The Quantum Zeno Effect
and
Interaction-free Measurements

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Contents

1 Abstract

This thesis will be about understanding the quantum Zeno effect and its use in improving interaction-free measurements. First, it will give a brief mathematical derivation of the non-exponential form of the survival probability for unstable systems when considering short times. There it will become clear, that frequent measurements can lead to an inhibition of transition between states. This effect is called the quantum Zeno effect. Then we will proceed to a presentation of Cook's proposal of experimental verification of the quantum Zeno effect and its realisation by Itano et al. in an experiment based on Cook's idea. Afterwards there will be an explanation of the basic concept of an interaction-free measurement. At last it will be shown, how interaction-free measurements can be improved by use of the quantum Zeno effect.

2 Introduction - Or where the Zeno Paradox got its name from

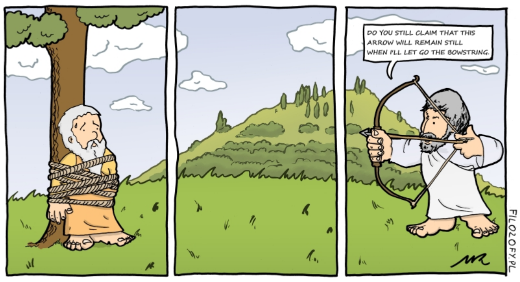

Figure 2.0.1: Comical view on Zeno's Arrow Paradox
source: `http://filozofy.blox.pl/resource/93_Strzala_blog_e.jpg`

Already very early in history, greek philosopher Zeno asked the question whether all motion is just illusory. To explain his reasoning, he introduced the famous arrow paradox. It states, that if one considers an arrow flying through space and one furthermore just observes it at a certain instant of time, then one cannot distinguish between movement or rest. For developing his paradoxon, he assumed that time is made up of several discrete, durationless instants. Because no time elapses during this instant, the arrow will not be seen to move, it is therefore indistinguishable to a resting one. Even if one considers several ones, there is no instant in time, where the arrow will ever be seen to move. So Zeno asks if one can generally say, that there exists something as motion.[1]

Of course we know today, that there are multiple mistakes in Zeno's reasoning. If the instants of time he is talking about are not exactly of zero duration, one will be able to see the object at a different point in space at each moment of observance. But even if the duration of these intstants of time is zero and one adds them up, one will never recieve a total time interval with a nonzero duration.

Nevertheless, the Zeno effect is named after Zeno's arrow paradox, because it describes the phenomenon of a possible inhibition of transition between states, under the premise that the regarded system is measured frequently enough. This is in correspondance to this idea of several instants of time, where the arrow is never seen to move. (Does not change its state.)

3 Decay of unstable systems - short time derivation

3.1 Classically

For reasons of simplicity, let's consider an unstable system, such as an unstable particle. Classically, this particle is assigned a constant decay probability per unit time Γ, which is just the inverse of the particle lifetime τ_E. Note, that the decay probability is neither dependent on environmental influences, nor the total number of unstable systems considered. Now focusing on N of such systems at a time t, the decay probability of those N systems in a certain time interval δ_t is governed by:

$$-dN = N\Gamma dt = \frac{N}{\tau_E}dt \tag{3.1}$$

If the equation is partially integrated, this leads to:

$$N(t) = N_0 e^{-\frac{t}{\tau E}} \tag{3.2}$$

The classical probability of nondecay is then given by

$$P(t) = \frac{N(t)}{N_0} = e^{-\frac{t}{\tau E}} \tag{3.3}$$

If one now performs a taylor expansion of the nondecay probability for short times, one can easily see that the approximation will possess a term linear in t.

$$P(t) = e^{-\frac{t}{\tau E}} \approx 1 - \frac{t}{\tau_E} + \mathcal{O}(t^2) \tag{3.4}$$

Quantum mechanicly, the short time approximation of the survival probability will lead to a different result[5].

