First midterm test in Advanced Econometrics
Tentative answers

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April 18, 2013

1. For the following ARMA models, determine the relevant characteristic polynomials and conclude whether the processes are stable. What distinguishes a stable process from a stationary process?

   (a) $X_t = \varepsilon_t$
   (b) $\Delta X_t = \varepsilon_t$
   (c) $X_t = 0.5X_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$
   (d) $X_t = \varepsilon_t + 2\varepsilon_{t-1}$

   Answers: In Verbeek's notation, we have $\theta(z) = \alpha(z) = 1$ for (a), $\theta(z) = 1 - z$ and $\alpha(z) = 1$ for (b), $\theta(z) = 1 - 0.5z$ and $\alpha(z) = 1 + 0.5z$ for (c), and $\theta(z) = 1$ and $\alpha(z) = 1 + 2z$ for (d). Models (a) and (c) clearly define stable processes, with all roots, if any, larger than one (none in (a), and 2 and -2 in (c)). Similarly, the process defined in (b) is clearly unstable, with the only root at unity. Model (d) is a bit more complicated, with the characteristic polynomial apparently 'unstable' (if we want to use this terminology on polynomials), while the thus defined MA process is stable anyway. The wording varies slightly across authors in time series, with recommended usage being the one used by Helmut Lütkepohl: stationary processes have time-constant first and second moments, while stable processes approach these time-constant moments if started from arbitrary starting conditions as time increases. A different word for 'stable process' is 'asymptotically stationary process'.

2. A friend of yours wishes to fit an ARMA model to her data. She tries out all ARMA(p, q) models for $0 \leq p \leq 4$ and $0 \leq q \leq 4$, and she obtains a minimum AIC at $p = q = 2$ and a minimum BIC at $p = 2, q = 0$, a maximum AIC at $p = 4, q = 3$ and a maximum BIC at $p = q = 4$. Which ARMA model would you suggest to use, or would you recommend running more tests? In the latter case, describe which tests you would suggest.

   Answers: Whether you follow the recommendation by AIC or by BIC, you should always minimize these criteria never maximize them. Otherwise, there is no definite answer, and preferences are often subjective. Both AIC and BIC do not work extremely well in small samples, and particularly AIC is often modified or 'corrected', as it tends to choose too complex models. Recommendations for using additional tests also vary across authors. Not much additional information can be obtained from any tools that are intrinsically linked to correlation: F tests, Q tests, or correlograms are unlikely to assist in model selection after the choice by an information criterion. Tests on coefficient stability and of course unit-root tests may make more sense. Durbin-Watson tests should not even be mentioned, they are invalid.

3. You wish to test for unit roots in trending data using the Dickey-Fuller test, but you only have access to a software that does not explicitly provide that test, just regression analysis. You also have access to tables of significance points.

   (a) In a preliminary lag order search, you choose an AR(3) model as having the best fit to your data. Indicate the regression that you would have to run now, and also indicate where in the typical regression printout you would find the test statistic that you should compare to your table of significance points.

   (b) What distinguishes the DF–μ from the DF–τ test?

   (c) Assuming that the DF–test rejects, what is your conclusion concerning the generating process for your data?

   Answers: The regression to be run is

$$\Delta Y_t = \mu + aY_{t-1} + \beta_1 \Delta Y_{t-1} + \beta_2 \Delta Y_{t-2} + u_t,$$

   and the crucial statistic is the t–value of the coefficient a or $\hat{a}$. Note that this regression model with two lags corresponds to an AR(3) model for Y_t. This is the DF–μ version. In the DF–τ version, an additional time trend should be inserted among the regressors. Rejection by the DF test appears to imply a stationary (DF–μ) or a trend-stationary (DF–τ) data-generating process. Note, however, that some authors consider two-sided DF tests, with rejection in the right tail implying an explosive process.

4. A (white-noise) process (Y_t) follows the ARCH model

$$\mathrm{var}(Y_t|\mathcal{I}_{t-1}) = \sigma_t^2 = 1 + 0.5Y_{t-1}^2$$

and has been started in the distant past, such that it can be regarded as stationary. Evaluate the unconditional variance $\mathrm{var}Y_t$.

Answers: Applying the expectation operator on this definitional equation yields

$$\mathrm{E}\{\mathrm{var}(Y_t|\mathcal{I}_{t-1})\} = \mathrm{var}(Y_t) = 1 + 0.5\mathrm{E}(Y_{t-1}^2) = 1 + 0.5\mathrm{var}(Y_{t-1}),$$

and thus $\mathrm{var}Y_t = 2$ for stationary Y.