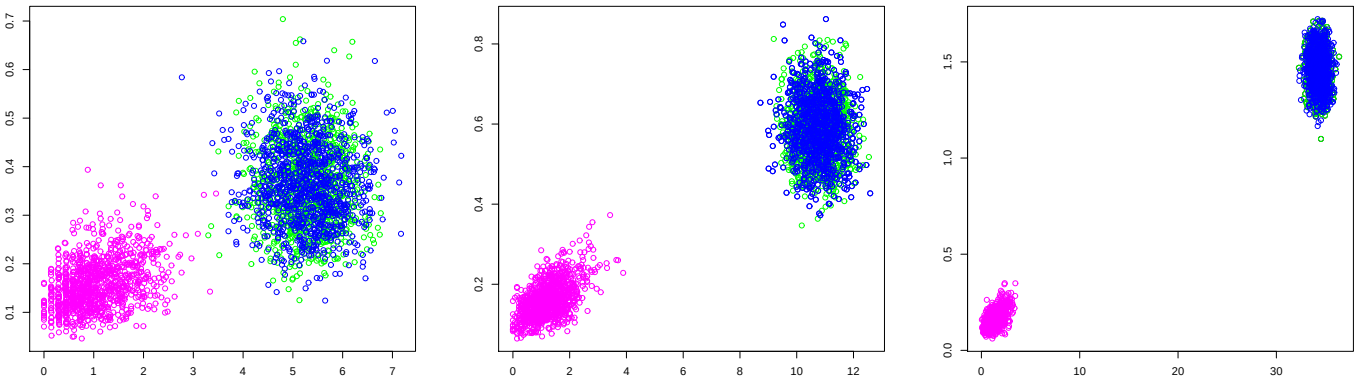
is asymptotically distributed as $|N(0,1)|$. $\hat{\sigma}$ and $\widehat{\mathrm{MAD}}$ are moments estimates for increments required for re-scaling. ζ_1 is defined in accordance with $K_T^*(0)$ and uses $3T/4$ instead of T.

The distance from the saltire ζ_2

The *median distance from the saltire* in graphical coordinates depends on the ratio of the increments to the extension of the bin, i.e. the extremum over the sample. For random walks and related processes, the extremum is known to expand at the rate of $T^{0.5}$. ζ_2 is defined as the median distance times $T^{0.5}$.

The asymptotic growth rate of random walk extrema has been used in non-parametric unit-root tests, e.g. AES. Its distribution is sensitive to autocorrelation in increments. In contrast to ζ_1 (which is corrected by a ratio of moment characteristics), it is invariant to distributional assumptions.

Visualization of discriminatory power

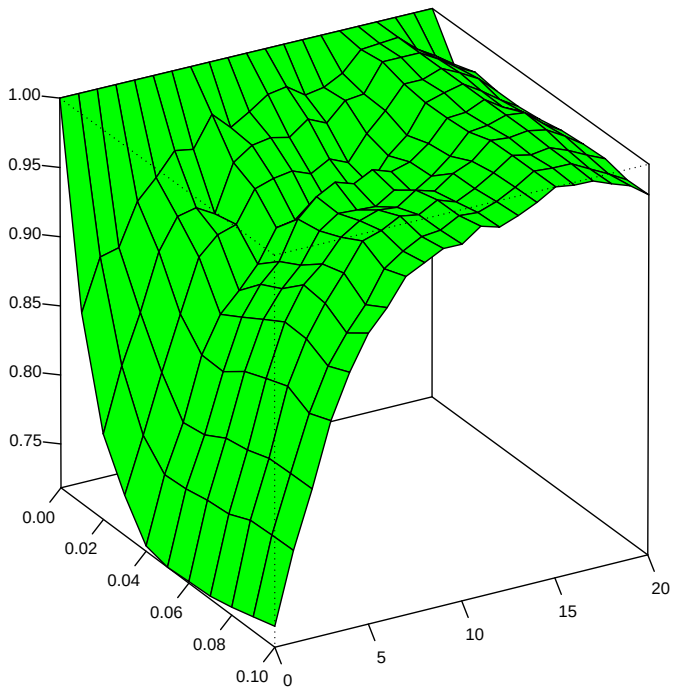


1000 realizations of the test statistics ζ_1 and ζ_2 based on a SRW (magenta), a random walk (blue), and a white noise (green). 25, 100, and 1000 years.

