Residual entanglement of accelerated fermions is not nonlocal

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Entanglement in motion

If we consider **two observers**, where one of them **accelerates uniformly**, two scenarios naturally arise. The state can be entangled w.r.t. bosonic or fermionic modes [2]. Due to the **Unruh effect** we have a **degradation of entanglement**. The difference of bosonic and fermionic modes is that the **entanglement does not vanish for the fermionic case** in the infinite acceleration limit.

So the question at hand is:
What is the operational meaning of the nonvanishing entanglement of fermions [1]?

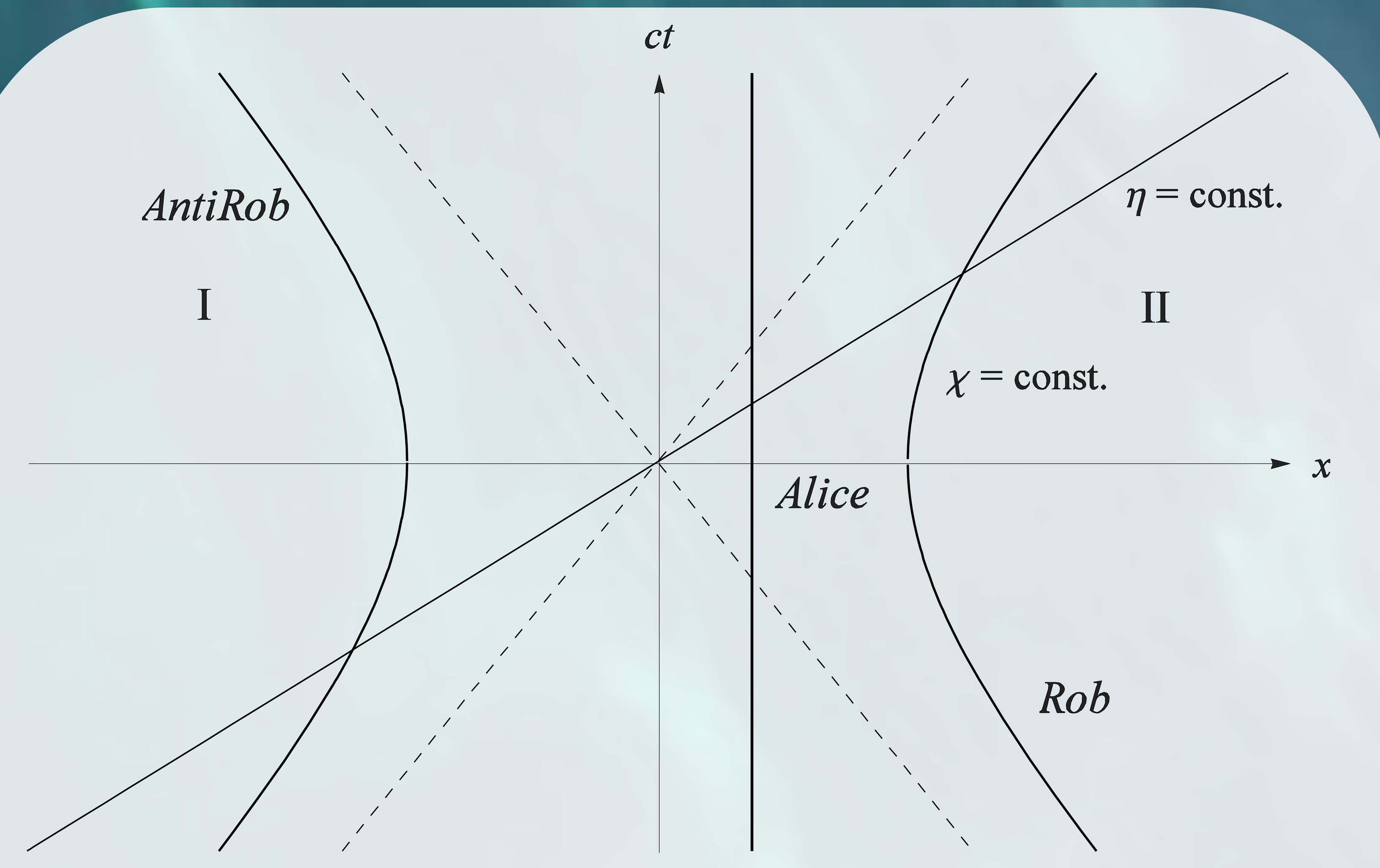

Fig. 1: The uniformly accelerated observers are confined to their respective Rindler wedges I ($|t| < x$) and II ($|t| < -x$). These wedges are causally disconnected from each other. Their worldlines are hyperbolas, which correspond to to lines of constant $\chi = \frac{c^2}{a}$, where a is their proper acceleration and $0 < \chi < \infty$.

