# Entanglement in Particle Physics

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# Contents

- Composite quantum systems, pure or mixed states
  nonlocal – contextual features, entanglement
  basis for quantum information, communication and computation

- Aim: to understand features of entanglement, quantum correlations
  phenomenological → conceptual → mathematical aspects

- Elementary particles – massive, internal symmetries, decay
  *K*-mesons strangeness
  *B*-mesons beauty

- Bell inequalities for *K*-mesons
  BI for flavour variation & relation to *CP* violation, BI for time variation

- Stability of quantum system
  understand decoherence – entanglement loss

- Outlook for future experiments

# Part I

## Entanglement and Bell inequalities

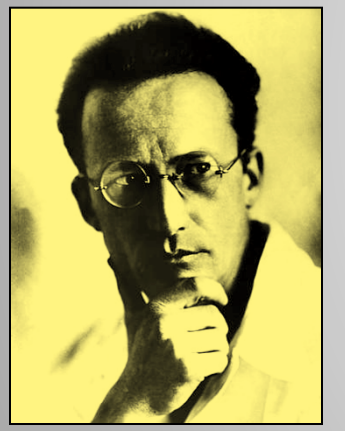
# Schrödinger's Cat

Superposition of quantum states:

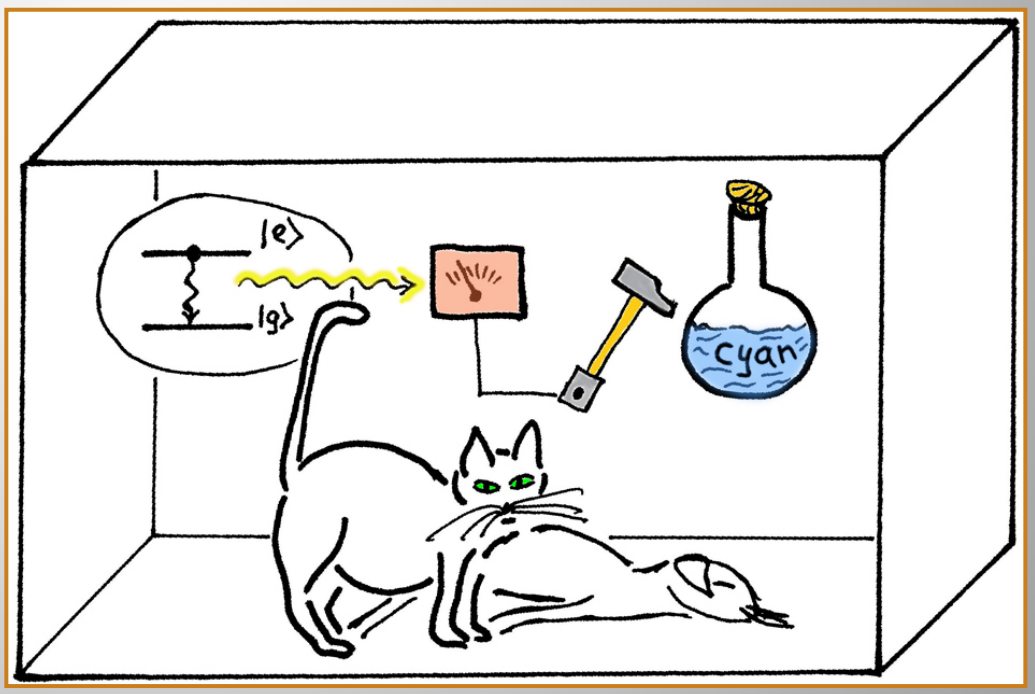
quantum particle can be in several different states simultaneously **!**

Quantum entanglement: superposition of subsystems classically not possible **!!**

**E. Schrödinger:** "verschränkte Zustände … the whole is in a definite state, the parts individually taken are not." 1935



**Paradox:**
look into
dead **?** or alive **?**




Schrödinger's cat
superposition of 2 states:
dead and alive

cat entangled with decaying atom

$$|\text{cat}\rangle = |\text{dead}\rangle|g\rangle + |\text{alive}\rangle|e\rangle$$

