Third and final test in Introductory Econometrics

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1. In the simple linear regression $y_i = \beta_0 + \beta_1 x_i + u_i$, all Gauss-Markov assumptions hold, except that $\mathrm{E}u_i^2 = \mathrm{E}(u_i^2|x) = \lambda z_i$, i.e. there is heteroskedasticity, with the error variance proportional to an observed positive variable z with unknown proportionality factor λ.[12 points]

   (a) If you estimate the regression using OLS, will the regression coefficient estimates $\hat{\beta}_1$ be unbiased?

   (b) Will their variance estimates obtained from $\hat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$ be unbiased? How does the matrix $\mathbf{X}$ look like here?

   (c) Describe how you can construct a regression

   $$y_i^* = \beta_0 w_i + \beta_1 x_i^* + u_i^*$$

   that fulfills all Gauss-Markov conditions. In particular, indicate how w and y^*, x^* are defined. What is the variance $\mathrm{E}u_i^{*2}$?

2. If a regression is run in a time-series environment rather than over an independent cross section, things change. It is important to distinguish clearly between the static and the dynamic regression:[12 points]

   (a) State the conditions (we called them TS.1–TS.5) that are required for a Gauss-Markov Theorem to hold. These are tuned to a static time-series regression.

   (b) Which of the conditions cannot hold in a dynamic regression? How is it typically replaced (we called this condition TS.3') in dynamic regressions?

   (c) Unbiasedness and consistency are two important properties for an estimator. Which of the two is no more valid for OLS in a dynamic regression, even if TS.3' holds, and which is still valid if TS.1,TS.2, TS.3' hold?

3. A researcher is interested in whether hamburgers are responsible for obesity. To investigate this issue, for 1000 randomly chosen humans, information on their weight W, on father's weight WF, mother's weight WM, and on the annual intake of hamburgers H is collected. The OLS regression $W_i = \beta_0 + \beta_1 WF_i + \beta_2 WM_i + \beta_3 H_i + u_i$ yields an insignificant $\hat{\beta}_3$, which worries the researcher who does not like hamburgers and desperately wants to find evidence for a positive effect. [12 points]

   (a) How would you interpret u? Which maybe unobserved effects are contained in u?[Hint: very few people live on a pure hamburger diet]

   (b) Could it make sense to assume that u is correlated with H? Which of the effects that you have identified in (a) could be responsible for this correlation?

   (c) Someone has suggested the intake of diet coke DC as an instrument for H. Which two conditions should this instrument fulfill? Do you think that they hold here? If they hold, will the IV estimator with DC as an instrument for H, and WF, WM assumed exogenous be consistent?

   (d) It turns out that WF and WM are positively correlated, as birds of one feather flock together. Does this correlation make your IV estimator inconsistent?

4. For the following tests, write down the null and alternative hypotheses. You can use words or symbols, but unusual notation must be explained. [9 points]:

   (a) The Breusch-Godfrey test;

   (b) The Hausman test for endogeneity in an IV regression;

   (c) The F–test for the last three regression coefficients in a regression with 5 regressors.