Forecast combination based on multiple encompassing tests in a macroeconomic DSGE-VAR system

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The general research question

Does forecast averaging based on multiple encompassing tests (a tool suggested by Harvey & Newbold, 2000) assist in improving predictions for real output—a main variable of interest in macroeconomic forecasting—in a macroeconomic system with a realistic generation design?

Knowledge of the data-generation mechanism has the advantage that exact comparisons can be evaluated via Monte Carlo. The relevance depends crucially on whether the design is realistic.

Two approaches for realistic designs

- A core macreoeconomic VAR that is adapted to observed data from main economies: considered in Costantini & Kunst (2011, *Journal of Forecasting*);
- DSGE structures that are considered by current macroeconomists to be adequate descriptions of actual economies: DSGE structure by Smets and Wouters (2003), DSGE-VAR structure by Del Negro *et al.* (2007): this project.

General assumption: the forecaster does not know the generating design and applies comparatively simple time-series methods.

The main idea of the algorithm

Suppose there are M (usually model-based) out-of-sample forecasts $\hat{Y}_t^{(k)}$, $k = 1, \ldots, M$, for a vector variable Y.

Compile all M forecasts over a training sample $t = N - n, \ldots, N - 1$. These yield forecast errors $e_t^{(k)} = Y_{jt} - \hat{Y}_{jt}^{(k)}$ for a component Y_j in focus.

Run encompassing regressions to check whether any of the M forecasts encompasses its rivals. Eliminate the encompassed models.

Construct a new forecast by uniform averaging over all models that remain. Use this forecast for observation $t = N$.

The encompassing regressions

Start with $j = 1$. Run the homogeneous regression

$$e_t^{(1)} = \sum_{k=2}^{M} a_k (e_t^{(1)} - e_t^{(k)}) + u_t.$$

Consider the regression F–statistic. If the F–test does not reject its null, model # 1 forecast-encompasses its rivals.
Then, run comparable regressions for $e_t^{(j)}$, $j = 2, \ldots, M$:

$$e_t^{(j)} = \sum_{k=1, k \neq j}^{M} a_k (e_t^{(j)} - e_t^{(k)}) + u_t.$$

The decision rule

- If all M F–tests or none of them reject their null hypotheses, the forecast will be a uniformly weighted average of all models.
- If only some F–tests reject their null, only those models that encompass their rivals will be used in an otherwise uniform average.

The four rival models

1. A univariate AR model for the targeted output series;
2. A bivariate VAR model for output and inflation;
3. A bivariate VAR model for output and nominal interest;
4. A factor-augmented VAR (FAVAR) model for output and linear combinations of the remaining observed variables.

Lag orders are determined via AIC or BIC. Inflation and interest were chosen in accordance with Costantini & Kunst (2011).

The FAVAR model

Two-step algorithm:

1. The number of factors is determined via BIC(3) according to Bai and Ng (2002), with the maximum set at two or three: typically, the maximum factor dimension is selected. Factors are then calculated via principal components analysis;
2. A VAR model is estimated for the output series together with the factor variables that were determined in the first step.

DSGE model according to Smets& Wouters

Optimal intertemporal allocation of consumption with external habit formation:

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}\mathrm{E}_t[\hat{C}_{t+1}] - \frac{1-h}{(1+h)\sigma_c}(\hat{R}_t - \mathrm{E}_t[\hat{\pi}_{t+1}]) + \frac{1-h}{(1+h)\sigma_c}\varepsilon_t^b.$$

New Keynesian Phillips curve for the real wage with partial indexation:

$$\begin{aligned}\hat{w}_t &= \frac{\beta}{1+\beta}\mathrm{E}_t[\hat{w}_{t+1}] + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t[\hat{\pi}_{t+1}] - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t \\ &+ \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\{1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\}\xi_w}\{\hat{w}_t - \sigma_l\hat{L}_t \\ &- \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1}) + \varepsilon_t^l\} + \eta_t^w.\end{aligned}$$

DSGE model: capital and investment

Physical capital accumulates according to:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau \hat{I}_{t-1}.$$

Investment evolves with adjustment costs:

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t[\hat{I}_{t+1}] + \frac{\varphi}{1+\beta}\hat{Q}_t + \varepsilon_t^i.$$