Weighted average of ζ_1 and ζ_2

The statistics ζ_1 and ζ_2 process different information. A combination such as $\zeta_1 + c\zeta_2$ may have higher power than individual tests. Relative scales would suggest $c = 7$, some simulations rather indicate $c = 17$.

Does a weight of 17 maximize test power?



Relative power depending on the weight c in $\zeta_1 + c\zeta_2$ for $T = 100$ and generating model $x_t = \phi x_{t-4} + \varepsilon_t$ with $\phi \in [0.9, 1]$. Weight c on the x–axis and $1 - \phi$ on the y–axis. Optimal c may be around 17. Note, however, that this is a special direction of alternatives.

Significance points for the tests

	$T = 100$			$T = 400$			$T = 4000$		
	1%	5%	10%	1%	5%	10%	1%	5%	10%
ζ_1	2.10	1.53	1.27	2.12	1.55	1.29	2.15	1.58	1.29
ζ_2	0.25	0.20	0.17	0.23	0.19	0.17	0.22	0.18	0.16
$\frac{\zeta_1+17\zeta_2}{18}$	0.32	0.25	0.22	0.31	0.25	0.22	0.31	0.24	0.21
$\tilde{\zeta}_1$	1.66	1.18	0.94	1.93	1.37	1.11	2.06	1.49	1.20
$\frac{\tilde{\zeta}_1+17\zeta_2}{18}$	0.29	0.22	0.19	0.30	0.22	0.19	0.29	0.23	0.19

Rejection frequencies at 25 years and at 100 years

	$T = 100$			$T = 400$		
ϕ	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*
1.00	0.050	0.050	0.050	0.050	0.050	0.050
0.99	0.074	0.083	0.086	0.184	0.268	0.267
0.98	0.102	0.121	0.131	0.367	0.575	0.577
0.97	0.140	0.171	0.188	0.548	0.798	0.806
0.96	0.183	0.229	0.253	0.712	0.914	0.927
0.95	0.229	0.283	0.319	0.825	0.965	0.978
0.94	0.279	0.339	0.388	0.895	0.986	0.993
0.93	0.326	0.394	0.452	0.940	0.995	0.998
0.92	0.377	0.446	0.520	0.967	0.997	0.999
0.91	0.427	0.497	0.585	0.984	0.999	1.000
0.90	0.473	0.544	0.643	0.992	1.000	1.000

Generating model is $x_t = \phi x_{t-4} + \varepsilon_t$. Significance level is 5%. $\phi = 1$ is the null.