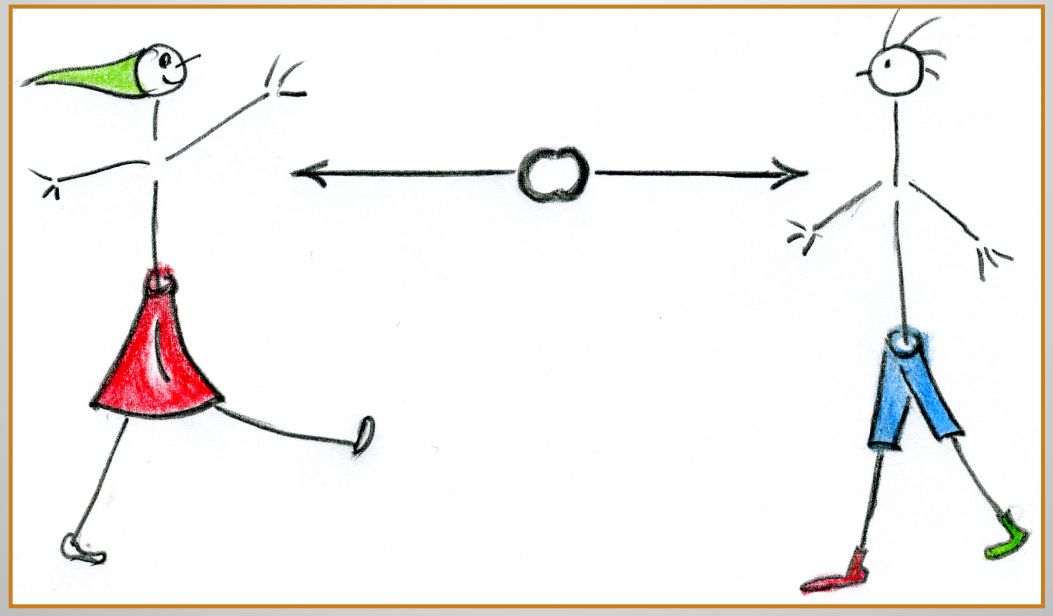
# Entanglement

**combination of 2 quantum systems → strange phenomenon:**

**quantum information: Alice „knows“ about Bob without contact**

**not classically explainable → spooky**

**quantum state of 2 systems**



**Alice** **Bob**

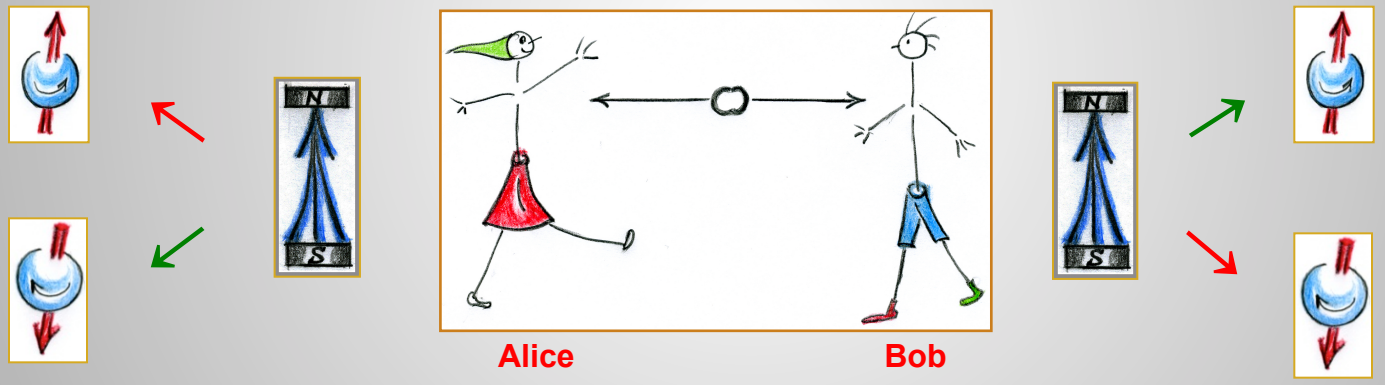
spin measured by Alice and Bob

# EPR Paradoxon

Einstein – Podolsky – Rosen 1935

*Completeness of theory:*
Every element of physical reality must have counterpart in physical theory **!**

*Physical reality:*
If we can predict with certainty a physical quantity, without disturbing, then it is real **!**




if Alice then Bob will find or vice versa

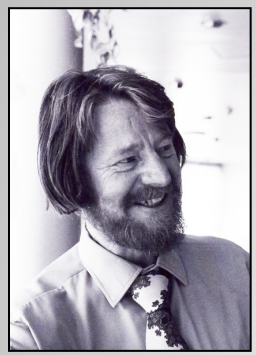
EPR conclude spin at Bob is real
reality not contained in QM

**EPR: QM incomplete !**

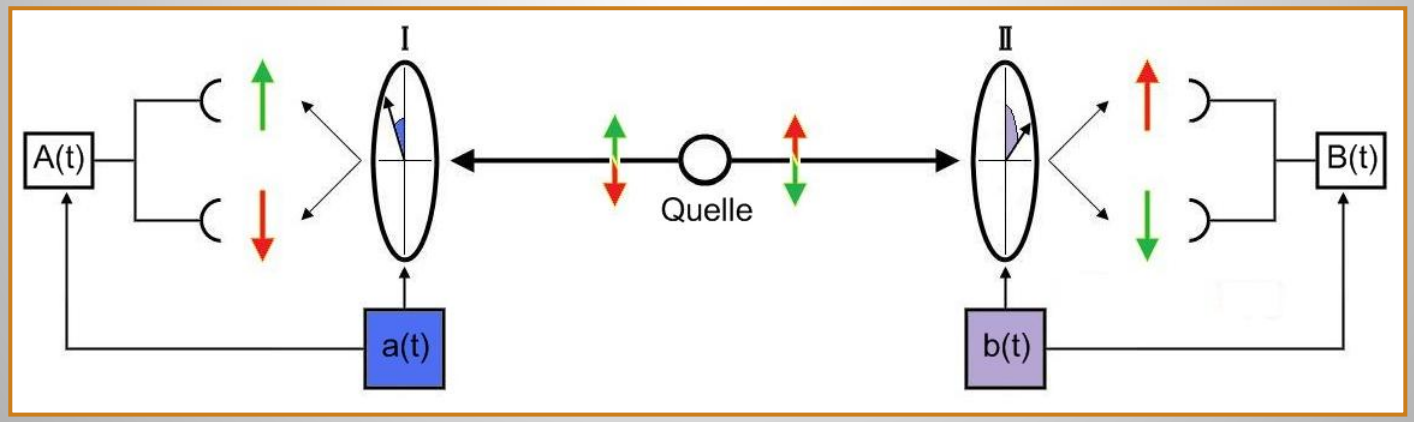
however **Bohr: QM complete !**

# Bell's Theorem

Bell's Theorem 1964



J.S. Bell: *" In a certain experimental situation all LRT (local realistic theories) are incompatible with quantum mechanics. "*