Real value of installed capital:

$$\hat{Q}_t = -(\hat{R}_t - \mathrm{E}_t[\hat{\pi}_{t+1}]) + \frac{1-\tau}{1-\tau+\bar{r}^k}\mathrm{E}_t[\hat{Q}_{t+1}] + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}\mathrm{E}_t[\hat{r}^k_{t+1}] + \eta_t^q.$$

DSGE model: intermediate good producers

Competitive intermediate goods producers maximize the present value of expected future profits facing a production function:

$$\hat{Y}_t = \phi\varepsilon_t^a + \phi\alpha\hat{K}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{L}_t.$$

Their labor demand equation is given by:

$$\hat{L}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{K}_{t-1}.$$

Intermediate goods producers face nominal rigidities, which implies the standard New Keynesian Phillips curve for inflation with partial indexation:

$$\begin{aligned}\hat{\pi}_t &= \frac{\beta}{1+\beta\gamma_p}\mathrm{E}_t[\hat{\pi}_{t+1}] + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} \\ &+ \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \varepsilon_t^a] + \eta_t^p.\end{aligned}$$

DSGE model: equilibrium and interest rate

The goods market equilibrium condition:

$$\hat{Y}_t = (1 - \tau k_y - g_y)\hat{C}_t + \tau k_y \hat{I}_t + \varepsilon_t^g.$$

Monetary policy follows a Taylor-type interest-rate rule:

$$\begin{aligned}\hat{R}_t &= \rho\hat{R}_{t-1} + (1-\rho)[\bar{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \bar{\pi}_t) + r_y\hat{Y}_t] + r_{\Delta\pi}(\hat{\pi}_t - \hat{\pi}_{t-1}) \\ &\quad + r_{\Delta y}(\hat{Y}_t - \hat{Y}_{t-1}) + \eta_t^r.\end{aligned}$$

Details on simulation design

Three-step algorithm:

1. 2000 replications of DSGE model trajectories of length 1100;
2. Bayesian DSGE-VAR estimation using the step-one trajectories as data;
3. 2000 replications drawn from the posteriors of step two.

The available 2000 replications can be broken into 10,000 samples of length at most 200: sufficient number for assessing sample sizes varying from 40 to 200.

Evaluation of prediction performance

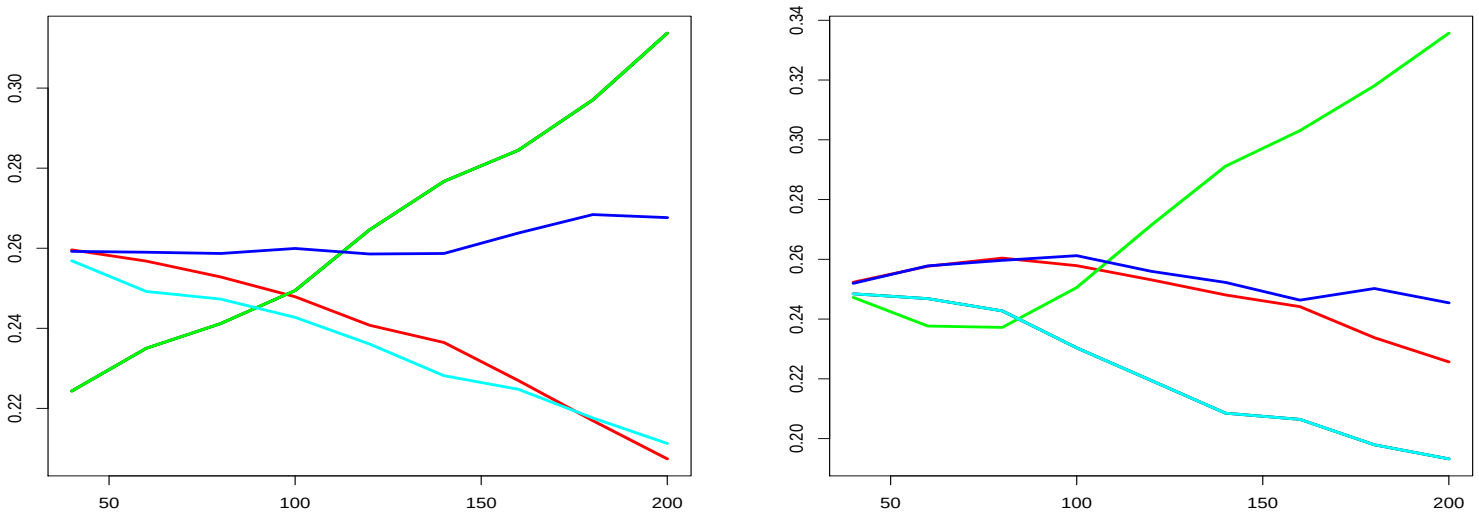
For any sample of size N, multiple encompassing tests are applied to out-of-sample forecasts for the observations numbered $3N/4$ to $N-1$. Performance on forecasting observation N is evaluated.