3.2 Quantum-mechanically

To obtain a law for the time evolution of a quantum state $|\psi(t)>$, the general form of the time-dependent Schrödinger-Equation is considered[5]:

$$i\hbar\partial_t|\psi_t>= H|\psi_t> \tag{3.5}$$

The formal solution in the Schrödinger picture is then given by

$$|\psi_t>= U(t,t')|\psi_{t'}> \tag{3.6}$$

with U being the time evolution operator. If one now futhermore assumes, that H does not depend on t and one considers the initial conditions

$$i\hbar\partial_t U(t,t') = HU(t,t') \tag{3.7}$$

$$-i\hbar\partial_t U(t,t') = U(t,t')H \tag{3.8}$$

$$U(t,t) = I \tag{3.9}$$

then U can be written as:

$$U(t,t') = e^{\frac{-iH(t-t')}{\hbar}} \tag{3.10}$$

Knowing this about the time evolution operator, one can proceed to describing the short-time behaviour of an unstable system quantum-mechanically. Therefore, $|\psi_0>$ is chosen to be the state of the named system at t=0. With use of (3.10) one can conclude that the temporal evolution is given by:

$$|\psi_t>= U(t)|\psi_0> \tag{3.11}$$

The probability of the regarded system still being in the state $|\psi_0>$ at a later time t is governed by the square modulus of the survival amplitude:

$$P(t) = | < \psi_0 | e^{-iHt} | \psi_0 > |^2 \tag{3.12}$$

If the exponential function is then expanded into a taylor-series up to the second order, one receives the following result for the approximated survival probability of the system:

$$P(t) \simeq 1 - \frac{t^2}{\tau_Z^2} \tag{3.13}$$

where

$$\tau_Z^{-2} = < \psi_0 | H^2 | \psi_0 > - < \psi_0 | H | \psi_0 >^2 \tag{3.14}$$

is called the "Zeno time". In contrast to the classical case, the survival probability is now quadratic in t. So theoretically, if t $\rightarrow 0$ this would lead to a vanishing decay rate and therefore to the inhibition of transition of the state $|\psi_0 >$ regarded here to some other state.

It is also interesting to have a look at what is happening if several measurements are performed. Let $\tau = \frac{T}{N}$ be equal intervals of time at which N measurements are performed. The probability of the system still being in its initial state shall be derived. Every single measurement can be seen as a projection of the system onto the state that represents the outcome of the measurement. With the measurement being performed, the temporal evolution starts anew. It will be seen, that the survival probability will increase.

The probability for the system to still be in its initial state at the time T (after N measurements) is given by

$$\begin{aligned} P_N(T) = [P(\tau)]^N &\simeq \left(1 - \frac{1}{\tau_Z^2}\left(\frac{T}{N}\right)^2\right)^N \\ &\sim 1 - \frac{1}{N}\frac{T^2}{\tau_Z^2} \end{aligned} \tag{3.15}$$

As one can see now, repeated measurements decrease the decay probability. This phenomenon is called the quantum Zeno effect. If furthermore the case of taking the limit of N $\rightarrow \infty$, so to say the case of a continuous measurement, is considered, one speaks of the "Zeno paradox" because of the following the result:

$$\lim_{N\rightarrow\infty} P_N(T) = 1 \tag{3.16}$$

Although the obtained result that the survival probability tends to 1 in the limit of N $\rightarrow \infty$ is mathematically correct, it is paradoxical because the limit of $N \rightarrow \infty$ is practically unattainable due to not being possible to realise this in an experiment. Numerous authors also showed, that the N $\rightarrow \infty$ limit is also unphysical due to various other reasons, like for example the restrictions given by Heisenberg's uncertainty principle or the time-energy uncertainty relation. [5]

Apart from the unphysicality of the N $\rightarrow \infty$ limit, the quantum Zeno effect will not be of relevance in most observed systems, like for example in macroscopic domains. Joos describes this quite efficiently in his book with the following words:"Staring at my pencil will not keep it from falling from my desk"[4]. Furthermore frequent measurements do not affect exponentially decaying systems. For an exponential survival probability $P(t) = e^{-\Gamma t}$, repeated measurements would simply lead to:

$$P_N(t) = \left(e^{-\Gamma \frac{t}{N}}\right)^N = e^{-\Gamma t} \tag{3.17}$$

4 Experimental Verification of the QZE

4.1 Theoretical Concept

To create a setting, where the Zeno effect can be tested experimentally, Itano et al. [2] turned to Cook's proposal. His idea was to take a single trapped ion and induce a transition from one energy level to another, then show, that frequent measurements increase the probability of finding the ion in the starting state.