**Alice**

**Bob**

# Bell Inequalities

Expectation value for combined spin measurement $E(a,b)$

**Inequality** for different directions of measurement:

$$S = |E(a,b) - E(a,b')| + |E(a',b') + E(a',b)| \leq 2$$

**CHSH-type**

In terms of probabilities P(a,b): $E(a,b) = -1 + 4\,P(a \Uparrow, b \Uparrow)$

$$P(a,b) \leq P(a,c) + P(c,b)$$

**Wigner-type**

**Inequalities satisfied in <u>each</u> local realistic theory !**

Comparison with quantum mechanics:

$E_{QM}(a,b) = -\cos(\alpha-\beta)$ → $S_{QM} = 2\sqrt{2} = 2.8 > 2$

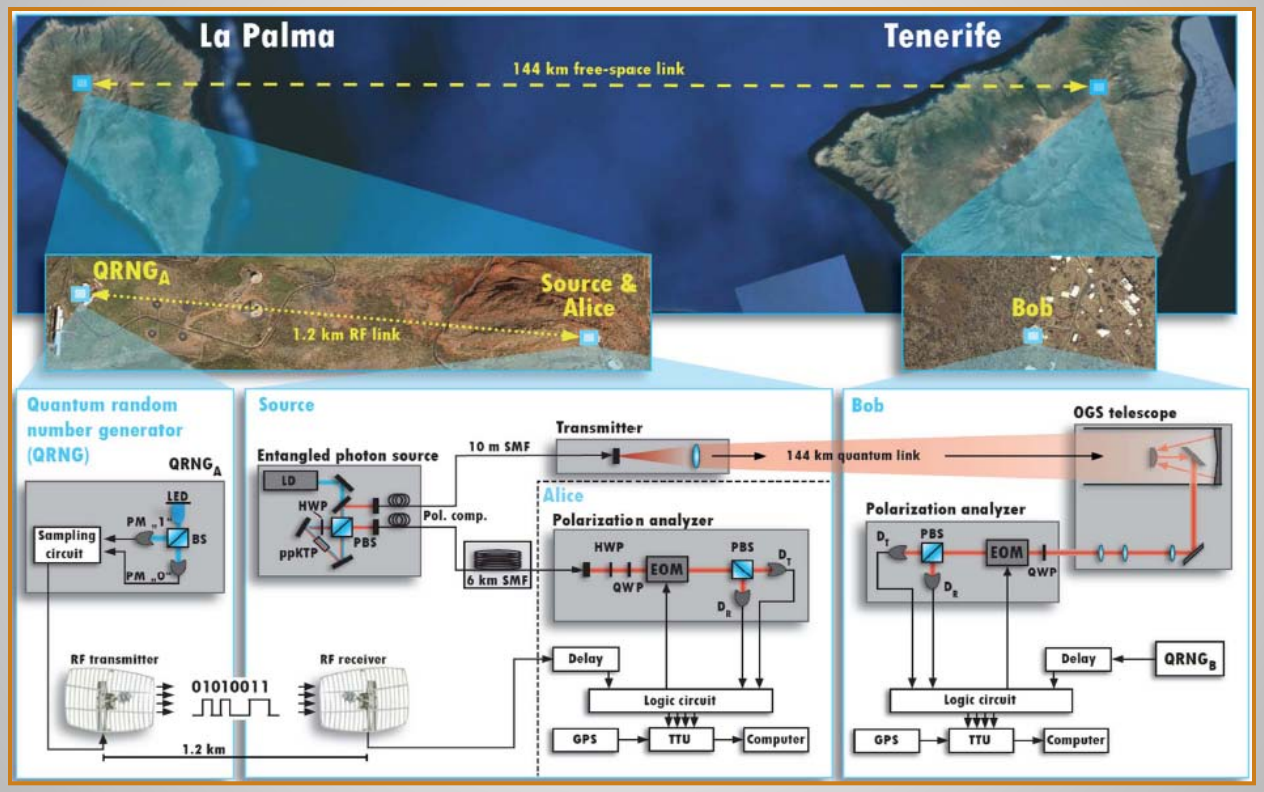
Experiment $S_{Exp} = 2.73 \pm 0.02$

Weihs, … , Zeilinger 1998, Aspect etal. 1982, Fry etal. 1976, Clauser etal. 1972

**Bell inequality violated in quantum mechanics and experiment !!**

# Tenerife Bell-Experiment

**Transmission of entangled photons over 144 km in free space**




**Bell parameter:**

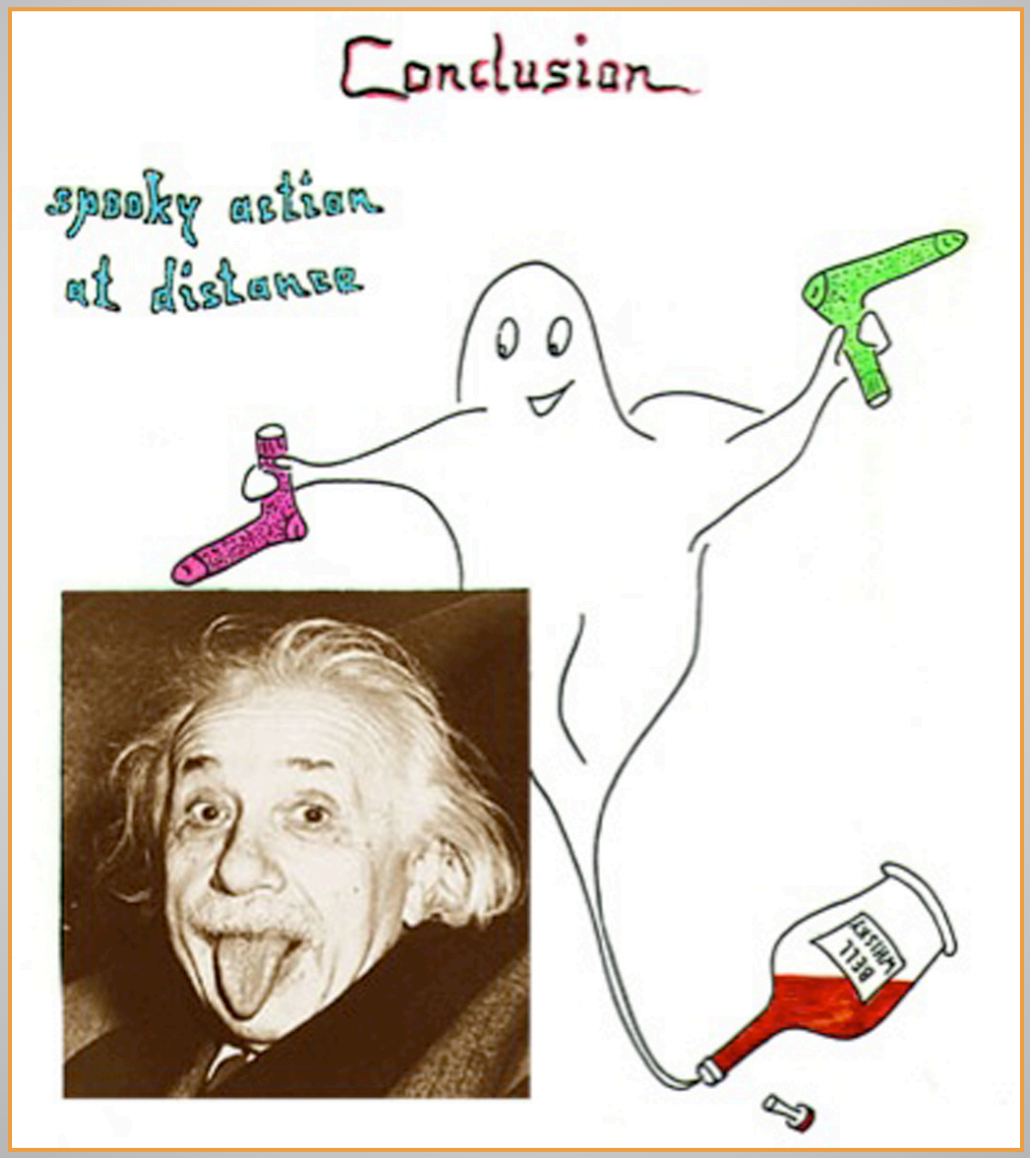
$S_{max} = 2.636$

$S_{exp} = 2.612 \pm 0.114$

Zeilinger, Ursin, et al., 2009

# Conclusion

Reality ?




Reality ?

# Part II

## Bell inequalities for strange mesons

# Strange Mesons I

***K* – meson** Kaon

bound state of quark–antiquark ($q\bar{q}$)

u ... up
d ... down
s ... strange