Here we show:

- Weights allotted to the four rival models in one-step prediction;
- MSE ratio of test-based average forecast relative to uniform average for significance levels varying from 1% to 10%.

Other accuracy measures have also been evaluated: mean absolute errors (MAE) and percentage best.

Weights allotted to rival models



Left: AIC, right: BIC; 3 factors, encompassing test at 1% level. Red: univariate, green: FAVAR, blue: with interest rate; cyan: with inflation. Sample size N on the x–axis.

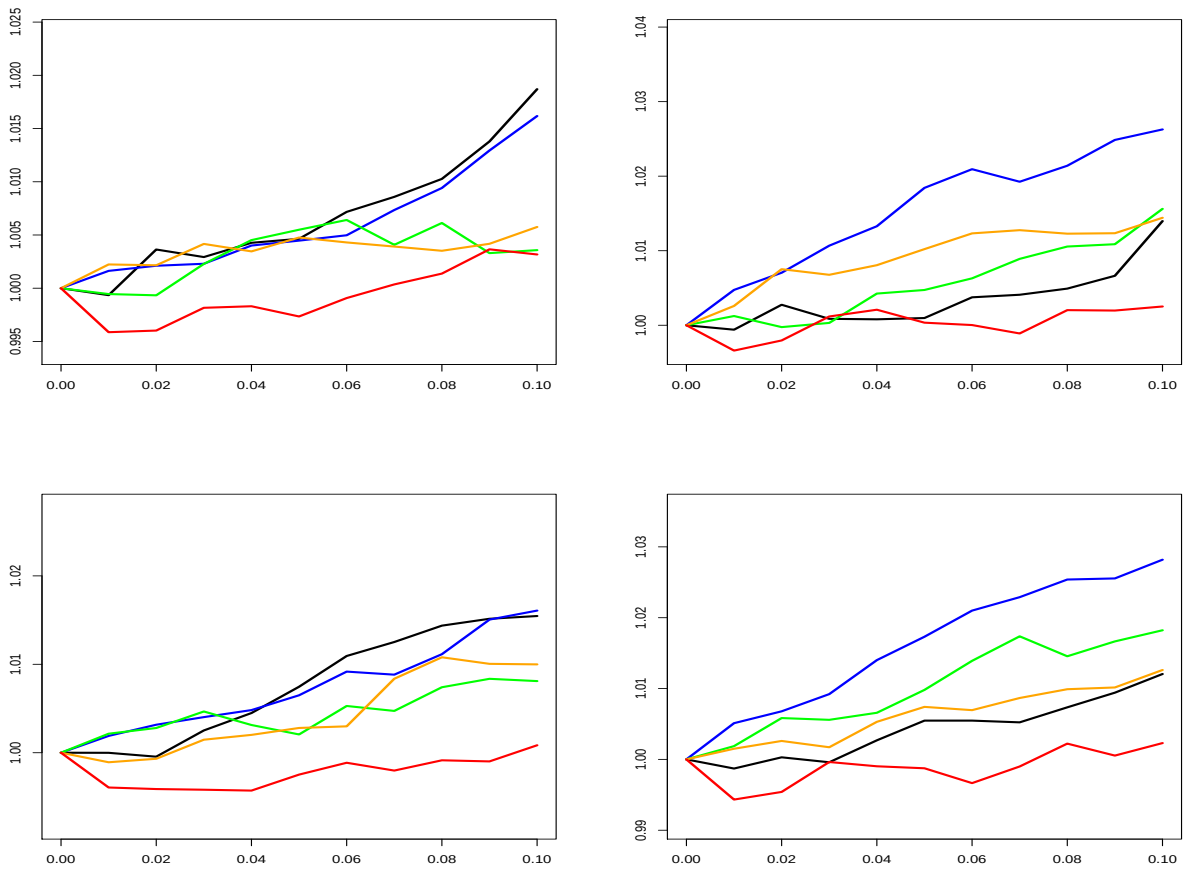
FAVAR dominates for larger samples

- At $N = 40$, the univariate AR is often best. The encompassing test yields only slight deviations from uniform weighting;
- At $N = 200$, the FAVAR model yields best predictions, and it is also allotted a larger weight by the encompassing procedure;
- The bivariate VAR with inflation and the univariate AR lose ground as the sample size increases.