Cook developed his reasoning by considering the following level structure:

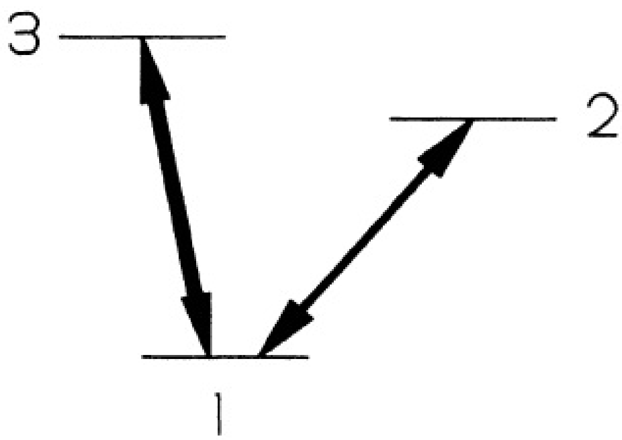

Figure 4.1.1: Picture of energy level structure proposed by Cook to demonstrate the QZE
source: [2]

The diagram shows three levels. Level 1 is the assumed ground state of the ion, level 2 is considered to be a metastable state, with the presumption that spontaneous decay from 2→1 is negligible. Level three is one of the possible dipole transition energy levels. [1]

Let's suppose the regarded ion is in level 1 at the time t=0. A perturbation in the form of a radiofrequency field with the resonance frequency $\omega_{1,2} = \frac{E_2 - E_1}{\hbar}$ is applied. With use of the Jaynes-Cummings-Model (for detailed derivation

[1] Electric dipole transitions underlie a certain selection rule for the quantum numbers n,m,l. If for example one considers a transition from a state $|n, m, l\rangle \rightarrow |n', m', l'\rangle$ based on a hydrogen-like atom, the criteria $\Delta m = 0, \pm 1, \Delta l = \pm 1, \Delta n = n - n'$ have to be met.[11]

see [12]), which describes the dynamics of the interaction of the considered atom and radiofrequency field, one receives the following probabilities P_1 and P_2 for the system to be in level 1, respective level 2 after a measurement is performed:

$$P_1(t) = cos^2\left(\frac{\Omega\tau}{2}\right) \tag{4.1}$$

$$P_2(t) = sin^2\left(\frac{\Omega\tau}{2}\right) \tag{4.2}$$

Here, Ω denotes the Rabi frequency, which is the frequency of oscillations between ground and excited state. If a measurement is performed after a short period of time, meaning $\Omega\tau << 1$, the approximated probabilities read[2]:

$$P_1 \simeq 1 \tag{4.9}$$

$$P_2 \simeq \frac{1}{4}\Omega^2\tau^2 \tag{4.10}$$

[2]The solutions gained here are of this form, because one considers the perturbation to have exactly the transition frequency of the 1→ 2 transition. For the general case, there would be a detuning term $\Delta = \omega - \omega_{1,2}$ and the solution of the corresponding e.o.m. of a density matrix σ_{ij} in the rotating wave frame with suitable initial conditions would read:

$$\sigma_{12} = -\Omega[\Delta(1 - cos(S\tau) + iSsin(S\tau]/2S^2 \tag{4.3}$$

$$\sigma_{11} = [\Delta^2 + \Omega^2 cos^2(S\tau/2)]/S^2 \tag{4.4}$$

$$\sigma_{22} = \Omega^2 sin^2(S\tau/2)/S^2 \tag{4.5}$$

with S being $S = (\Delta^2 + \Omega^2)^{1/2}$.
The short time limit of $S\tau << 1$ would lead to:

$$\sigma_{12} \simeq \frac{1}{2}i\Omega\tau + \frac{1}{4}\Delta\Omega\tau^2 \tag{4.6}$$

$$\sigma_{11} \simeq 1 - \frac{1}{4}\Omega^2\tau^2 \tag{4.7}$$

$$\sigma_{22} \simeq \frac{1}{4}\Omega^2\tau^2 \tag{4.8}$$

where $\sigma_{22} = P_2$, $\sigma_{11} = P_1$ and σ_{12} measures the amount of coherent superposition in σ_{ij}. [3]