$\bar{K}^0(\bar{d}s)$ S=-1 $K^0(d\bar{s})$ S=+1 strangeness

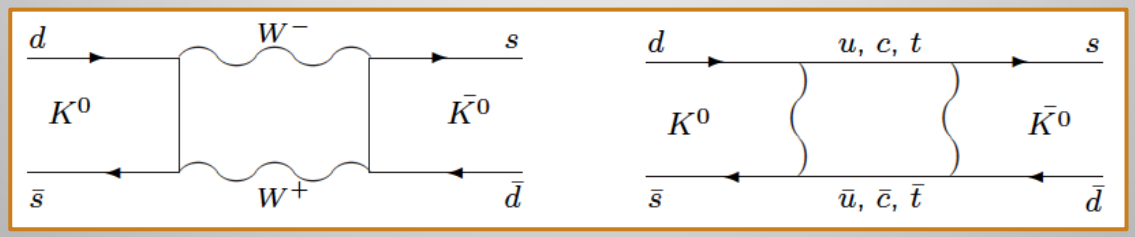
quasi–spin

*C* ... charge conjugation
*P* ... parity

mass: 497 MeV $J^P = 0^-$ pseudoscalar

strong interactions: *S*, *CP* conserving weak interactions: *S*, *CP* violating

$K^0 \leftrightarrow \bar{K}^0$ oscillation due to weak interactions $|\Delta S = 2|$

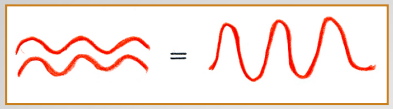


# Strange Mesons II

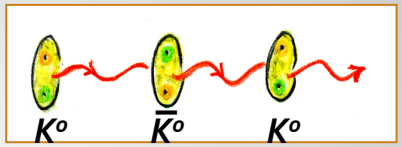
**Strange mesons**

selected by Nature to demonstrate fundamental principles of QM such as:

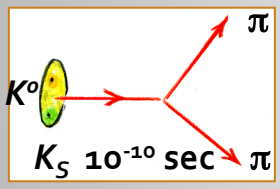
1) superposition

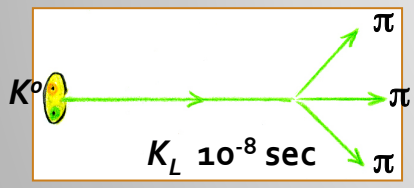


2) oscillation

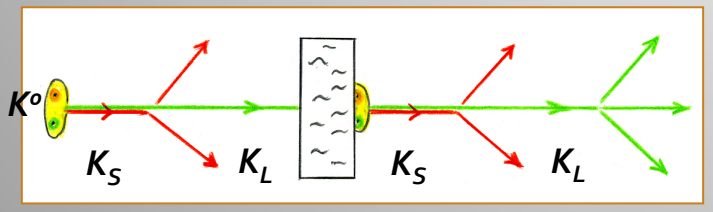



3) decay property




4) regeneration




**quantum states**

Strangeness eigenstates

$$S|K^0\rangle = +|K^0\rangle \qquad S|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$CP$-transformation $P$... parity $C$... charge conjugation

$$CP|K^0\rangle = -|\bar{K}^0\rangle \qquad CP|\bar{K}^0\rangle = -|K^0\rangle$$

$CP$ eigenstates

$$CP|K_1^0\rangle = +|K_1^0\rangle \qquad |K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$
$$CP|K_2^0\rangle = -|K_2^0\rangle \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$K^o = K_S + K_L$