Autocorrelated increments distort test properties

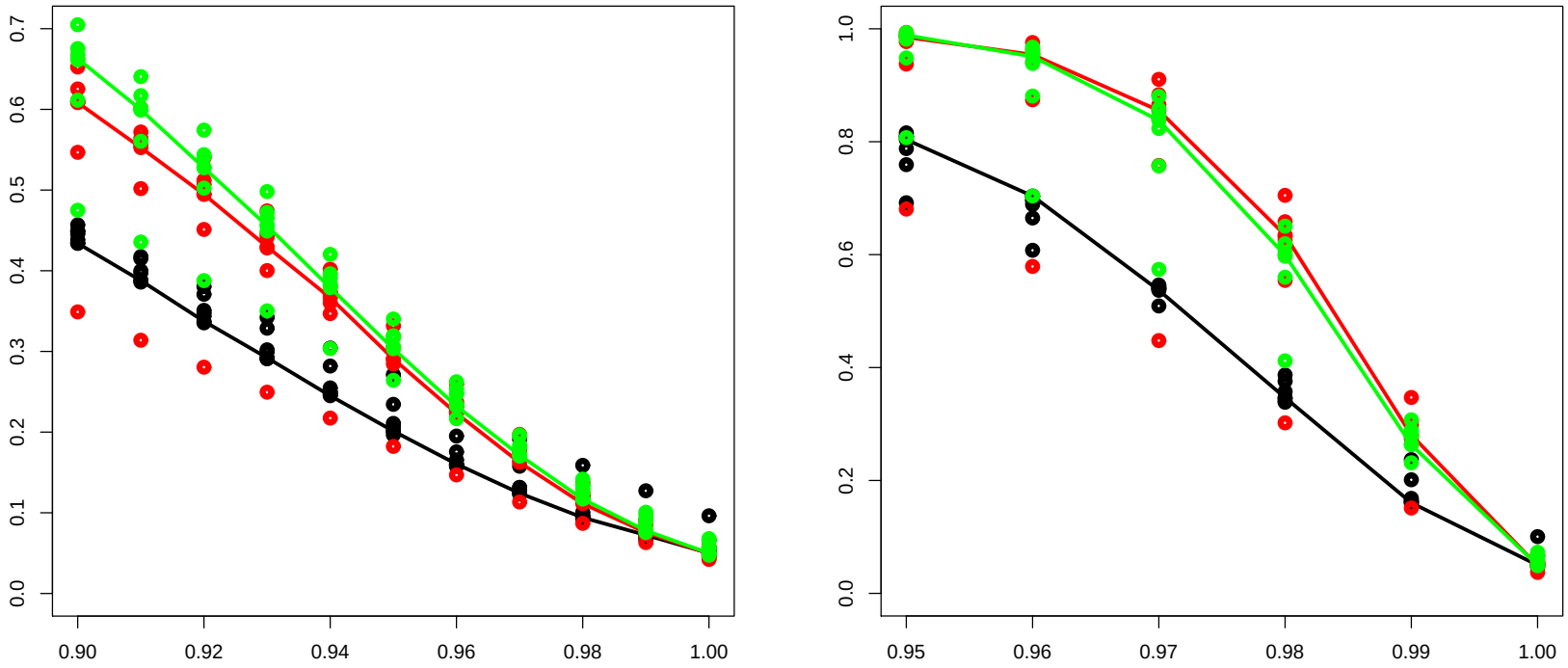
	$\theta = -0.5$			$\theta = 0$			$\theta = 0.5$		
ϕ	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*
1.0	0.041	0.042	0.043	0.050	0.050	0.050	0.044	0.035	0.036
0.98	0.084	0.099	0.101	0.102	0.121	0.131	0.083	0.096	0.094
0.96	0.137	0.185	0.190	0.183	0.229	0.253	0.143	0.195	0.193
0.94	0.204	0.288	0.296	0.279	0.339	0.388	0.227	0.314	0.324
0.92	0.283	0.390	0.406	0.377	0.446	0.520	0.325	0.444	0.467
0.9	0.364	0.485	0.510	0.473	0.544	0.643	0.431	0.555	0.600

Generating model is $x_t = \phi x_{t-4} + u_t$, $u_t = \theta u_{t-1} + \varepsilon_t$. $T = 100$. Significance level is 5%. $\phi = 1$ is the null.

The problem of autocorrelated increments

- Akonom (1993) shows that the level-crossings statistic can be made robust to autocorrelation in random-walk increments, replacing the variance by a long-run variance (spectrum at 0). García and Sansó (2006) use this result to robustify the nonparametric unit-root test of Burridge and Guerre. This result immediately transfers to the statistic $\tilde{\zeta}_1$ instead of ζ_1 for the seasonal test;
- The distribution of ζ_2 is sensitive to autocorrelation in increments, but there is no comparable generalization result. Even without adjustment, it is the more powerful component of the seasonality test.