What are the factors of the FAVAR?

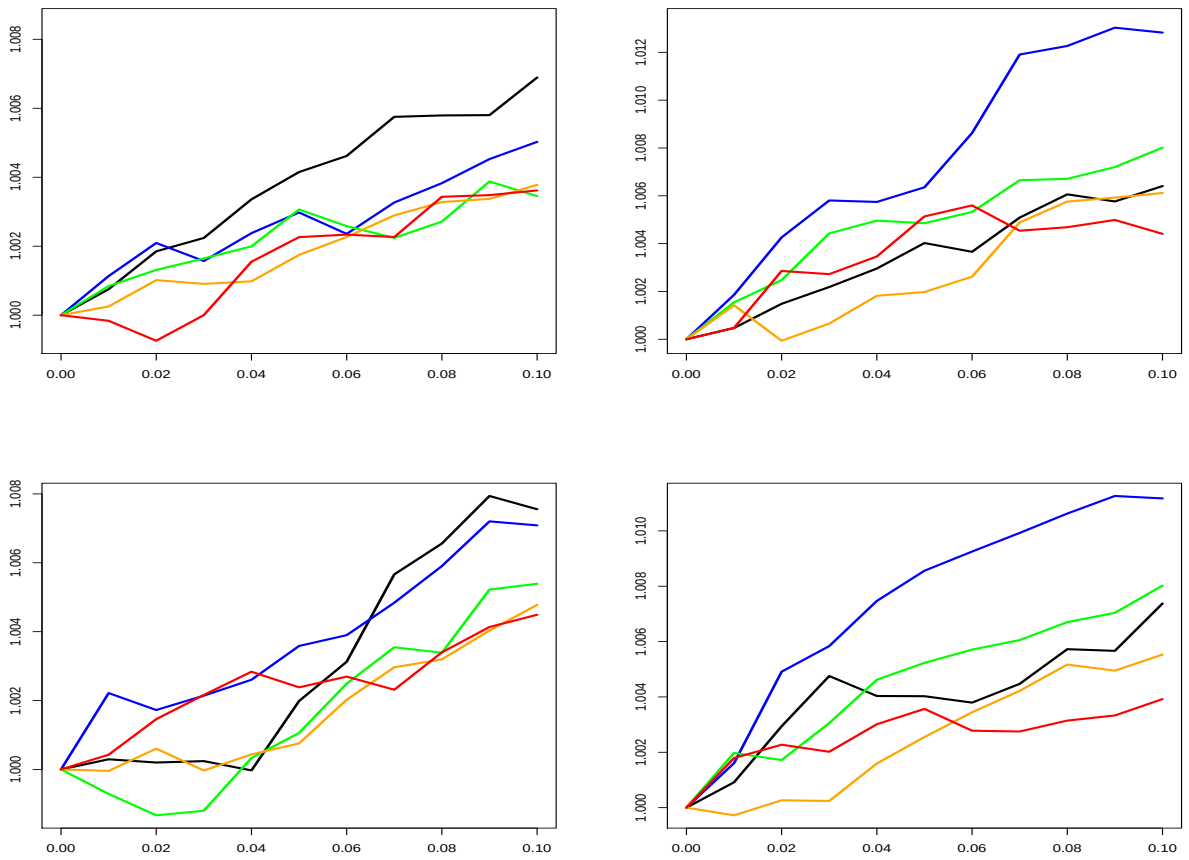
- Factor loadings and hence factors vary considerably across specifications and replications;
- The first factor typically mainly depends on investment I and the capital stock K;
- The second factor again depends mainly on capital K and investment I, with some contribution from wages W and the rental rate r^k;
- The third factor depends on consumption C and wages W, with some contribution from the labor force L and the real value of capital Q.

One-step forecasts, DSGE-VAR



Left: 2 factors; right: 3 factors; top: AIC; bottom: BIC. Significance level on the x–axis.