$K_L = K^o + \bar{K}^o$

absorbed in matter

# Kaon Decays

K–decay

$K_S \approx K_1^0 \longrightarrow 2\pi \qquad \tau_S \approx 10^{-10}$ sec short-lived

$K_L \approx K_2^0 \longrightarrow 3\pi \qquad \tau_L \approx 5 \cdot 10^{-8}$ sec long-lived

physical masses $m_S, m_L$ with $\Delta m = m_L - m_S = 3.5 \cdot 10^{-6}$ eV small

$CP$-violation $K_L \longrightarrow 2\pi$ small

short-lived, long-lived states $|\varepsilon| \approx 10^{-3}$ $CP$-violating parameter

$$|K_S\rangle = \frac{1}{N}\left(p\,|K^0\rangle - q\,|\bar{K}^0\rangle\right) \qquad p = 1+\varepsilon, \quad q = 1-\varepsilon$$

$$|K_L\rangle = \frac{1}{N}\left(p\,|K^0\rangle + q\,|\bar{K}^0\rangle\right) \qquad N^2 = |p|^2 + |q|^2$$

decaying states evolve in time $\longrightarrow$ Weisskopf-Wigner approximation

$$|K_{S\atop L}(t)\rangle = e^{-i\lambda_{S\atop L}t}|K_{S\atop L}\rangle \quad \text{with} \quad \lambda_{S\atop L} = m_{S\atop L} - \frac{i}{2}\Gamma_{S\atop L} \quad \text{and} \quad \Gamma_{S\atop L} \sim \tau_{S\atop L}^{-1}$$

$\Longrightarrow$ time evolution for $K^0$ and $\bar{K}^0$

# Kaon Oscillation

$K^0$ beam produced at $t = 0$

probability for finding $K^0$ or $\bar{K}^0$ at $t > 0$

$$\left|\langle K^0|K^0(t)\rangle\right|^2 = \frac{1}{4}\left\{ e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\,e^{-\Gamma t}\cos(\Delta m t)\right\}$$

$$\left|\langle \bar{K}^0|K^0(t)\rangle\right|^2 = \frac{1}{4}\left\{ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-\Gamma t}\cos(\Delta m t)\right\}$$

with $\Delta m = m_L - m_S$ and $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$ $\qquad \Gamma_{\substack{S\\L}} \sim \tau_{\substack{S\\L}}^{-1}$

$K^0$ beam oscillates with frequency $\Delta m/2\pi$

oscillation visible at $t \sim \tau_S$, $\Delta m\,\tau_S = 0.47$

$K_0$ beam at $t = 0$ $\Longrightarrow$ $\bar{K}^0$ equal to $K^0$ far away from source through $K_L$

# Kaonic Qubits

| | | | | |
|---|---|---|---|---|
| | $K_0 \sim \Uparrow$ | | $\bar{K}^0 \sim \Downarrow$ | 2-dim Hilbertspace |
| strangeness | $+1$ | up | $-1$ | down |

operators in "quasi-spin" space: $S \sim \sigma_3$ $CP \sim -\sigma_1$ ~~$CP$~~ $\sim \sigma_2$

Hamiltonian $H = M - \frac{i}{2}\Gamma = \frac{1}{2}\left(a \cdot \mathbb{1} + \vec{h} \cdot \vec{\sigma}\right)$ with $|K_{\substack{S \\ L}}\rangle$ eigenstates

## Analogy

| K-meson | spin-$\frac{1}{2}$ | photon |
|---|---|---|
| $\|K^0\rangle$ | $\|\Uparrow\rangle_z$ | $\|V\rangle$ |
| $\|\bar{K}^0\rangle$ | $\|\Downarrow\rangle_z$ | $\|H\rangle$ |
| $\|K_S\rangle$ | $\|\Rightarrow\rangle_y$ | $\|L\rangle = \frac{1}{\sqrt{2}}(\|V\rangle - i\|H\rangle)$ |
| $\|K_L\rangle$ | $\|\Leftarrow\rangle_y$ | $\|R\rangle = \frac{1}{\sqrt{2}}(\|V\rangle + i\|H\rangle)$ |

