Midterm test 'Vector Autoregressions'

November 2006

1. Consider the following VAR(2) process

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & 0.67 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} -1/4 & 0 \\ 0 & -1/4 \end{pmatrix} \begin{pmatrix} x_{t-2} \\ y_{t-2} \end{pmatrix} + u_t$$

   (a) Show that this defines a stable VAR process.

   (b) Represent this VAR(2) process as a VAR(1), and provide the matrix $\mathbf{A}$.

   (c) Why do we not say: a *stationary* VAR process?

2. A rather simple VAR(1) process is defined by

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + u_t.$$

   (a) Provide the (in general infinite) MA representation (Wold representation) for this process.

   (b) Assume that $\mathrm{E}(u_t u_t') = \mathbf{I}_2$. What is the impulse-response representation for this process?

3. The coefficients of a VAR(2) process are to be estimated from data.

   (a) Which estimation method would you use and why?

   (b) Represent the process of example 1 in the form $\mathrm{vec}(Y) = (Z' \otimes \mathbf{I}_2) \times \mathrm{vec}(B) + \mathrm{vec}(U)$. Indicate the structure of the matrix $(Z' \otimes \mathbf{I}_2)$ and of the vectors. [Hint: the model is estimated without a constant, therefore the column of ones does not occur. It is maybe best to start constructing the matrix using the values x_0, y_0, x_{-1} and y_{-1}, such that the first row becomes $(x_0, 0, y_0, 0, x_{-1}, 0, y_{-1}, 0)$]

   (c) Can you make a statement concerning Granger causality for any of the two directions $X \to Y$ and $X \to Y$?