One-step forecasts, DSGE



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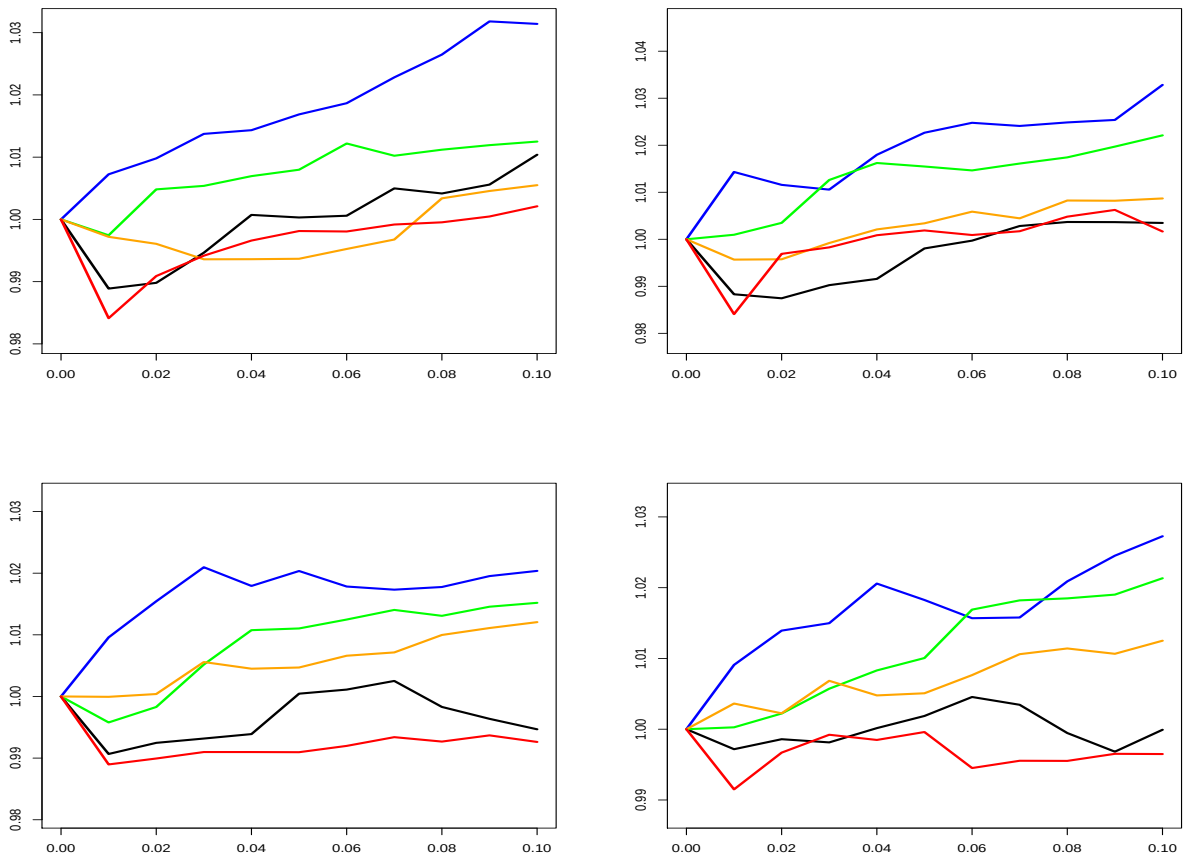
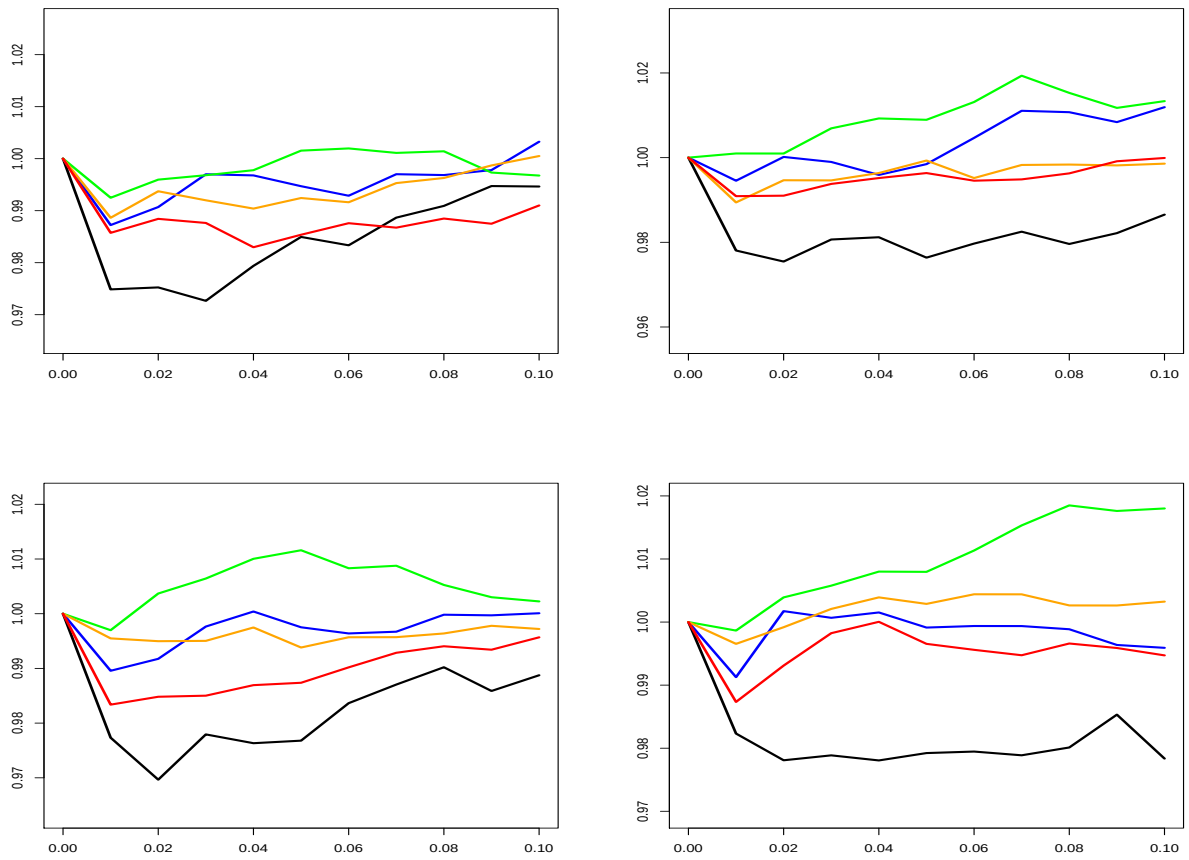
Two-step forecasts, DSGE-VAR



