

First test in Macro-econometrics

Robert M. Kunst

May 2, 2016

1. Stationarity.[8 points]

- (a) What conditions does a process (X_t) have to fulfill in order to be covariance-stationary?
- (b) How are the white noise and the random walk related to each other?
- (c) What is a first-order autoregressive process? What are the conditions under which the AR(1) process is stationary?
- (d) What distinguishes a stable from a stationary process?

2. Characteristic polynomials [6 points]

- (a) For the following AR(2) processes, provide the characteristic AR polynomial $\phi(z)$, determine the polynomial zeros (roots) and state whether they fulfill the conditions for the AR(2) process to be stable (asymptotically stationary):

i. $X_t = 0.9X_{t-2} + \varepsilon_t$;

ii. $X_t = 4X_{t-1} - 4X_{t-2} + \varepsilon_t$.

- (b) What about the MA(2) process

$$X_t = \varepsilon_t + 2\varepsilon_{t-1} \quad ?$$

Provide its characteristic MA polynomial $\theta(z)$, determine its roots, and state whether it is stable (or even stationary).

3. You analyze an empirical data set of 100 time series observations. You encounter the described features. Please outline in brief what model (AR, MA, ARMA, and possibly what lag orders) you would like to estimate in each case. [8 points]
- (a) Sample ACF and PACF mainly show insignificant values, some isolated points stick through the 95% confidence boundaries;
 - (b) The first two values of the sample ACF are large and significant, all other values are insignificant. The PACF shows a smooth but partly irregular decay;
 - (c) The first four values of the sample PACF are large and significant, the others are insignificant. The ACF shows a smooth and geometric decay;
 - (d) Several among the first ten ACF as well as PACF values are moderately large and significant, for larger lags you notice a convergence of both functions toward zero.

4. You wish to test for unit roots in trending data using the Dickey-Fuller test, but you only have access to a software that does not explicitly provide that test, just regression analysis. You also have access to tables of significance points.[7 points]
- (a) In a preliminary lag order search via AIC, you choose an AR(3) model as having the best fit to your data. Indicate the regression that you would have to run now, and also indicate where in the typical regression printout you would find the test statistic that you should compare to your table of significance points.
 - (b) Given that the DF-test rejects, what is your conclusion concerning the generating process for your data?

5. Which of the following ARCH models, if any, fulfills the condition for stability and potential covariance stationarity (then, they are covariance stationary, assuming that they have been started at $t = -\infty$):[6 points]

(a) $E(\varepsilon_t^2 | \mathcal{I}_{t-1}) = 1 + 0.4\varepsilon_{t-1}^2 + 0.4\varepsilon_{t-2}^2$;

(b) $E(\varepsilon_t^2 | \mathcal{I}_{t-1}) = 1 - 0.4\varepsilon_{t-1}^2 + 0.4\varepsilon_{t-2}^2$;

(c) $E(\varepsilon_t^2 | \mathcal{I}_{t-1}) = 1 + 0.4\varepsilon_{t-1}^2 + 0.6\varepsilon_{t-2}^2$.