Furthermore it is assumed, that the transition from $3 \rightarrow 1$ is strongly allowed and that an atom being in level 3 can not decay to level 2, but only back to level 1. The measurements are performed via the application of optical pulses, which drive the $1 \rightarrow 3$ transition, cause the wave function to collapse and project the ion either onto level 1 or 2.

So now there are two possible cases: Either the ion is projected onto level 1 at the beginning of the pulse, then it cycles between the energy levels 1 and 3. During this cycle it emits several photons, until the pulse is turned off and the ion returns to level 1 [3]. Or the ion can be projected onto level two, where it wouldn't scatter any photons. And because of the assumed negligibility of the spontaneous $2 \rightarrow 1$ decay, the ion doesn't leave the level during the measurement.

If a second measurement is applied after the first one, the results can slightly differ, because during the time between the measurement pulses, the wave function evolves.

Now to Cook's proposal of experimental verification of the Zeno Effect: His idea was, to drive the $1 \rightarrow 2$ transition with a so called π pulse. The π pulse is a pulse of duration $T = \frac{\pi}{\Omega}$. When applied to an ion sitting in level 1 at $\tau = 0$, this yields the following result for the probabilities 1 and 2 at $\tau = T$:

$$P_1(T) = 0 \tag{4.11}$$

$$P_2(T) = 1 \tag{4.12}$$

So here it is easy to see that the π pulse indeed drives the transition from 1 $\rightarrow$ 2. The level population after n measurement pulses are applied, can be determined by considering the vector representation of a two-level system.

$$R_1 = \rho_{12} + \rho_{21} \tag{4.13}$$

$$R_2 = i\,(\rho_{12} - \rho_{21}) \tag{4.14}$$

$$R_3 = \rho_{22} - \rho_{11} \tag{4.15}$$

In their paper, Itano et al. refer to a more detailed derivation for this special vector R, where the authors Rabi, Ramsay and Schwinger look at rotating coordinates in magnetic resonance problems and for simplification assume the

[3] the time of its return is approximately equal to the lifetime of level 3

considered system to be of spin $\frac{1}{2}$. But this is actually not necessary here. If one simply takes the common Bloch-representation of a density matrix for spin $\frac{1}{2}$ particles

$$\rho = \frac{1}{2}\left(1 + \vec{a}\vec{\sigma}\right) \tag{4.16}$$

and takes a closer look at the coefficients a_i which are the entries of the Bloch-vector and determined via the expectation value

$$a_i = tr\left(\rho\sigma\right) \tag{4.17}$$

then the calculation of the a_i leads to:

$$a_1 = tr(\rho\sigma_1) = tr\begin{pmatrix}\rho_{12} & \rho_{11}\\ \rho_{22} & \rho_{21}\end{pmatrix} \tag{4.18}$$

$$a_2 = tr(\rho\sigma_2) = tr\begin{pmatrix}i\rho_{12} & -i\rho_{11}\\ i\rho_{22} & -i\rho_{21}\end{pmatrix} \tag{4.19}$$

$$a_3 = tr(\rho\sigma_3) = tr\begin{pmatrix}\rho_{11} & -\rho_{12}\\ \rho_{21} & -\rho_{22}\end{pmatrix} \tag{4.20}$$

So for the vector R ($=\vec{a}$) one gets:

$$\vec{R} = \begin{pmatrix}\rho_{12} + \rho_{21}\\ i\left(\rho_{12} - \rho_{21}\right)\\ \rho_{11} - \rho_{22}\end{pmatrix} = \begin{pmatrix}\rho_{12} + \rho_{21}\\ i\left(\rho_{12} - \rho_{21}\right)\\ P_1 - P_2\end{pmatrix} \tag{4.21}$$

The only thing different is the opposite sign in the R_3 component, which is chosen to be reversed due to the initial conditions. This means, that level 1 is simply assumed to geometrically correspond to the pure state $\vec{a} = (0, 0, -1)$ on the Bloch sphere.