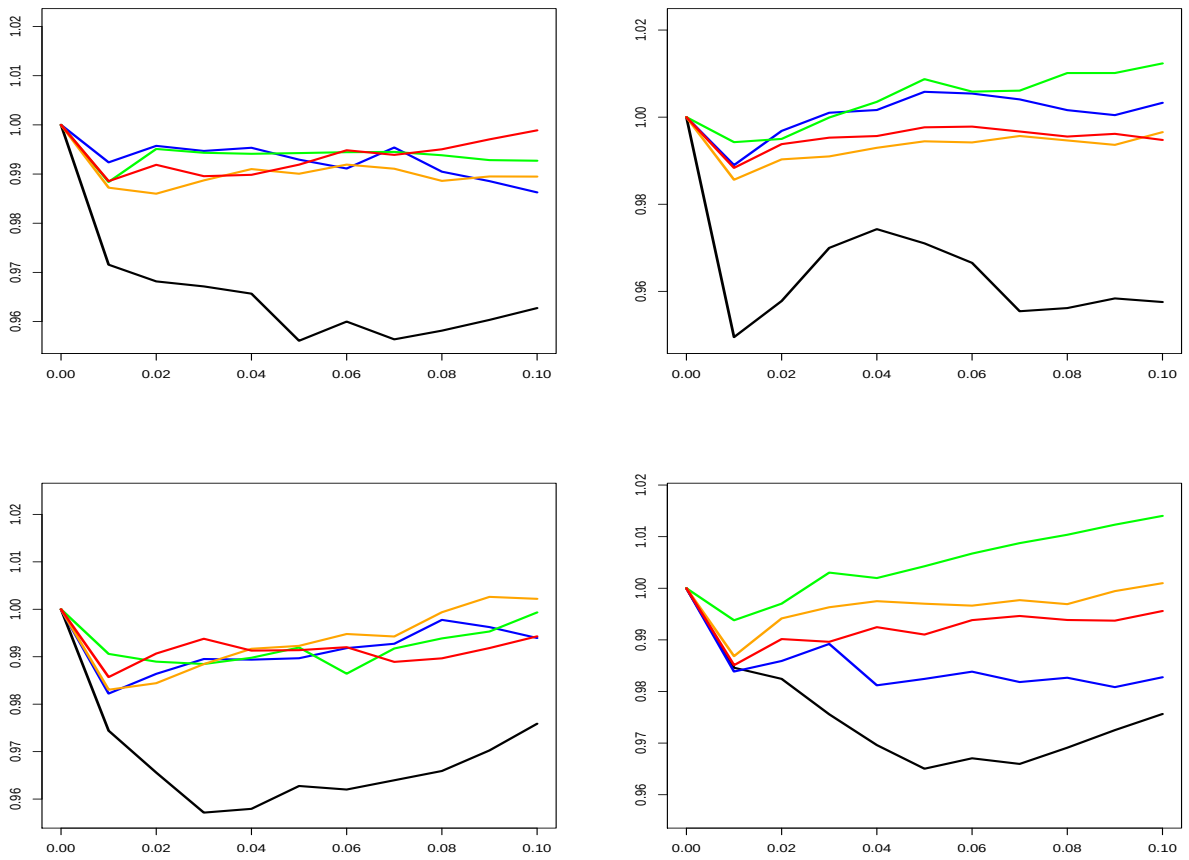
Left: 2 factors; right: 3 factors; top: AIC; bottom: BIC. Significance level on the x–axis.