# Production of entangled Kaons

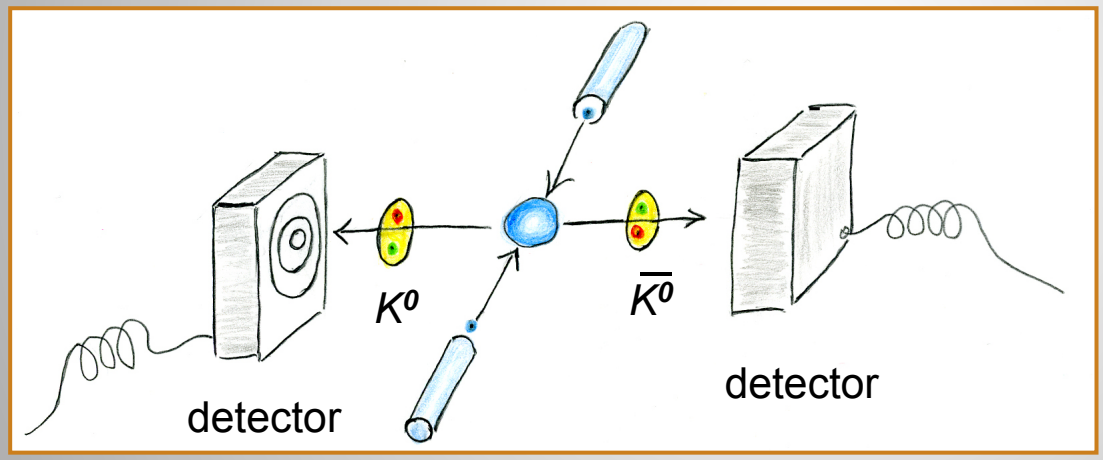
**Matter – Antimatter collisions**

**$e^+e^-$ collider DA$\Phi$NE in Frascati**

KLOE-2 experiment Di Domenico et al.

vector meson ($s\bar{s}$): $\phi(1020) \rightarrow K_L K_S$ 34%

**Test of quantum coherence**




**Joint expectation value**

$E(a.b) \rightarrow E(k_a, t_l, k_b, t_r)$

dependent on ↑ ↑

quasi-spin and time

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle_l \otimes |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l \otimes |K^0\rangle_r\}$$

production of Bell state at t = 0

# Kaonic Bell Inequality of Wigner-type

**Local Realistic Theories satisfy BI with choice:**
- **fix quasi-spin — vary time**
- **vary quasi-spin — fix time**

Consider case:

**fix time** → **vary quasi-spin of kaon** → rotation in quasi-spin space

for BI we need 3 different "angles" — quasi-spins: $|K_S\rangle$, $|\bar{K}^0\rangle$, $|K_1\rangle$

**Bell inequality of Wigner-type** F. Uchiyama 1997

$$P(K_S, \bar{K}^0) \leq P(K_S, K_1) + P(K_1, \bar{K}^0)$$

P ... probability

However, it contains unphysical *CP*-even state $|K_1\rangle$

**But !**

BI ⟹ inequality for physical *CP* parameter — experimentally testable **!!**

How come **?**

# Experimental Inequality

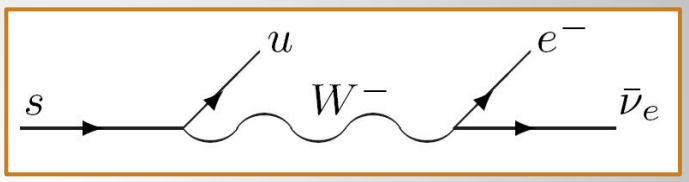
**BI** ⇒ optimal **inequality** for weights p, q of state $|K_S\rangle$

$$|K_S\rangle = 1/N\,(p|K^0\rangle - q|\bar{K}^0\rangle)$$
$$|K_L\rangle = 1/N\,(p|K^0\rangle + q|\bar{K}^0\rangle)$$

$$|p| \leq |q|$$ experimentally testable !

$p = 1 + \varepsilon,\ q = 1 - \varepsilon$
$N^2 = |p|^2 + |q|^2,\ |\varepsilon| \approx 10^{-3}$

Experiment: decay of *K*-mesons




Semileptonic decay of strange mesons:

$$K^0(d\bar{s}) \longrightarrow \pi^-(d\bar{u})\ l^+\ \nu_l \qquad \bar{s} \longrightarrow \bar{u}\ l^+\ \nu_l$$
$$\bar{K}^0(\bar{d}s) \longrightarrow \pi^+(\bar{d}u)\ l^-\ \bar{\nu}_l \qquad s \longrightarrow u\ l^-\ \bar{\nu}_l$$

⇒

$l^+$ tags $K^0$ in $K_L$ state — $|p|^2$ ... probability for $K^0$ in $K_L$
$l^-$ tags $\bar{K}^0$ $l = \mu, e$ — $|q|^2$ ... probability for $\bar{K}^0$ in $K_L$

Charge asymmetry

$$\delta = \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}$$

# Charge Asymmetry

$|p| \leq |q|$ Bertlmann-Grimus-Hiesmayr 1999

→ **Bell inequality** for $\delta$

$\boldsymbol{\delta \leq 0}$ whereas: $\delta_{exp} = (3.27 \pm 0.12) \cdot 10^{-3}$

**BI experimentally violated !**

consider 2 BI's $\delta \leq 0$ and $\delta \geq 0$ when $\bar{K}^0 \rightarrow K^0$, $p \leftrightarrow q$

→ $\boldsymbol{\delta = 0}$ *CP* conservation in contradiction to experiment !

## Conclusion

**LRT are only compatible with strict *CP* conservation in $\bar{K}^0 K^0$ mixing !**

$\delta \neq 0$ — $K^0\bar{K}^0$ entanglement

*CP* violation — nonlocal – contextual

# BI for unitary Time Evolution

Next, consider case:

**fix quasi-spin → vary time**

• for $t_l \neq t_r$: EPR–Bell correlation

**Joint expectation value**

E(a.b) → $E(k_a, t_l, k_b, t_r)$

**consider kaonic system:** choose strangeness S = +1 $k_a = k_b = K^0$

time evolution of states, respect unitary time evolution:

$$U(t,0)\,|K_{S,L}\rangle = e^{-i\lambda_{S,L}t}\,|K_{S,L}\rangle + |\Omega_{S,L}(t)\rangle$$