Adjusted test statistic $\tilde{\zeta}_1$ mitigates nuisance influence



Power function for $T = 100$ and $T = 400$. Black: $\tilde{\zeta}_1$; red: ζ_2; green: ζ^*.

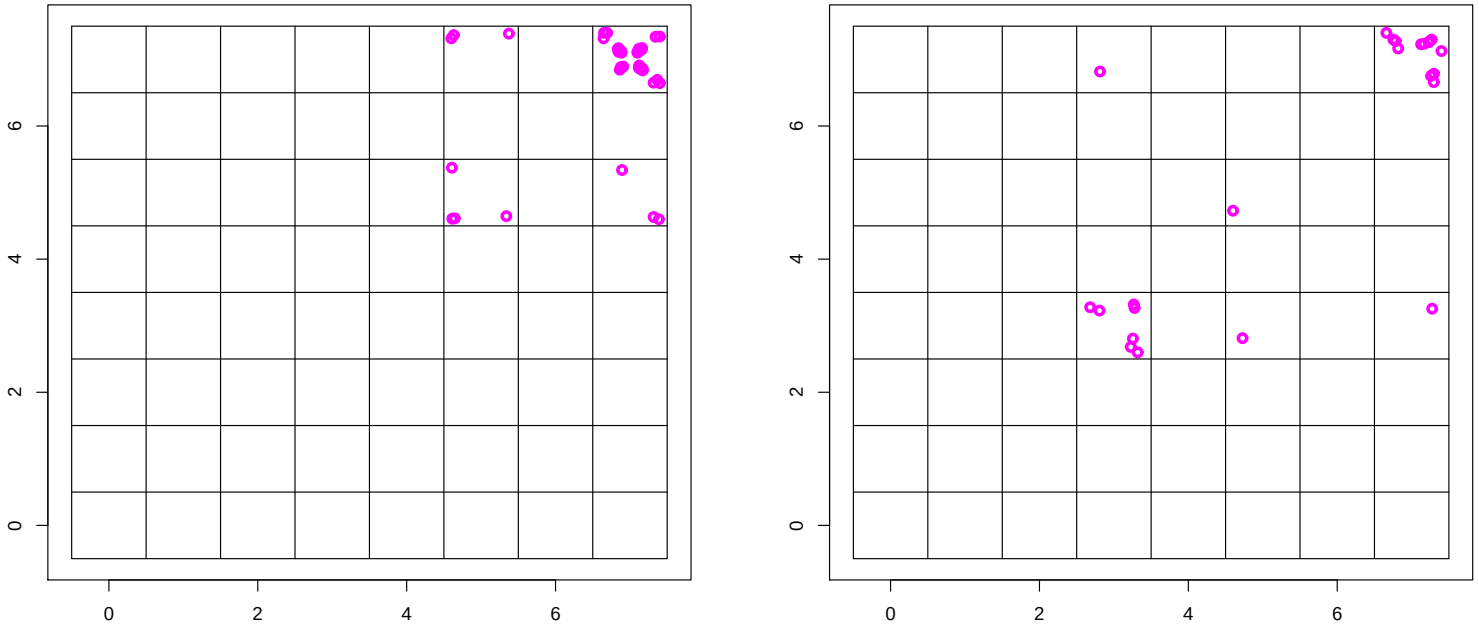
ζ_1 and ζ_2 convey different information

Potential arguments for *not* combining the tests ζ_1 and ζ_2:

	ζ_1 small	ζ_1 large
ζ_2 small	seasonal unit root	regime switching
ζ_2 large	deterministic season	no seasonality

