

# Second test in Macro-econometrics

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1. Concepts.[11 points]

- (a) Assume the variable  $(X_t)$  is an  $n$ -variate time-series variable. How do we define (covariance) stationarity of such a variable?
- (b) Assume the  $n$ -variate variable  $(X_t)$  follows a vector autoregressive process of order  $p$ , i.e. VAR( $p$ ). What is the stability condition for this case? [We remember that a variable is called stable if it becomes stationary as the sample gets larger]
- (c) When do we call two I(1) variables cointegrated?

2. A vector autoregression of order 2, VAR(2). [12 points]

- (a) The dynamic behavior of two macroeconomic aggregates  $X$  and  $Y$  follows a VAR(2) model

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 1.5 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} X_{t-2} \\ Y_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Write the model in its error-correction form  $\Delta x_t = \mathbf{\Pi}x_{t-1} + \mathbf{\Gamma}\Delta x_{t-1} + \varepsilon_t$  for  $x = (X, Y)'$ , and particularly determine the matrix  $\mathbf{\Pi}$ . If  $X$  and  $Y$  cointegrate, this impact matrix should have a rank of 1. Can you confirm this?

- (b) Try and find a factorization of  $\mathbf{\Pi} = \alpha\beta'$ . Provide the cointegrating vector  $\beta$ . [Hint: we assume that explosive roots and unit roots other than one are not present in this system.]
- (c) Have a look at the second equation in the system, the one that determines  $Y_t$ . This looks like a univariate AR(2) model, and it has a characteristic polynomial. Can you confirm that this polynomial has a unit root?

3. Panels [12 points] Someone wishes to estimate the influence of financial development on corruption in a panel of 24 African countries for annual data 2000–2016. The relationship is presumed to be measurable by two index variables, a financial development index  $FD$  and a corruption index  $C$ . A first idea is the static model (with  $\varepsilon$  containing potential country effects)

$$C_{it} = \beta_0 + \beta_1 FD_{it} + \varepsilon_{it}$$

- (a) Would you instinctively rather consider a fixed-effects or a random-effects model for this problem? Why? If a Hausman tests rejects, does this support your choice?
- (b) Somebody suggests you should also use another regressor that is easily obtained, a dummy variable  $F$  that is 1 for former French colonies and 0 for other countries. What is the problem with this regressor in fixed-effects estimation?
- (c) The fact that your static model has strongly serially correlated residuals suggests considering the dynamic model

$$C_{it} = \beta_0 + \beta_1 C_{i,t-1} + \beta_2 FD_{it} + \beta_3 F_{it} + \varepsilon_{it}$$

How would you interpret the coefficient  $\beta_1$ ? Why should we not naively estimate this model by standard random-effects or fixed-effects methods? Briefly describe one of the solutions for this problem that has been recommended in the literature.