Jittered phase diagrams for seasonal patterns in time series

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Presented at the Nordic Econometric Meeting 2013, Bergen

Outline

The basic classification of seasonal time series

Seasonality may be generated by seasonal unit roots or by deterministic patterns. Two classes of hypothesis tests exist:

- Tests with seasonal unit roots as their null hypothesis: Dickey, Hasza, Fuller (1984), Hylleberg, Engle, Granger, Yoo (1990);
- Tests with deterministic seasonality as their null: Canova & Hansen (1995), Tam & Reinsel (1997), Caner (1998).

Conflicting decisions have been studied by Hylleberg (1995) and Kunst & Reutter (2002).

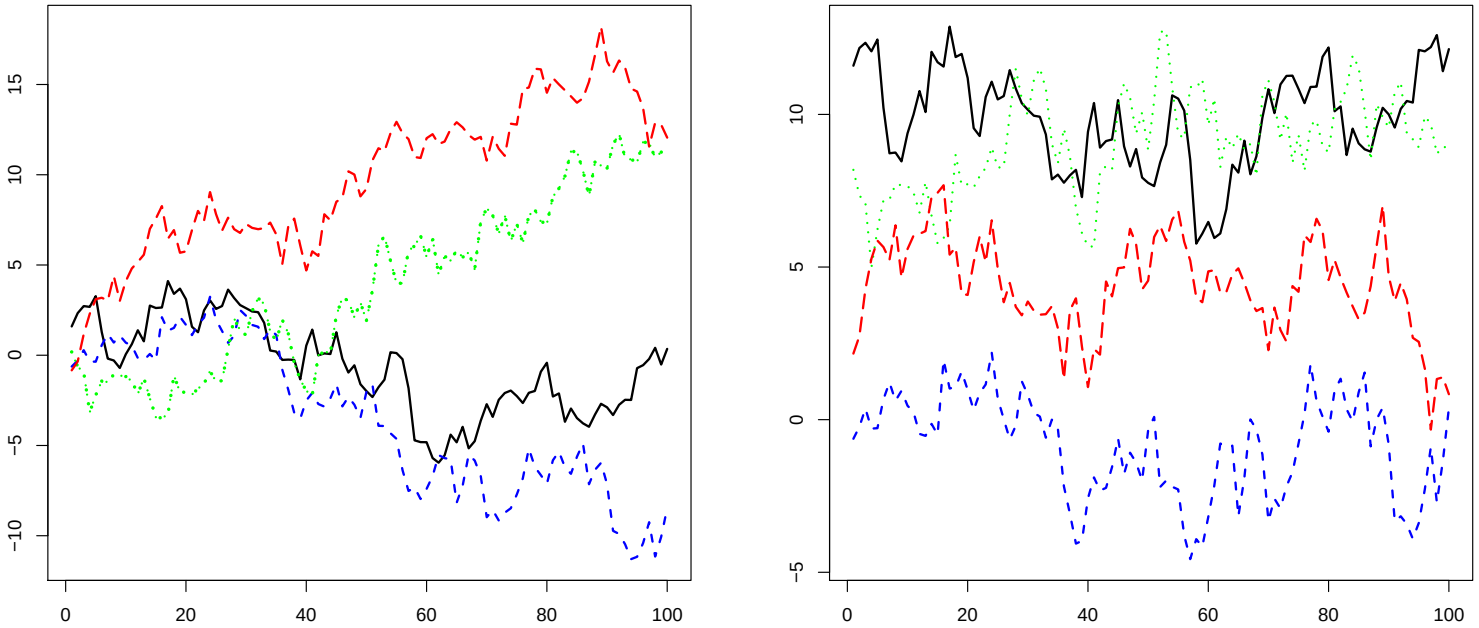
Properties of the two classes

Processes with *deterministic seasonality* rarely change the main qualitative features of their seasonal patterns. Even when they do so, patterns return to their original shapes ('summer remains summer, winter remains winter'): *pattern reversion*;

Processes with *seasonal unit roots* rarely change the main qualitative features of their seasonal patterns. If they do so, patterns often will not return to original shapes ('winter becomes summer'): *pattern persistence*;

Processes with weak seasonality frequently change the main qualitative features.