Three-step forecasts, DSGE-VAR



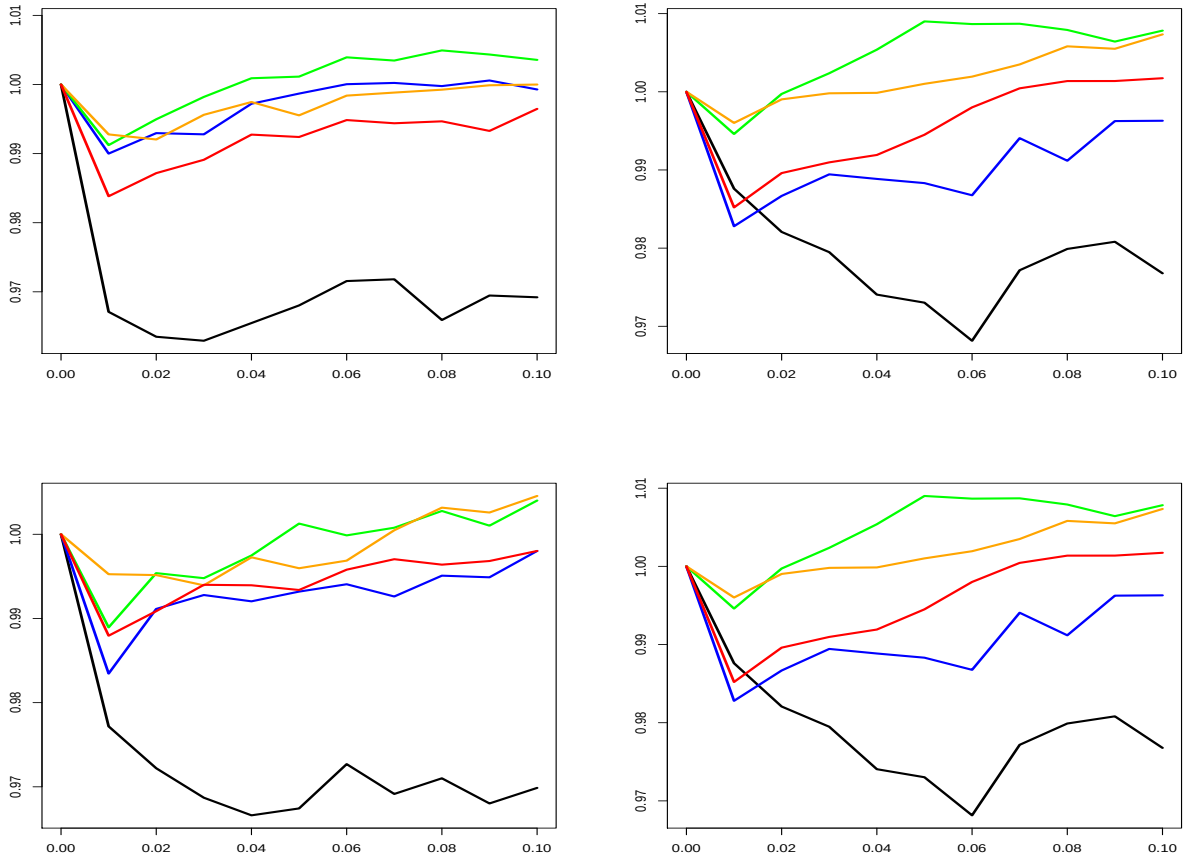
Left: 2 factors; right: 3 factors; top: AIC; bottom: BIC. Significance level on the x–axis.