Rindler coordinates

Our two observers will be **Alice** and **Rob**, who will be the one to **uniformly accelerate**. His coordinates are related to the coordinate frame at rest of Alice.

$$ct = \chi \sinh\left(\frac{a\eta}{c}\right), x = \chi \cosh\left(\frac{a\eta}{c}\right)$$

These Rindler coordinates do **not cover the whole Minkowski spacetime**. Thus we need more sets of coordinates, e.g.

$$ct = -\chi \sinh\left(\frac{a\eta}{c}\right), x = -\chi \cosh\left(\frac{a\eta}{c}\right)$$

for region I. For both relevant regions (I,II) the coordinates are defined over the whole domain $(-\infty, \infty)$. Therefore, we can find **independent canonical field quantizations**.

Connection between inertial and accelerated observer

We can expand our field modes into a complete set of solutions of the **Dirac equation in Minkowski- or Rindler coordinates**.

$$[i\gamma^\mu(\partial_\mu - \Gamma_\mu) + m]\phi = 0$$

Hence the field operator ϕ can be expressed as

$$\phi_M = N_\mathrm{M} \sum_k \left(c_{k,\mathrm{M}}\, u^+_{k,\mathrm{M}} + d^\dagger_{k,\mathrm{M}}\, u^-_{k,\mathrm{M}}\right),$$

$$\phi_R = N_\mathrm{R} \sum_j \left(c_{j,\mathrm{I}} u^+_{j,\mathrm{I}} + d^\dagger_{j,\mathrm{I}} u^-_{j,\mathrm{I}} + c_{j,\mathrm{II}} u^+_{j,\mathrm{II}} + d^\dagger_{j,\mathrm{II}} u^-_{j,\mathrm{II}}\right).$$

By finding the transformation between those two solutions we can **compare accelerated and inertial observers**. This can be done with the **Bogoljubov transformation**.

$$u^+_{j,\mathrm{M}} = \sum_k \left[\alpha^\mathrm{I}_{jk} u^+_{k,\mathrm{I}} + \beta^\mathrm{I}_{jk} u^-_{k,\mathrm{I}} + \alpha^\mathrm{II}_{jk} u^+_{k,\mathrm{II}} + \beta^\mathrm{II}_{jk} u^-_{k,\mathrm{II}}\right]$$

The coefficients α, β connect our modes. The annihilation and creation operators can also be related in the different coordinate sets.

For fixed acceleration, we can find combinations of Minkowski modes which transform into monochromatic Rindler modes. These so called **Unruh modes** have the **annihilation operators**

$$C_{k,\mathrm{R/L}} \equiv \left(\cos r_k c_{k,\mathrm{I/II}} - \sin r_k d^\dagger_{k,\mathrm{II/I}}\right), \quad \tan r_k = e^{-\pi c\Omega/a}$$

Or more generally, if we not use the **single mode approximation**

$$c^\dagger_{k,\mathrm{U}} = q_\mathrm{L}(C^\dagger_{\Omega,\mathrm{L}} \otimes \mathbb{1}_\mathrm{R}) + q_\mathrm{R}(\mathbb{1}_\mathrm{L} \otimes C^\dagger_{\Omega,\mathrm{R}}).$$

Bell inequalities and accelerated states

Let us consider the following state

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|0_\omega\rangle_\mathrm{A} |0_\Omega\rangle_\mathrm{U} + |1_\omega\rangle^\epsilon_\mathrm{A} |1_\Omega\rangle^\pm_\mathrm{U}\right)$$

If we now **accelerate Rob's part** of the state we have to transform his part with respect to the acceleration. This is done by using the above mentioned **Bogoljubov transformations**. Because the **Rindler coordinates** do **not cover the whole space** we have to **trace over the region** we do not have access to, thus arriving at the **density matrix**

$$\rho^+_{AR+} = \frac{1}{2}\begin{pmatrix} \cos^2 r_\Omega & 0 & 0 & q_R^* \cos r_\Omega \\ 0 & \sin^2 r_\Omega & 0 & 0 \\ 0 & 0 & |q_L|^2 \cos^2 r_\Omega & 0 \\ q_R \cos r_\Omega & 0 & 0 & |q_R|^2 + |q_L|^2 \sin^2 r_\Omega \end{pmatrix}.$$

We already know from previous works that this state **remains entangled** even for **infinite acceleration** ($r_\Omega \to \frac{\pi}{4}$). The **concurrence** in this case cannot exceed $C = \frac{1}{\sqrt{2}}$.
To calculate the **violation of a Bell inequality** we use a theorem of Horodecki et.al.[3]. Thus the **Bell-CHSH expectation value** becomes

$$\langle \mathcal{B}_{max} \rangle_{\rho^+_{AR+}} = 2\sqrt{2}|q_R| \cos r_\Omega$$