Traditional visualization: time plots per quarter



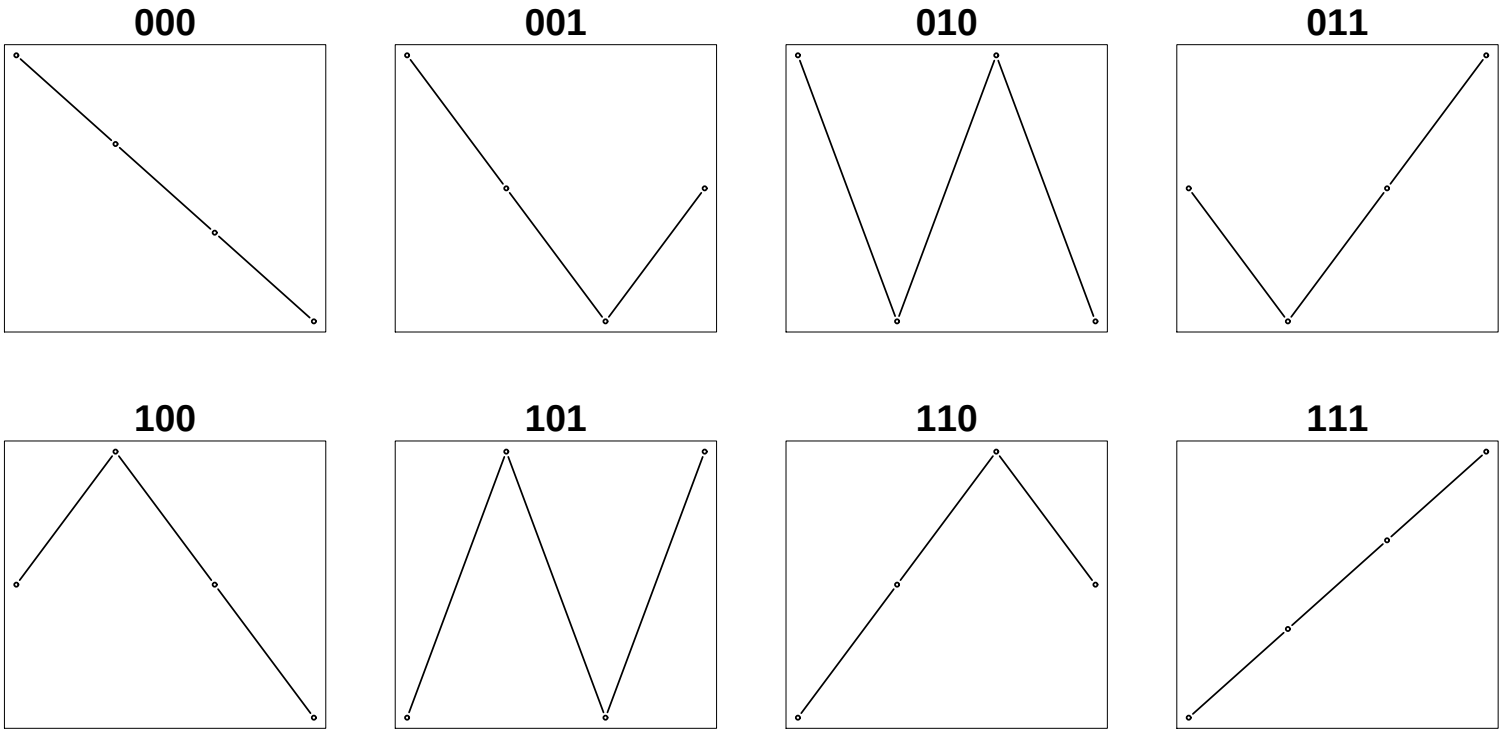
Time plots per quarter for a Gaussian seasonal random walk $x_t = x_{t-4} + \varepsilon_t$ (SRW, left) and a deterministic seasonal process (right).

Return to the original problem

The feature of concern is not really the existence of seasonal unit roots. Rather, it may be of interest whether a change in the seasonal pattern precedes a permanent shift to a different shape or not.

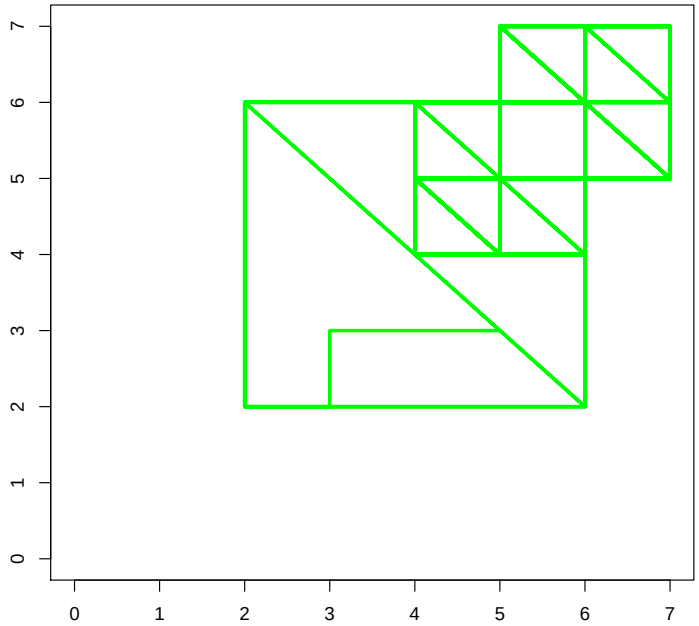
It may thus be convenient to focus on the discretized notion of distinctive seasonal shapes and their transition. The concomitant visualization may assist in the discrimination problem, even in cases where hypothesis tests do not offer a clear conclusion.

The eight seasonal patterns for quarterly data



Rising (1) and falling (0) quarters yield binary representations of numbers 0 to 7. 8 not 16 classes: trend not in focus.

Discrete phase plots



Phase diagram for pattern classes. Generating model is a quarterly Gaussian SRW for 10,000 years. The visualization is unsatisfactory.