Four-step forecasts, DSGE-VAR



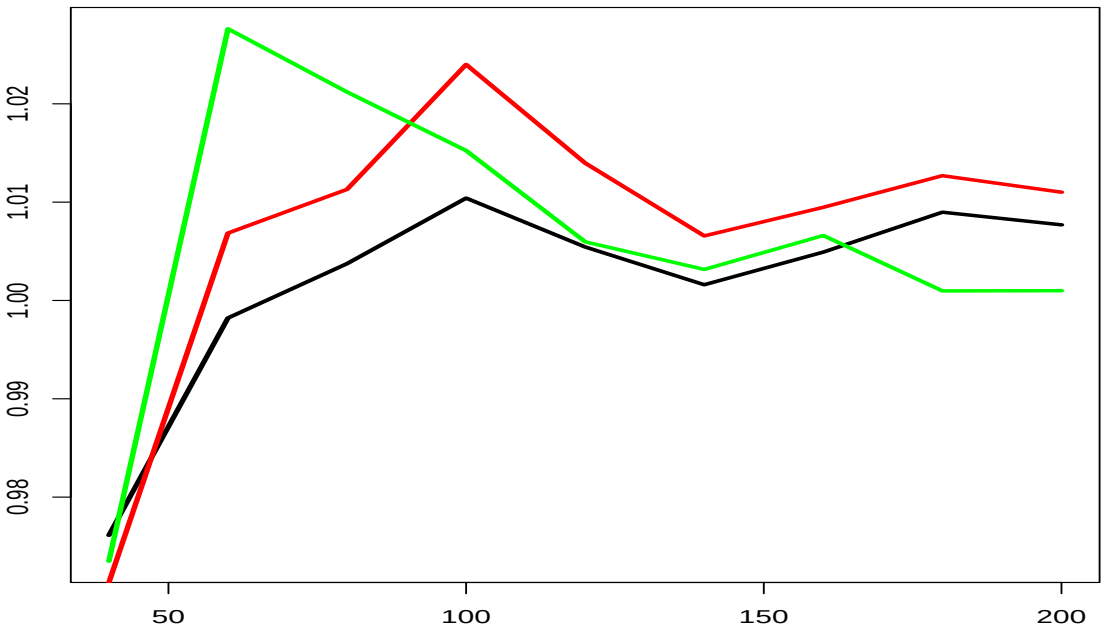
Left: 2 factors; right: 3 factors; top: AIC; bottom: BIC. Significance level on the x–axis.

Four-step forecasts, DSGE



Left: 2 factors; right: 3 factors; top: AIC; bottom: BIC. Significance level on the x–axis.

AIC or BIC, 2 or 3 factors?



MSE ratios relative to AIC 3 factors: black for AIC 2 factors; red for BIC 2 factors; green for BIC 3 factors. N on the x–axis.

AIC or BIC, 2 or 3 factors: summary

- BIC order selection tends to outperform AIC order selection for very small samples. At large N, AIC and BIC perform comparably;
- Excepting very small samples, 3 factors tend to be preferable to 2 factors. The BIC(3) typically always selects the maximum factor dimension.

Review of main results

- The encompassing procedure becomes most powerful at larger prediction horizons;
- The simulations suggest that the best significance levels for the encompassing test are rigorous, at 1%;
- In many constellations, the relative effect of the encompassing procedure is U–shaped in response to the sample size: good for small and larger samples, minor benefits around $N = 100$;
- The DSGE-VAR design assigns larger benefits to encompassing than the VAR design used in Costantini & Kunst (2011) and also than the pure DSGE design.

Thank you for your attention