$|\Omega_{S,L}(t)\rangle$ state of all decay products

**Joint expectation value in QM** *CP* violation neglected

$$E(t_l;t_r) = -\cos(\Delta m \Delta t)\cdot e^{-\Gamma(t_l+t_r)} + \frac{1}{2}(1-e^{-\Gamma_L t_l})(1-e^{-\Gamma_S t_r}) + \frac{1}{2}(1-e^{-\Gamma_S t_l})(1-e^{-\Gamma_L t_r}).$$

Terms from decay products

# Bell Inequality – general form

insert expectation value into BI

$$S = |E_{K^0K^0}(t_l, t_r) - E_{K^0K^0}(t_l, t_r')| + |E_{K^0K^0}(t_l', t_r) + E_{K^0K^0}(t_l', t_r')| \leq 2$$

However: **NO violation of BI** for ***all*** possible times $(t_l, t_r)$

**Reason:**
interplay

kaon decay ⟷ strangeness oscillation

Ghirardi-Grassi-Weber 1991

dependent on ratio $\mathbf{x = \Delta m / \Gamma}$ $\quad \Delta m = m_L - m_S$ , $\Gamma$ … decay width

**NO violation** for $0 < x < 2$ $\quad$ Experiment: $x^{exper}_{kaon} = 0.95$

**similarly: *B*–mesons, *D*–mesons**

## Conclusion

**We cannot use time-variation of BI (CHSH type) to exclude LRT !**

**Question:** Can we overcome this fact **?** **Yes !**

# BI for decaying Systems

**Bell inequality**

$$-2 \leq S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \leq +2$$

converted into **"witness" form**

B. Hiesmayr, A. Di Domenico etal. 2013

$$\min_{\text{all sep}} S[\rho_{sep}] \leq S[\rho] \leq \max_{\text{all sep}} S[\rho_{sep}]$$

**Expectation value** — quantum mechanical correlations

$$E^{QM}(a,b)\,[\rho] = \mathrm{Tr}\,(O_a \otimes O_b\, \rho)$$

**S – function of BI**

$$S^{QM}(a,a';b,b') = \mathrm{Tr}\,\{\,[O_a \otimes (O_b - O_{b'}) + O_{a'} \otimes (O_b + O_{b'})]\,\rho\}$$

$O_a$ … appropriate operators corresponding to decaying systems, e.g. kaons calculable inclusive *CP* violation

# Experimental Test of BI

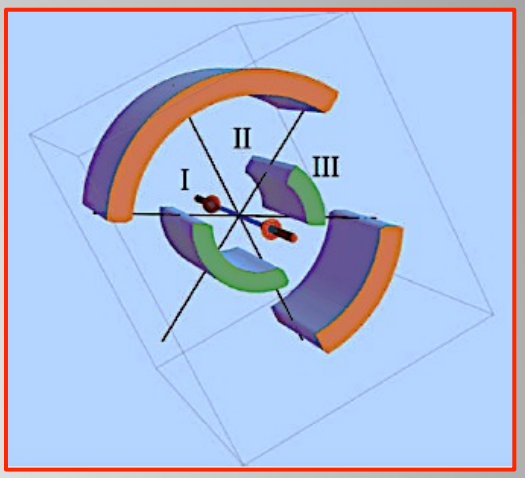
Intrinsic decay property also affects separable states

**Classical boundary for decaying system**

$$\left| \min/\max_{\text{all sep}} S\,[\rho_{sep}] \right| \leq 2$$

2 for $\Gamma \rightarrow 0$ stable systems

**leads to violation of Bell inequality !**




experimental set-up

We have got a tool to distinguish between LRT and QM !

production of Bell state $\rho^- = |\psi^-\rangle\langle\psi^-|$

**Example:** Kaonic system

3 different times corresponding to 3 Bell angles

$t_a = 0$, $t_b = t_{a'} = 1{,}34\ \tau_S$, $t_{b'} = 2{,}80\ \tau_S$

$$\min_{\text{all sep}} S\,[\rho_{sep}] = -0{,}58$$

$$S\,[\rho^-] = -0{,}69$$

$$\min S\,[\rho_{sep}] - S\,[\rho^-] = 0{,}11$$

11 %

Experimentally feasible at DAΦNE with KLOE-2 detector

# Part III

## Decoherence of entangled beauty

# Beauty Mesons

**Neutral $B$ – meson** $m_B = 5.3$ GeV $J^P = 0^-$

$B^o$ $(d\bar{b})$, $\bar{B}^o$ $(\bar{d}b)$ bound state of quark-antiquark

+1 -1 b ... beauty or bottom

QM formalism analogous to $K$ – meson $s \rightarrow b$

- $B^o \leftrightarrow \bar{B}^o$ oscillation
- $B$ decay: $B_H$ ... heavy state $B_L$ ... light state

Time evolution $|B_H(t)\rangle, |B_L(t)\rangle$ or $|B^o(t)\rangle, |\bar{B}^o(t)\rangle$

according to Wigner-Weisskopf approximation

$B$ – meson in contrast to $K$ – meson

- $\Delta m = m_H - m_L = 3 \cdot 10^{-4}$ eV large
- $\Delta\Gamma = \Gamma_H - \Gamma_L \approx 0$ small $\Gamma_{B^o}^{-1} = \tau_{B^o} = 1{,}5 \cdot 10^{-12}$ sec