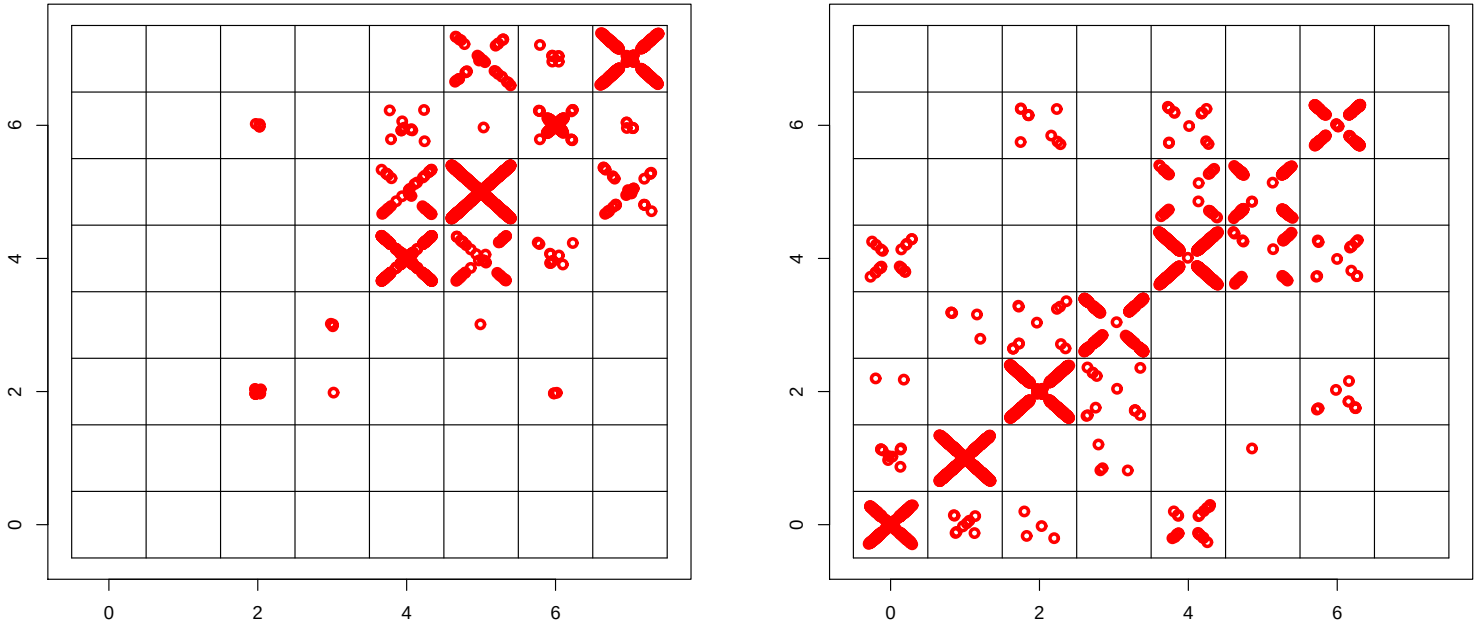
Jittering the phase plots

Observations are not just allotted to the bins $m = 0, \ldots, 7$ but are spread over the intervals $[m - 0.4, m + 0.4]$ according to the following rule:

- Observations are allotted uniformly $(0.5 : 0.5)$ to either $[m - 0.4, m)$ or to $(m, m + 0.4]$;
- Within the intervals, the outer limits $m \pm 0.4$ are set by the maximum 'depth' (largest increase or decrease in any quarter), and points are positioned relative to this maximum.

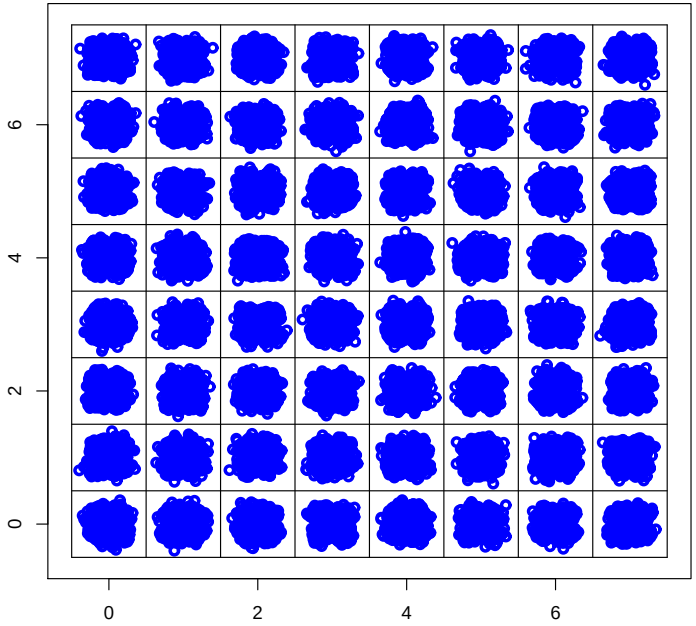
Shallow points are in the centers of the bins, deep points are in the extremes. For seasonal random walks, classes are entered through centers, and corners are approached as the residence in a bin continues.

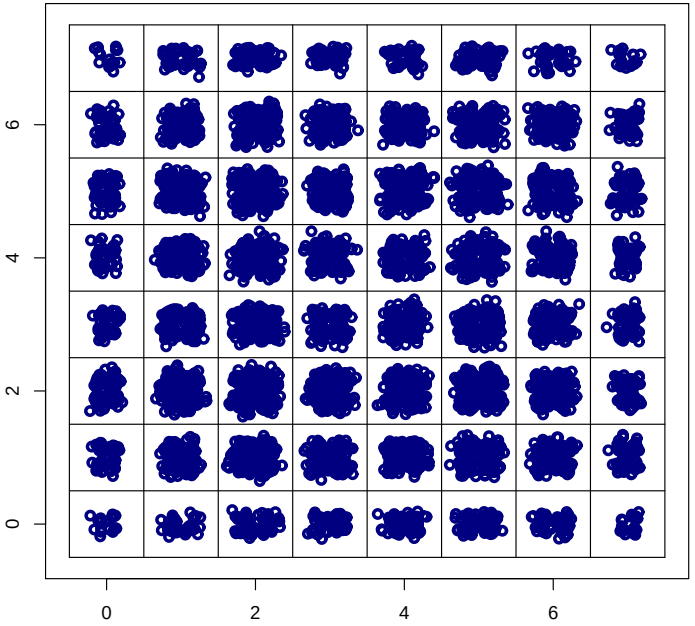
Jittered phase plots of seasonal random walks



10,000 years of $x_t = x_{t-4} + \varepsilon_t$, Gaussian errors.