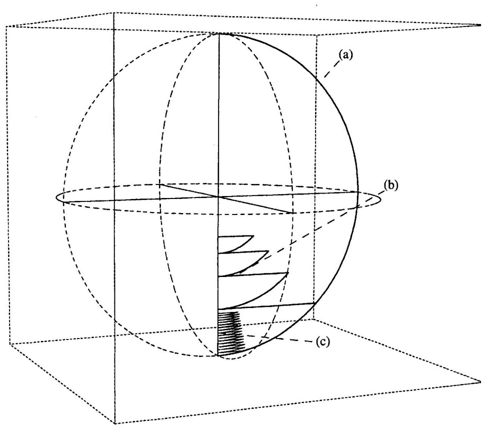

Figure 4.1.2: Bloch sphere
source: [4]

Path a in the picture describes the unitary time evolution of the starting state. Without performing any measurements, the temporal evolution will lead the Bloch vector from the south pole to the north pole during one half period of a Rabi oscillation. Path b shows the time evolution under the application of 4 measurements (interruptions of the unitary time evolution process), which correspond to a projection on the z-axis. After each projection, this process starts anew, but with a shorter starting vector. It is clearly visible, that path c, with 16 measurements performed, indicates the Zeno effect, because the time evolved state barely moves away from the starting state. [4]

The equation of motion for R is given by:

$$\frac{dR}{dt} = \omega \times R \tag{4.22}$$

for $\omega = (\Omega, 0, 0)$ the e.o.m. leads to the following two equations:

$$\frac{dR_2}{dt} = -\Omega R_3 \tag{4.23}$$
$$\frac{dR_3}{dt} = \Omega R_2 \tag{4.24}$$

the solution of the e.o.m. will therefore be of the form:

$$R_3 = -cos(\Omega\tau) \tag{4.25}$$
$$R_2 = sin(\Omega\tau) \tag{4.26}$$

For $\tau = 0$, R will be R$= (0, 0, -1)$[2].

The application of the first measurement pulse at $\tau = \pi/(n\Omega)$ projects the ion onto one of both levels (1 or 2). For the density matrix this means, that the coherences (ρ_{21} and ρ_{12}) will be set to 0. The populations ρ_{11} and ρ_{22} will not be changed. So R will be of the form:

$$R = [0, 0, -cos(\pi/n)] \tag{4.27}$$

One can now express the probability P_2 as follows:

$$P_2 = R_3 + P_1 \tag{4.28}$$
$$= R_3 + (1 - P_2) \tag{4.29}$$
$$P_2 = \frac{1}{2}(1 + R_3) \tag{4.30}$$

Plugging in the R from above for the case of n measurements having been performed, one can conclude:

$$P_2(T) = \frac{1}{2}[1 - cos^n(\pi/n)] \tag{4.31}$$

In the limit of $n \to \infty$ this tends to zero. So what has actually happened, is that the transition from $1 \to 2$ has been hindered by frequent measurements.

4.2 Verification of the QZA - Experiment by Itano, Wineland, Heinzen and Bollinger

For their experiment, Itano et al. used $^9Be^+$-ions. Level 1 and 2 were the $(m_I, m_J) = (\frac{3}{2}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ hyperfine sublevels of the ions in the ground state $2s^2S_{1/2}$. Level 3 was the $(\frac{3}{2}, \frac{3}{2})$ sublevel of the $2p^2P_{3/2}$ state. The third level can only decay to level 1. And as proposed by theory, spontaneous level 2$\rightarrow$ 1 decay is negligible.

They stored the $^9Be^+$-ions in a penning trap for several hours. In order to laser cool, optically pump and detect the ions, a radiation field of 313 nm was generated by a laser to drive the 1$\rightarrow$3 transition. When the radiation was nearly resonant with this transition, approximately $\frac{16}{17}$ of the ions were pumped to sublevel $(\frac{3}{2}, \frac{1}{2})$, the rest was at sublevel $(\frac{3}{2}, -\frac{1}{2})$.

