First test in Introductory Econometrics

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1. Consider the basic Gauss-Markov assumptions for the simple linear regression model:[6 points]

    (a) Please list the four assumptions that are required in the *simple* linear regression model for the unbiasedness property of the OLS estimator.

    (b) What is the additional assumption that is needed for OLS to be BLUE?

    (c) Why do we demand that not all observations of the regressor are identical?

2. Of course, we remember the formula for the OLS slope estimate in the simple linear regression model

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

In a specific regression, we know that the following holds:[6 points]

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = 200, \quad \frac{1}{n}\sum_{i=1}^{n} x_i y_i = 100, \quad \bar{x} = \bar{y} = 0$$

(a) Determine the slope estimate $\hat{\beta}_1$ and the intercept estimate $\hat{\beta}_0$.

(b) Now assume that $\bar{y} = 1$ but still $\bar{x} = 0$. Determine the new intercept and slope estimates (Hint: $\hat{\beta}_1$ remains unchanged).

(c) Why do these betas have hats on their heads, whereas those in the previous problem do not?

3. A traditional research topic in development economics is the relationship of political freedom and economic development. Presume cross-section data on various countries are available for a political freedom index Y and on *per capita* gross domestic product X.[6 points]

   (a) From a scatter plot, you see that indeed Y appears to grow with X on average. Thus, you regress Y on X using OLS. It appears, however, that residuals tend to be positive for the poorest and for the richest countries and to be negative for many intermediate cases. Which of the five Gauss-Markov conditions appears to be violated (assuming residuals have similar properties as errors)?

   (b) Does this imply that OLS is biased? What does it mean that OLS is biased?

   (c) Given this information, do you have a suggestion for a better regression using the same data?

4. Consider the multiple linear regression model $y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + u_i$. Imagine that Y stands for household consumption, X_1 for household income, and X_2 for household wealth.

    (a) What assumption must hold for the conditional expectation of the error term $\mathrm{E}(u|X_1, X_2)$? What does this imply for $\mathrm{E}u$?

    (b) Somebody says that β_1 describes the marginal reaction of household consumption to a change in household income. Is this statement correct? How is household wealth assumed to behave in this thought experiment?

    (c) We know that the two regressors need not be uncorrelated with each other in multiple regression. Do you think that income and wealth are correlated? If yes, will this correlation be positive or negative?