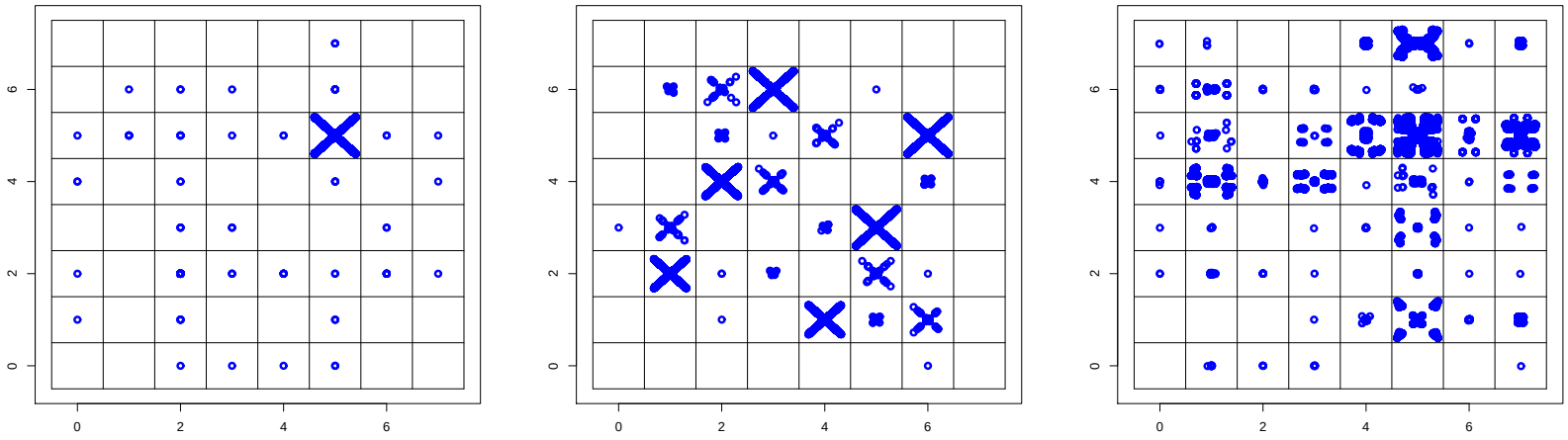
Jittered phase plot of non-seasonal processes





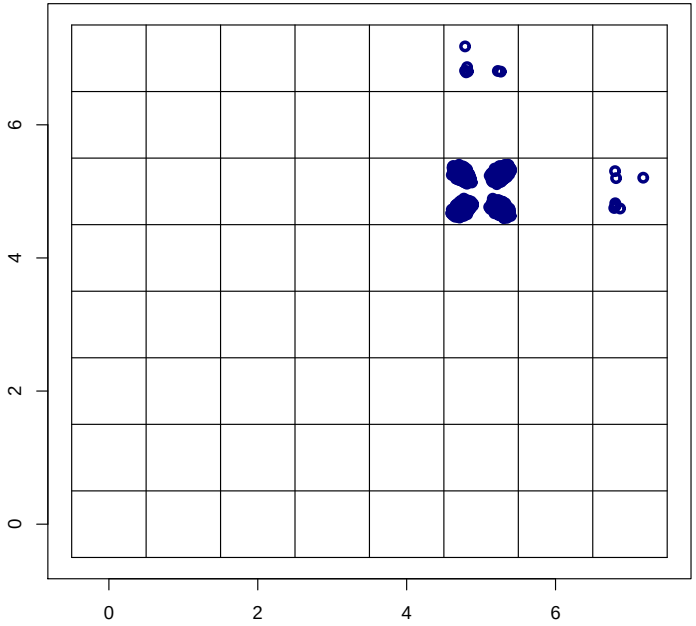
Random walk and white noise.

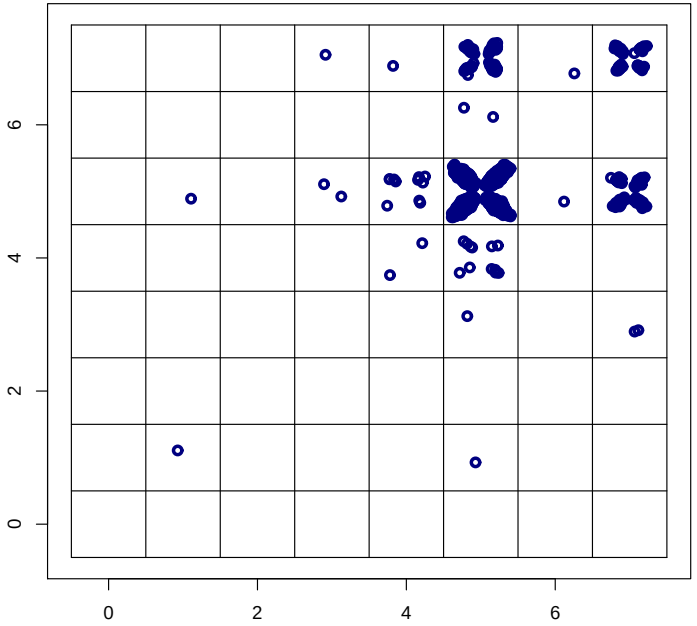
Seasonality at unexpected frequencies



$x_t = x_{t-j} + \varepsilon_t$ for $j = 2, 3, 6$.