At the start of the measurement, the 313-nm radiation was left on for 5 seconds, leading to a preparation of most of the $^9Be^+$-ions in level 1. Afterwards, the radiation was turned off and an on-resonant π pulse in the form of a 320.7 MHz radiofrequency field was turned on for T=256ms. During the π pulse, n measurements were performed, with n being 1, 2, 4, 8, 16, 32 and 64. That means, that the 313-nm pulses were timed in intervals of length T/n. At the end of the π pulse, the 313-nm field was turned on to prepare the state. Itano et al. recorded the number of photons in the first 100 ms, which gave a signal vaguely proportional to the number of ions sitting in level 1. After the calibration of the signal, their results showed an experimental verification of the quantum Zeno effect. For an increasing number of measurement pulses during the application of the π pulse, the experiment showed a decrease of the transition probability.

5 Interaction-free Measurements and Improvement by use of the Quantum Zeno Effect

On the following pages, the general idea of interaction-free measurements will be discussed. Then it will be shown, how one can make use of the quantum Zeno effect to improve interaction-free measurements.

5.1 Basic concept of Interaction-free Measurements

In 1993, Elitzur and Vaidman [6] developed a simple construct to determine, if an object is located along a certain path. The exeptionally unsusual thing about their method is, that the presence of the object can be discovered without the interaction of a photon with the object.

For their reasoning, they considered the following scheme:

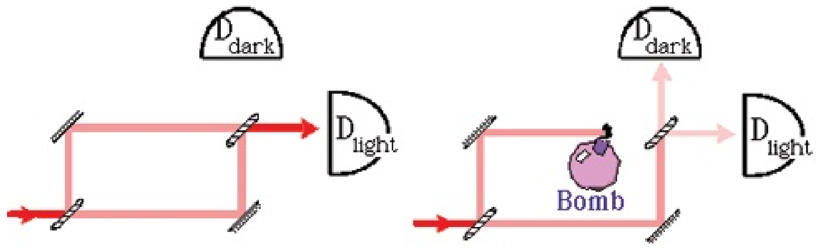

Figure 5.1.1: Interaction-free measurement scheme EV source:http://physics.illinois.edu/people/kwiat/interaction-free-measurements.asp

The picture shows a particle interferometer which is similar to a Mach-Zehender interferometer. The interferometer consists of two perfect mirrors and two beam-splitters with transmission coefficient $\frac{1}{2}$ .The chance for an incoming particle to be transmitted is 50%. The probability for reflection is given by $P_R = 1 - P_T = 50\%$.

First, one takes a look at the case, where no object is blocking any path. A single photon enters the interferometer, and either is transmitted or reflected by the first beamsplitter. The probability for the photon to take path one or two is exactly the same. With equal path lengths, there will be destructive interference for the upper detector port due to the wave-like properties of a particle. Therefore, detector D_{dark} won't ever respond. The probability for the photon to be detected at D_{light} will on the contrary be equal to P_{light}=1. [10]

If one turns to the case, where there's an object, which traditionally Elitzur and Vaidman assumed to be a bomb, blocking one path, the photon can collide with the bomb at a 50% chance. Or it can take the path that contains no object and pass through the second beam splitter. Due to the lack of interference occurences in the latter case, the photon can be detected either at D_{dark} or D_{light} with a 25% probability for each of both possibilities[10].

The following shall be a formalistic example for the evolvement of the photon state during its passage through the interferometer[6]:

A photon moving to the right will be denoted by $|1\rangle$, a photon moving up is expressed by the state $|2\rangle$. The mathematical operation that happens, when a photon collides with a beamsplitter is:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle] \tag{5.1}$$

$$|2\rangle \rightarrow \frac{1}{\sqrt{2}}[|2\rangle + i|1\rangle] \tag{5.2}$$

The beam splitter signifies a rotation in the x-y-plane around the angle $\frac{\pi}{2}$. Therefore the acquired phase-factor, which is usually denoted by $e^{i\Theta}$ simply becomes i.