Since we have a **local realistic state** if $\langle \mathcal{B}_{max} \rangle \leq 2$ **the accelerated state cannot be nonlocal in the infinite acceleration limit!**

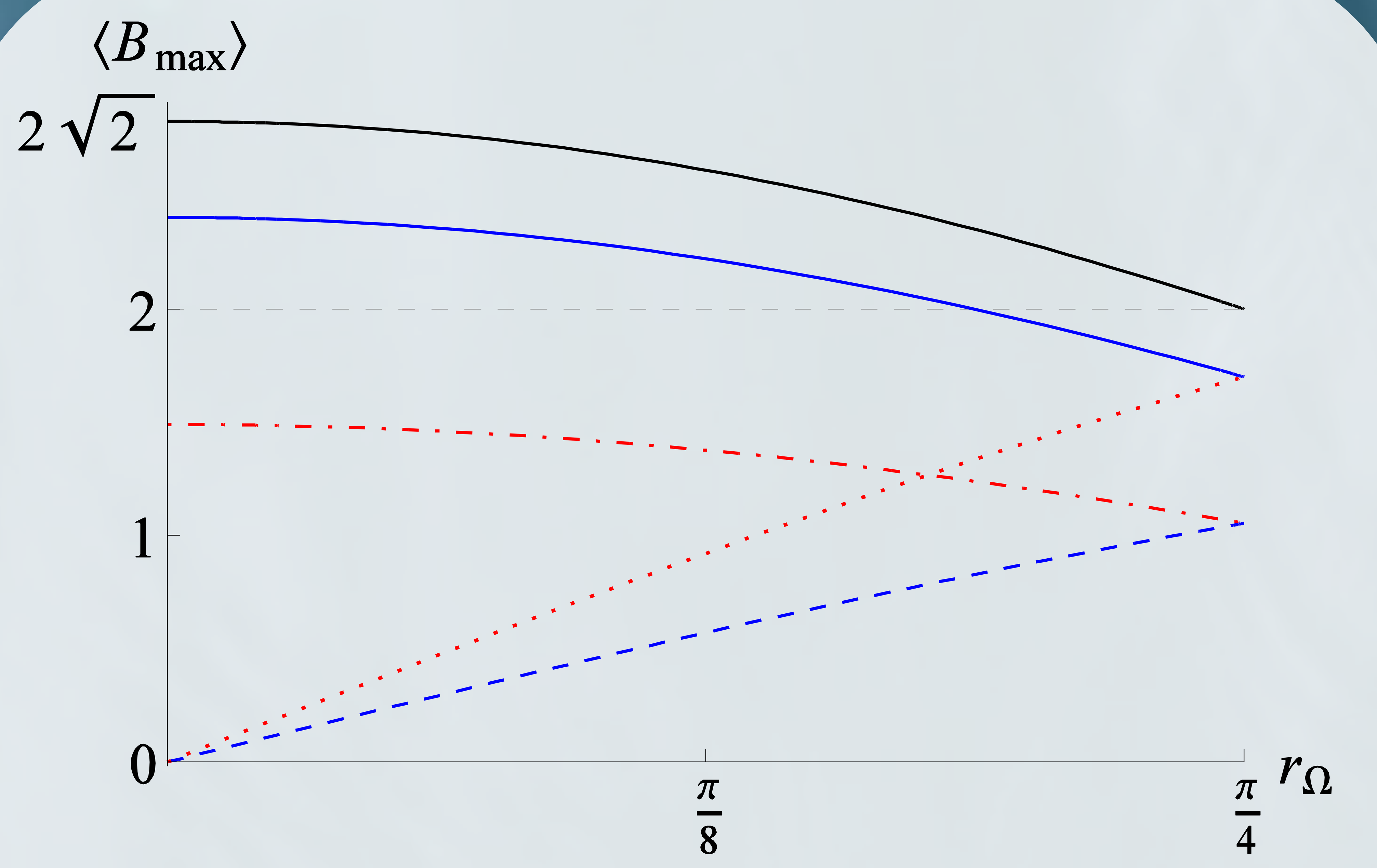

Fig. 2: Maximal Bell-CHSH parameter $\langle \mathcal{B}_{max} \rangle$ for the states ρ^+_{AR+} (blue solid, second from the top), ρ^+_{AR-} (blue dashed), $\rho^+_{A\bar{R}+}$ (red dot-dashed), $\rho^+_{A\bar{R}-}$ (red dashed) for $q_R = 0.85$ and ρ^+_{AR+} (purple solid, topmost) for $q_R = 1$.

No nonlocality remains in any of the reduced states in the limit $r \to \frac{\pi}{4}$!

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