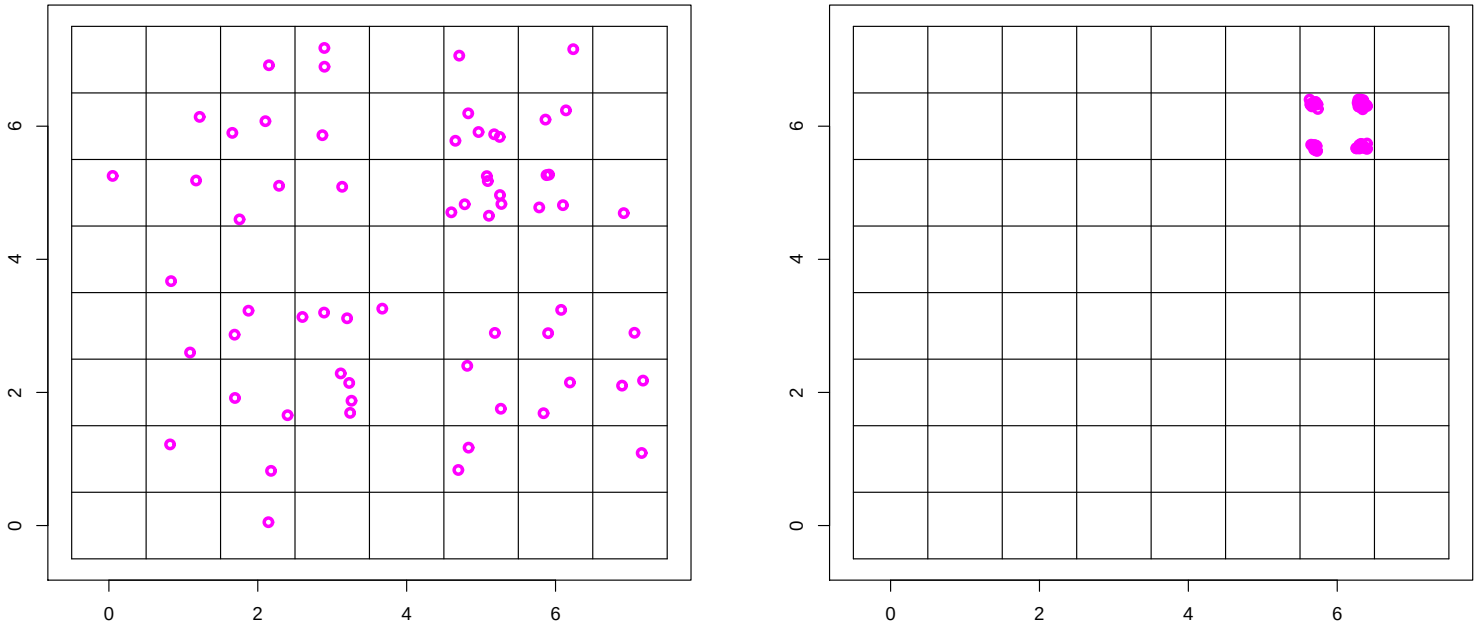
Testing of the northwest versus southeast corner corresponds to the design of Dickey, Hasza, and Fuller.