The operation of the perfect mirrors is given by:

$$|1\rangle \rightarrow i|2\rangle \tag{5.3}$$

$$|2\rangle \rightarrow i|1\rangle \tag{5.4}$$

In the undisturbed interferometer, the incident photon's state now evolves like this:

$$\begin{aligned} |1\rangle \xrightarrow{1^{st}BS} \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle] &\xrightarrow{mirror} \frac{1}{\sqrt{2}}[i|2\rangle - |1\rangle] \\ &\xrightarrow{2^{nd}BS} \frac{1}{2}[i|2\rangle - |1\rangle] - \frac{1}{2}[|1\rangle + i|2\rangle] = -|1\rangle \end{aligned} \tag{5.5}$$

As one sees, the photon leaves the interferometer and moves to the right, directly towards detector D_{light}.

On the other hand, when there's a bomb inbetween one of both possible photon-paths, the evolution of the photon state is described by:

$$\begin{aligned}|1\rangle \xrightarrow{1^{st}BS} \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle] &\xrightarrow{mirror} \frac{1}{\sqrt{2}}[i|2\rangle + i|scattered\rangle] \\ &\xrightarrow{2^{nd}BS} \frac{1}{2}[i|2\rangle - |1\rangle] + \frac{i}{\sqrt{2}}|scattered\rangle \qquad (5.6)\end{aligned}$$

Here, $|scattered\rangle$ denotes the state of the photon, which is scattered or absorbed by the object (bomb). The coefficients of the final state lead to the following probabilities:

$$P_{D_2} = \frac{1}{4} \qquad (5.7)$$

$$P_{D_1} = \frac{1}{4} \qquad (5.8)$$

$$P_{noclicks} = \frac{1}{2} \qquad (5.9)$$

This is exactly the same as already seen before.

The measurement efficiency for a lossless system, which is defined by the fraction η of measurements that can be interaction free is given by[7]:

$$\eta = \frac{P(det)}{P(det) + P(abs)} \qquad (5.10)$$

P(det) denotes the probability of the photon to be detected, whereas P(abs) is the probability of it being absorbed by the object (bomb). For the case considered here, the efficiency tends to 50%.

In general one can sum up the results of the interaction-free measurement scheme proposed by Elitzur and Vaidman as follows: When there was no object blocking one of both paths, detector D_{dark} never clicked. Therefore when one can observe the detection of a photon in detector D_{dark}, one knows, that there must have been an object placed in one of the paths. The single photon that is sent in the measurement apparatus, does not have to interact with the object itsself, to lead to the conclusion, that there is in fact an object present. Of course if it would take the path, where it interacts with the object, one would see an explosion in the case of the object being a bomb. The goal of the measurement scheme should now be to decrease the probability of the photon interacting with the bomb, which is exactly what Kwiat et al. focussed on.

5.2 Improvement of Interaction-free Measurements by using the Quantum Zeno Effect

For gaining a tool to make use of the Zeno effect and improve interaction-free measurements with it, one shall first turn to the question of how polarization rotators work. A polarization rotator simply rotates the polarization of an incoming photon by a certain angle ϕ. If one now considers an initially horizontally polarized photon passing through N polarization rotators, with each rotator changing the polarization by an angle $\phi = 90^o/N$ and then adds a horizontal polarizer at the end, the photon will not pass through the horizontal polarizer. Because of its vertical polarization after it has travelled through the N polarization rotators, it will not be detected.

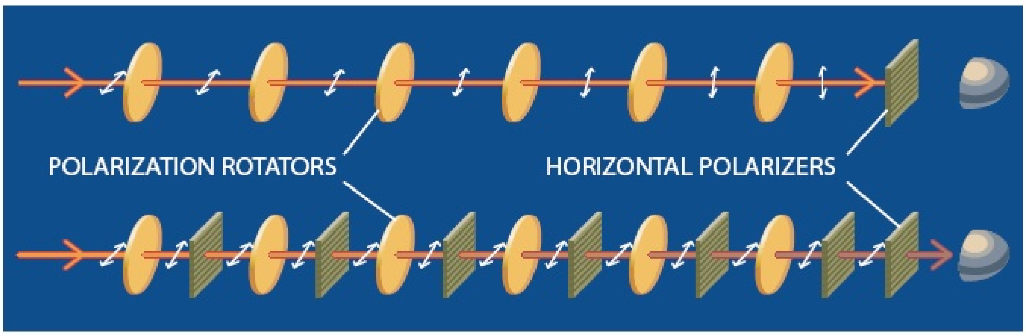

Figure 5.2.1: Polarization rotators and inserted horizontal polarizers source:[8]