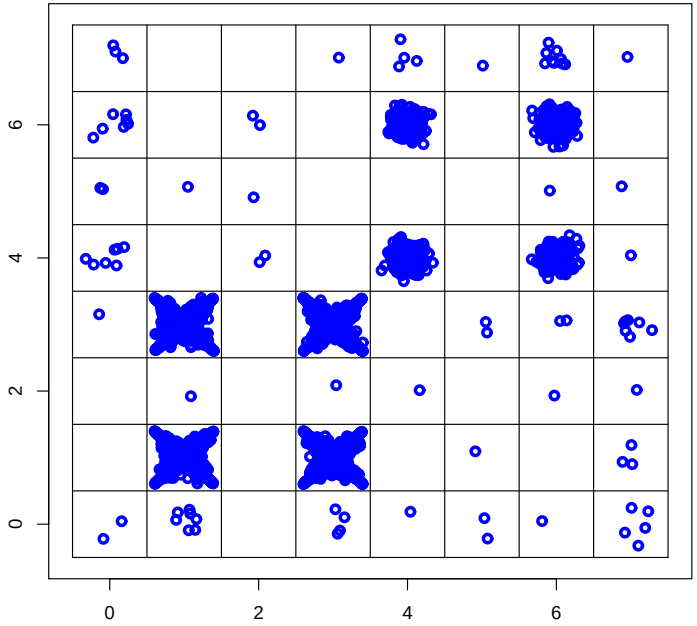
Jittered phase plot of deterministic seasonality





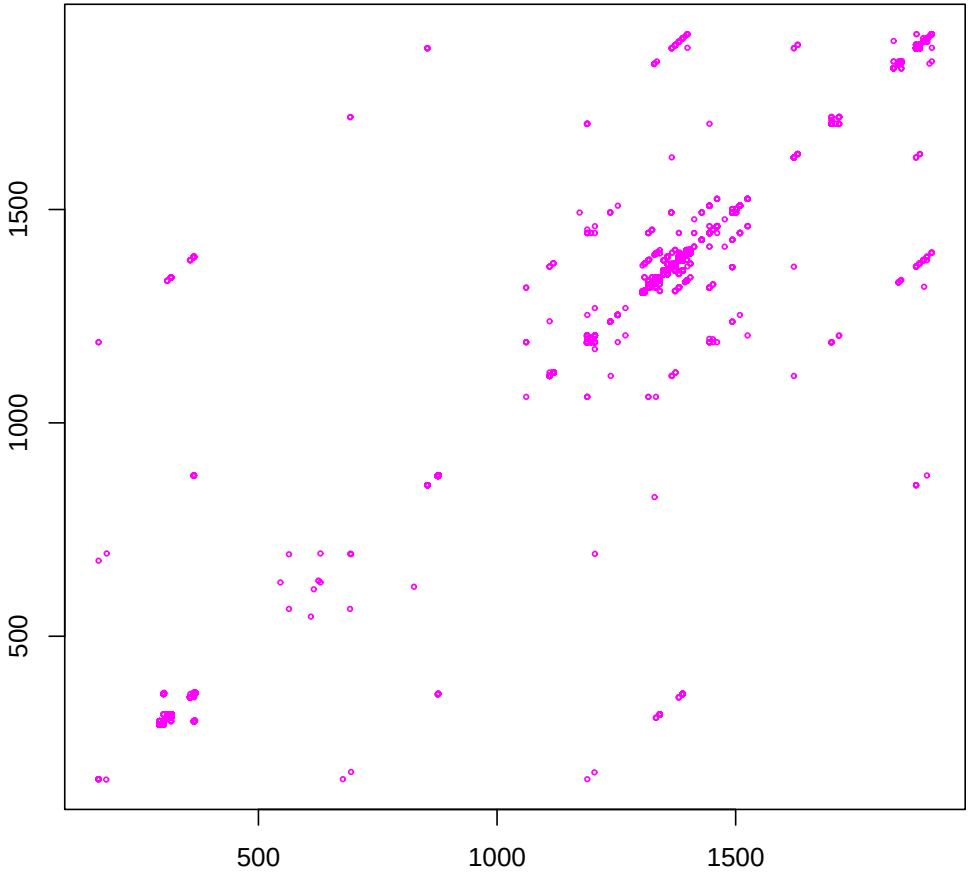
Generating process is $x_t = 0.4x_{t-4} + \sum_{j=1}^{4} d_t + \varepsilon_t$, with $(d_1, \ldots, d_4) = (0, 8, 3, 10)$.

Jittered phase plot for a periodically integrated process



Generating model is $x_t = \phi_s x_{t-4} + \varepsilon_t$ with $(\phi_1, \ldots, \phi_4) = (1, 0.25, 1.4)$.

Jittered phase plot of monthly SRW



What can be learnt from the jittered plots?

- Processes with seasonal unit roots generate X shapes in the bins (St. Andrew's crosses, saltires), other processes generate blurred crosses or blotches;
- Without jittering, the saltires are replaced by slashes, which is less attractive visually;
- The monthly version has too many classes (2,048), such that the visual impression within the bins is lost.

Two ideas for nonparametric tests

- Count the transitions between pattern classes;
- Consider the average distance from the saltire.

These ideas are reflected in the statistics ζ_1 and ζ_2.

The transition count statistic ζ_1

The statistic ζ_1 counts pattern transitions. It is closely related to the level-crossings test by Burridge & Guerre (1996). For seasonal random walks x_t, $x_{4t-j}, j = 0, \ldots, 3$ are random walks, and so are their differences and sums. When they cross levels, the pattern class changes.