Austrian and U.K. GDP



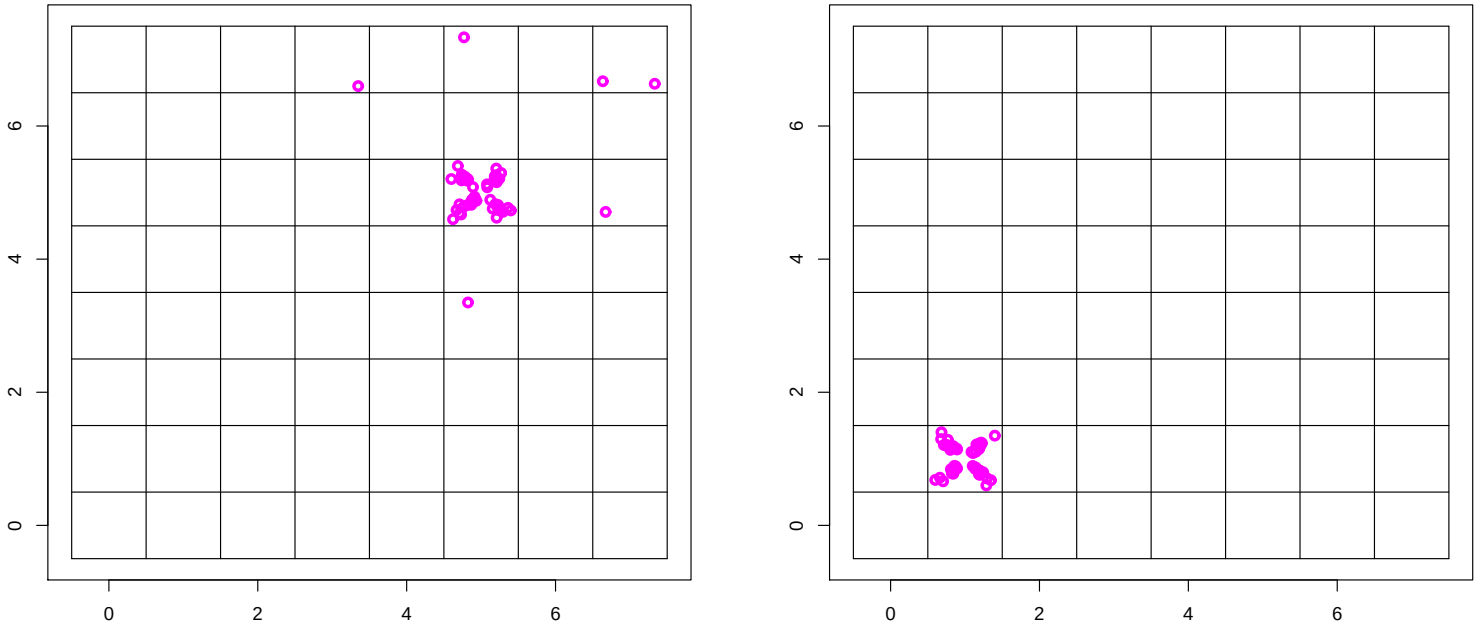
$T = 48$ for Austria, $T = 28$ for the United Kingdom: too short to be classified reliably. $(\tilde{\zeta}_1, \zeta_2, \zeta^*) = (0.35, 0.16, 0.17)$ for Austrian GDP (no rejection) and $(0.25, 0.19, 0.19)$ for U.K. GDP (ζ_2 rejects).

Heathrow data



Seasonality in precipitation is weak. Seasonality in temperature is deterministic. $(\tilde{\zeta}_1, \zeta_2, \zeta^*) = (2.60, 0.49, 0.60)$ for precipitation (clear rejection) and $(0, 0.21, 0.20)$ for temperature (ζ_2 rejects).

Austrian economics series



Industrial production and unemployment rate rarely if at all deviate from their typical seasonal patterns. $(\tilde{\zeta}_1, \zeta_2, \zeta^*) = (0.27, 0.17, 0.17)$ for production (near rejection for ζ_2) and $(0, 0.16, 0.15)$ for unemploment rate (no rejection).

Summary and conclusion

- Under standard conditions, the power of the considered ζ^* test can be lower than for a comparable parametric test. In small samples, it improves upon the component tests ζ_1 and ζ_2. In larger samples, ζ_2 alone competes with the combined test;
- By construction, ζ^* is robust to many monotonic transformations and can consider null and alternative hypotheses beyond the usual I(0)/I(1) framework;
- The long-run variance correction according to García and Sansó succeeds in removing most of the nuisance sensitivity in ζ_1. ζ_2 and thus ζ^* suffer from size distortion and from sensitivity to nuisance parameters that are not easily repaired. Nevertheless, ζ_1 is the weaker test or test component.

Thank you for your attention

References beyond the survey slide on unit-root tests

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