If there are instead horizontal polarizers inserted after each polarization rotator, the photon has a 66% chance to be detected. This can be seen by looking at the probability of absorption at the first horizontal polarizer, which is given by $P_{abs_1} = sin^2(15^o)$=16%. Analogously, the probability for the photon to pass through the horizontal polarizer is governed by $P_{trans_1} = cos^2(15^o)$. After N times, the chance of the photon being detected is simply given by

$P_{trans} = [cos^2(15^o)]^N$. In the case of N being N=6, this will yield a probability of detection of 66%. When N becomes large, the probability for the photon to be absorbed vanishes. So this can be seen as an optical version of the Zeno effect.

An implementation of the discussed matter can be realised in the following way:

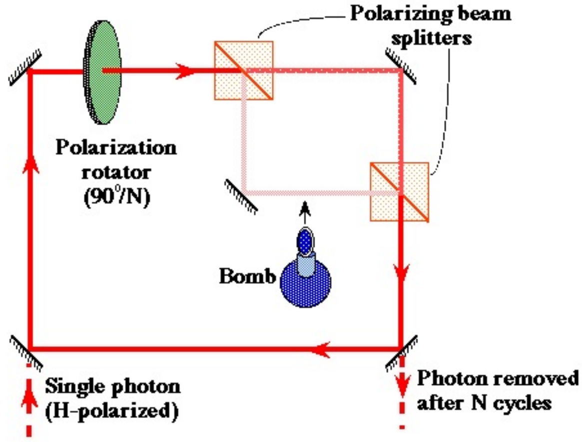

Figure 5.2.2: Interaction-free measurement scheme
source:http://physics.illinois.edu/people/kwiat/interaction-free-measurements.asp

Here, a polarization interferometer is simply included in the previous Mach-Zehnder interferometer. But instead of normal beam splitters, polarizing beam splitters are introduced. As a measurement basis, the H/V-basis is chosen. They transmit horizontally polarized photons and reflect the vertical ones. If again the case of N=6 is considered, then after the first cycle, the probability for the photon to take the lower path and be absorbed is $P_{abs_1} = sin^2(15^o)$=16%. If it is not being absorbed, the polarizing beam splitter projects the photon onto the horizontal polarization state and it takes the upper path. This process is repeated N times and after the Nth cycle, the photon is allowed to leave and its ploarization is measured. By the measurement of its polarization one can then tell if there has been an object blocking a path. For the case where there is no object located in between one of both paths, the detected photon would always be a vertically polarized one after the final measurement. This is due to the fact that the initial horizontal polarization is rotated stepwise to a vertical polarization.

With the implementation described above, which makes use of an optical form of the quantum Zeno effect, Kwiat et al. were able to observe measurement efficiencies of up to 73% (which correspond to detections of the horizontal polarized photons with an object present inbetween one of the paths). Furthermore they demonstrated a feasibility of efficiencies of up to 85%. [9]

6 Conclusion

The origins of the idea of inhibiting a certain transition are dated far, far back at the time of ancient Greek philosopher Zeno's productive period. The development of his famous arrow paradoxon set a ball rolling, that is still moving today. His idea of considering motion as just an illusion led to the question, if one can indeed really slow down certain kinds of transitions between quantum states, by just observing the system frequently enough. So to say, record it at several small instants of time and see, that the system does not change at all. When physicists like Peres, Cook and many others discovered, that such effects could actually be derived formally with the use of quantum mechanics, they lay the foundation for improving certain measurement schemes. By introducing an optical version of the Zeno effect, it was possible, to increase the efficiency of interaction-free measurements. Instead of seeing a bomb explode 50% of the time, one could suddenly take the Zeno effect as a useful tool to prevent interaction of the incident photon with the object, or at least make the chance of interaction sufficiently small.

List of Figures

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