The modified crossings count

$$K_T^*(0) = \frac{\hat{\sigma}}{\widehat{\mathrm{MAD}}} T^{-0.5} \sum_{t=1}^{T} I(X_{t-1} \leq x, X_t > x) + I(X_{t-1} > x, X_t \leq x)$$

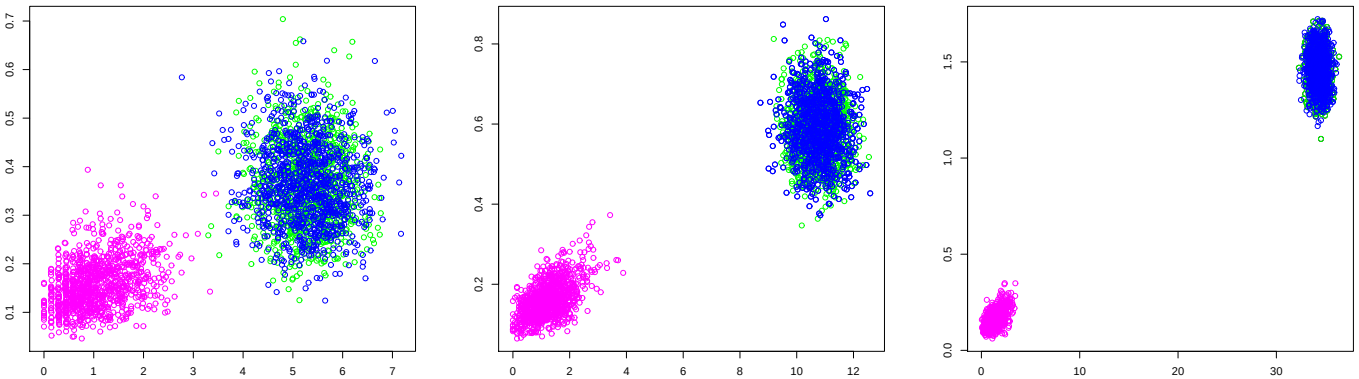
is asymptotically distributed as $|N(0,1)|$. $\hat{\sigma}$ and $\widehat{\mathrm{MAD}}$ are moments estimates for increments required for re-scaling. ζ_1 is defined in accordance with $K_T^*(0)$ and uses $3T/4$ instead of T.

The distance from the saltire ζ_2

The *median distance from the saltire* in graphical coordinates depends on the ratio of the increments to the extension of the bin, i.e. the extremum over the sample. For random walks and related processes, the extremum is known to expand at the rate of $T^{0.5}$. ζ_2 is defined as the median distance times $T^{0.5}$.

The asymptotic growth rate of random walk extrema has been used in non-parametric unit-root tests, e.g. Aparicio, Escribano, Sipols (2006). Its distribution is sensitive to autocorrelation in increments.

Visualization of discriminatory power



1000 realizations of the test statistics ζ_1 and ζ_2 based on a SRW (magenta), a random walk (blue), and a white noise (green). 25, 100, and 1000 years.

Weighted average of ζ_1 and ζ_2

The statistics ζ_1 and ζ_2 process different information. A combination such as $\zeta_1 + c\zeta_2$ may have higher power than individual tests. Relative scales suggest $c = 7$.

Significance points for the tests

	$T = 100$			$T = 400$			$T = 4000$		
	1%	5%	10%	1%	5%	10%	1%	5%	10%
ζ_1	2.10	1.53	1.27	2.12	1.55	1.29	2.15	1.58	1.29
ζ_2	0.25	0.20	0.17	0.23	0.19	0.17	0.22	0.18	0.16
$\frac{\zeta_1+7\zeta_2}{2}$	1.73	1.34	1.16	1.73	1.33	1.15	1.73	1.34	1.15

Rejection frequencies at 25 years and at 100 years

	$T = 100$			$T = 400$		
ϕ	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*
1.00	0.050	0.050	0.050	0.050	0.050	0.050
0.99	0.074	0.079	0.080	0.179	0.237	0.238
0.98	0.104	0.114	0.126	0.370	0.540	0.518
0.97	0.141	0.163	0.176	0.557	0.770	0.746
0.96	0.182	0.218	0.234	0.706	0.896	0.878
0.95	0.229	0.273	0.295	0.820	0.954	0.953
0.94	0.276	0.331	0.361	0.895	0.982	0.983
0.93	0.325	0.386	0.425	0.944	0.991	0.995
0.92	0.379	0.442	0.496	0.970	0.997	0.999
0.91	0.428	0.497	0.556	0.984	0.999	0.999
0.90	0.476	0.541	0.617	0.993	0.999	1.000

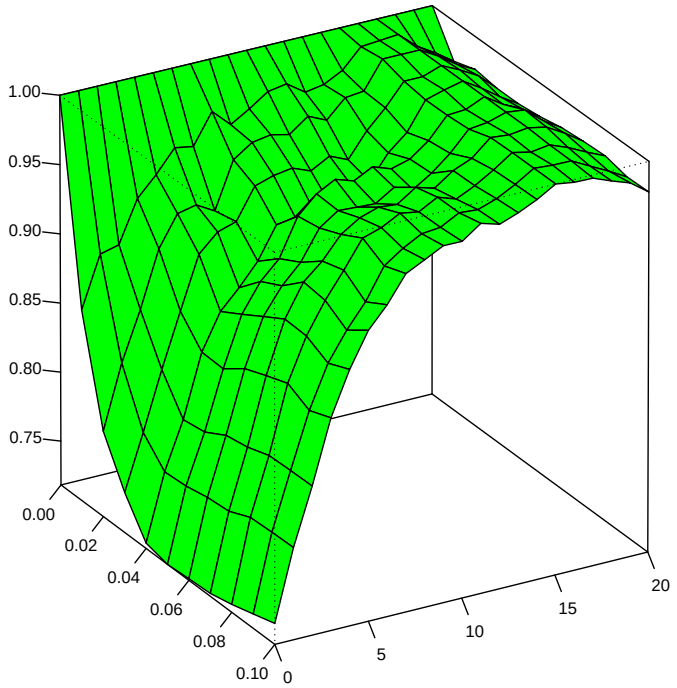
Generating model is $x_t = \phi x_{t-4} + \varepsilon_t$. Significance level is 5%. $\phi = 1$ is the null.

Autocorrelated increments distort test properties

	$\theta = -0.5$			$\theta = 0$			$\theta = 0.5$		
ϕ	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*	ζ_1	ζ_2	ζ^*
1.0	0.033	0.030	0.032	0.050	0.050	0.050	0.070	0.071	0.075
0.98	0.069	0.076	0.078	0.104	0.114	0.126	0.130	0.152	0.161
0.96	0.129	0.147	0.160	0.182	0.218	0.234	0.216	0.266	0.284
0.94	0.209	0.241	0.261	0.276	0.331	0.361	0.315	0.380	0.420
0.92	0.295	0.340	0.378	0.379	0.442	0.496	0.420	0.490	0.546
0.9	0.395	0.441	0.504	0.476	0.541	0.617	0.505	0.578	0.658

Negative (positive) autocorrelation causes negative (positive) size bias and affects the power properties correspondingly.

Does a weight of 7 maximize test power?



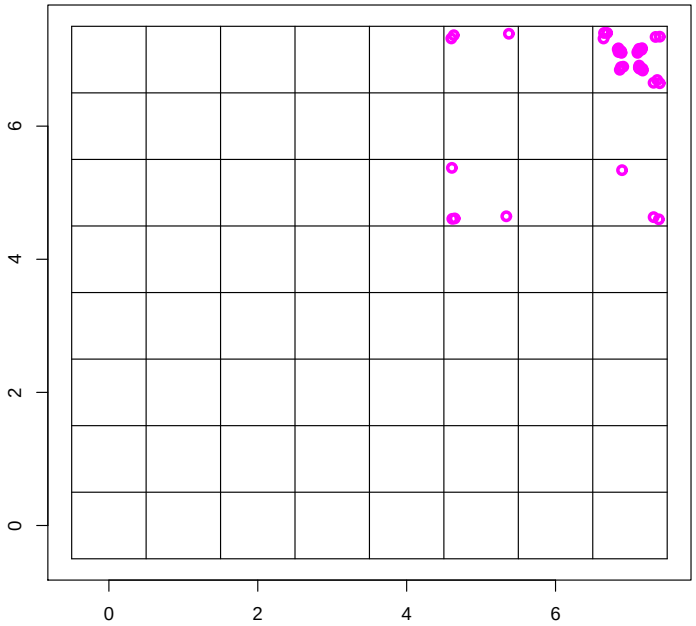
Power depending on the weight c in $\zeta_1 + c\zeta_2$ for $T = 100$ and $\phi \in [0.9, 1]$. Weight c on the x–axis and $1 - \phi$ on the y–axis. Optimal c may be around 16 rather than 7. Note, however, that this is a special direction of alternatives.

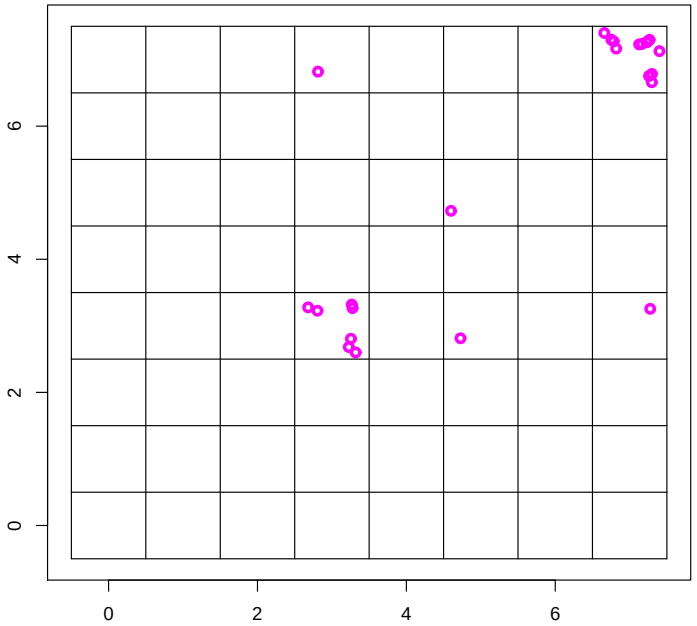
ζ_1 and ζ_2 convey different information

	ζ_1 small	ζ_1 large
ζ_2 small	seasonal unit root	regime switching
ζ_2 large	deterministic season	no seasonality

Testing of the northwest versus southeast corner corresponds to the design of Dickey, Hasza, and Fuller.

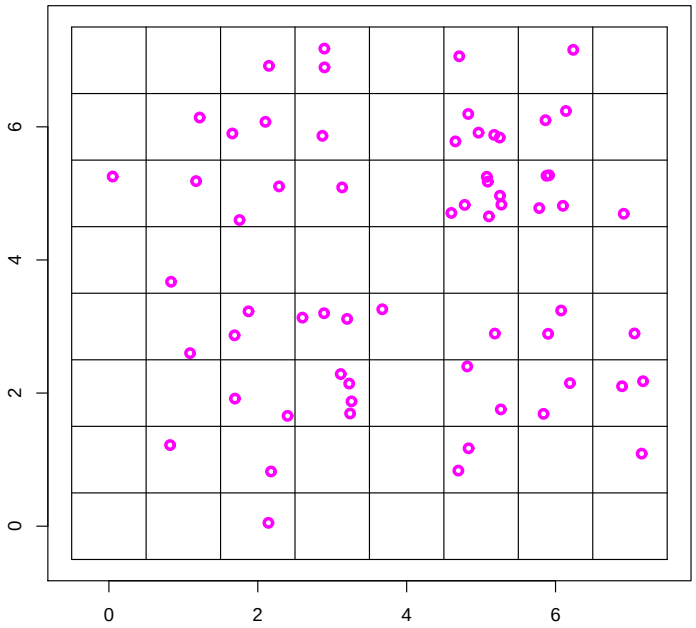
Austrian and U.K. GDP

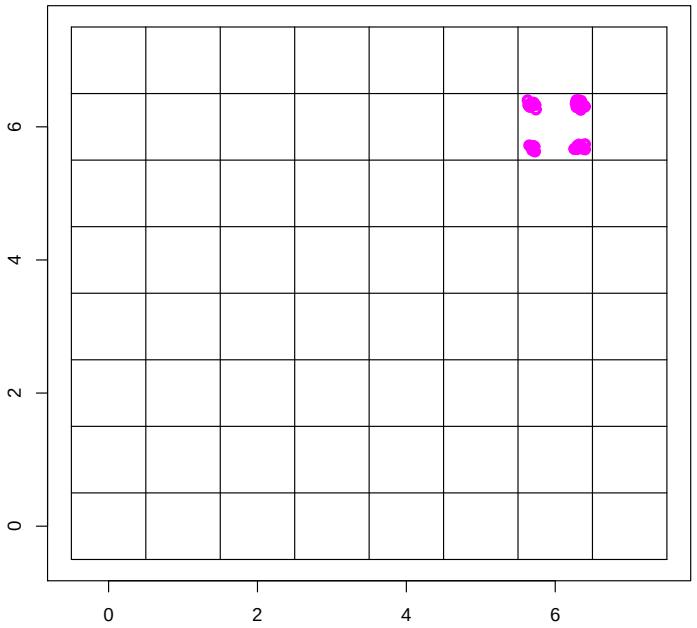




$T = 48$ for Austria, $T = 28$ for the United Kingdom: too short to be classified reliably.

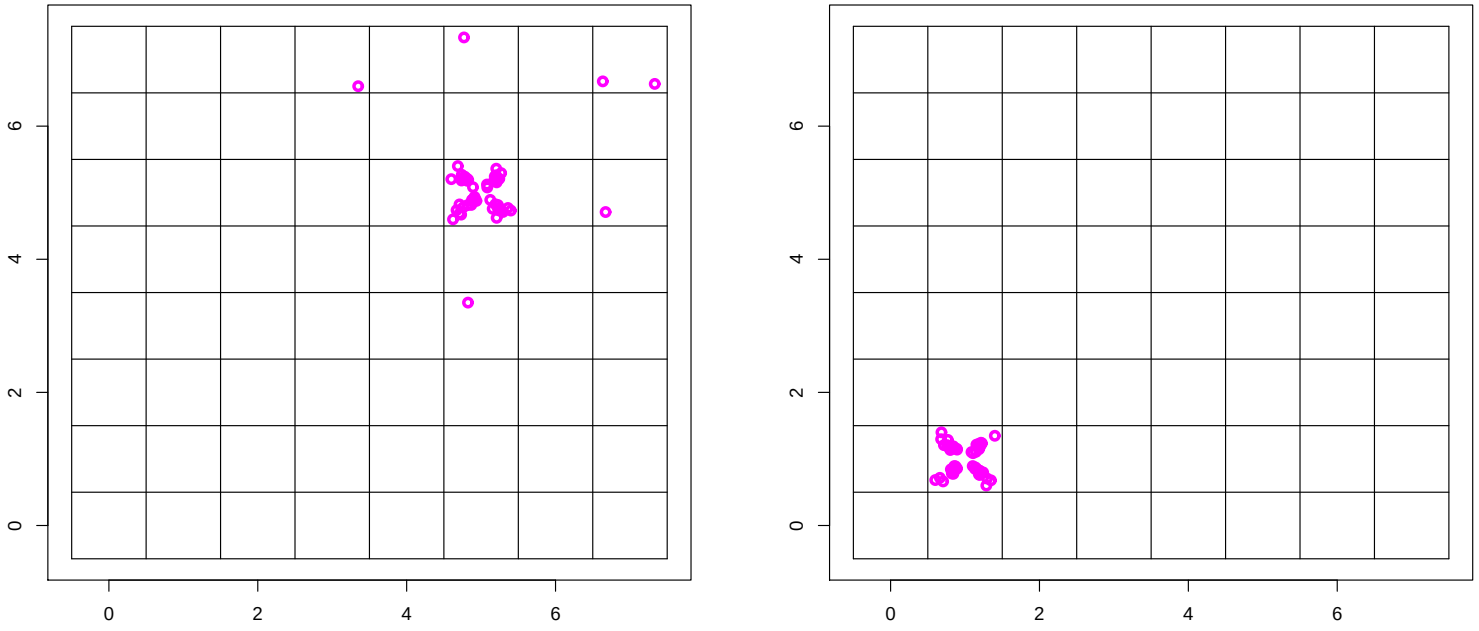
Heathrow data





Seasonality in precipitation is weak. Seasonality in temperature is deterministic.

Austrian economics series



Industrial production and unemployment rate rarely if at all deviate from their typical seasonal patterns.

Summary and conclusion

- The visualization by jittered phase plots is an appealing and potentially helpful tool that is meant to be used together with (not replacing) traditional hypothesis tests;
- Good discrimination requires samples that are slightly longer than those that are typically available;
- The general impression from application to various data sets is that good seasonal unit-root processes are rare in practice.

Thank you for your attention