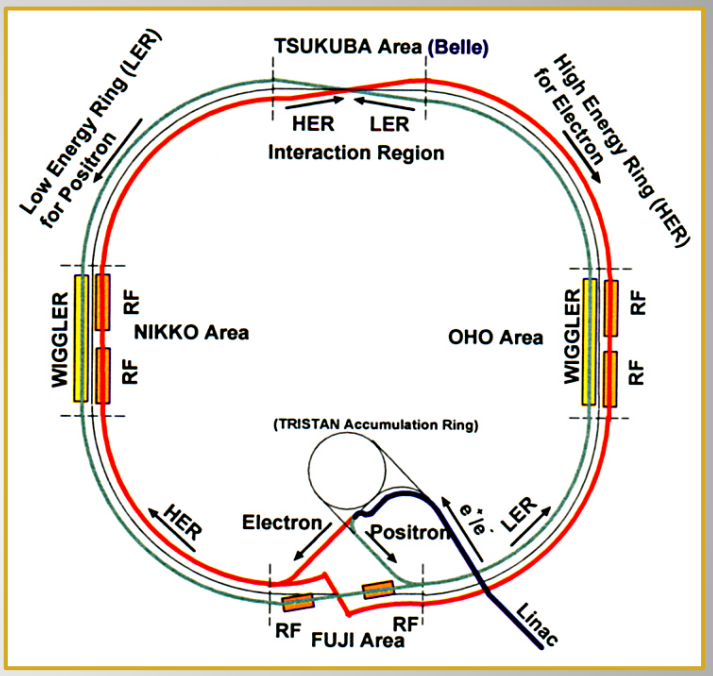
# Production of *B*–Mesons

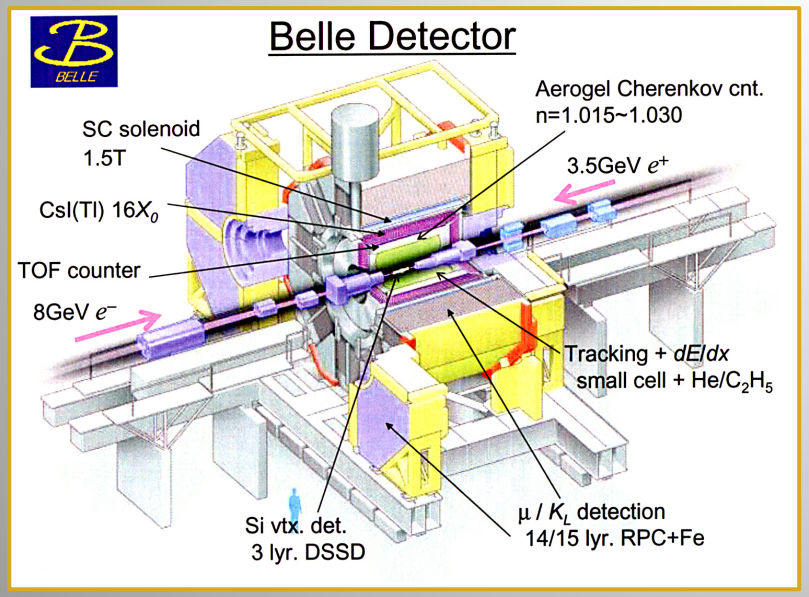
**$e^+e^-$ collider at KEK, Tsukuba, Japan**

Asymmetric ring: $e^-$ 8.0 GeV $e^+$ 3.5 GeV

to study *CP* violation
Kobayashi – Masukawa NP 2008

**Quantum experiments**
Apollo Go et al.
HEPHY, Richter, Vienna




***B* factory**

$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^o \bar{B}^o$

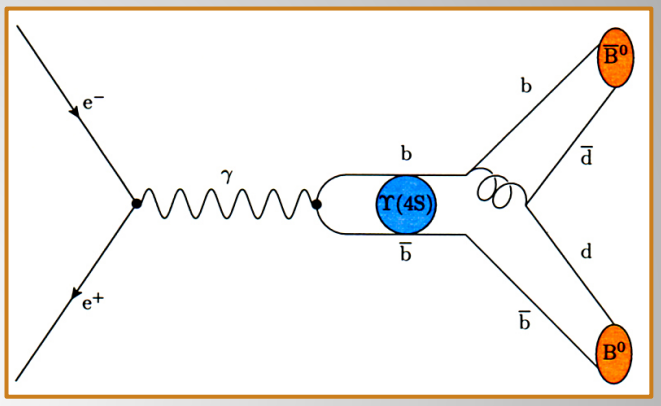
10.58 GeV

# Creation of entangled *B*–Mesons

**Resonance** $\Upsilon(4S)$ at 10.58 GeV
nearly at threshold of $B^0\,\bar{B}^0$ production

➡ ***B* factory**

Creation of entangled state of *B* mesons



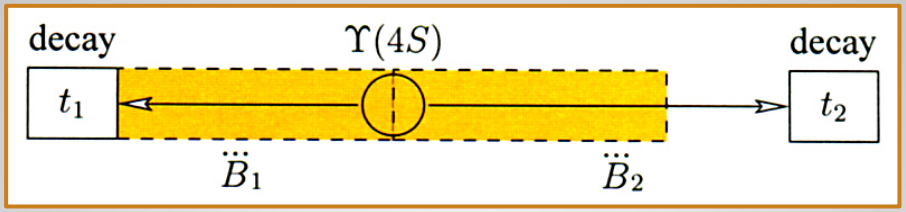

**Quantum state — entangled beauty**

$$|\psi(t_l,t_r)\rangle \;=\; \frac{N_{HL}}{\sqrt{2}}\left\{\,|B_H(t_l)\rangle_l \otimes |B_L(t_r)\rangle_r \;-\; |B_L(t_l)\rangle_l \otimes |B_H(t_r)\rangle_r\right\}$$

**Question:**
How to measure possible decoherence in entangled beauty **?**

# Decoherence Parameter

**Test of quality of entanglement** via decoherence option




**Probability** to detect beauty $|\, b_1 \,\rangle_l$ on left side and beauty $|\, b_2 \,\rangle_r$ on right side

$$\Big|\langle b_1|\langle b_2|\,\psi(t_l,t_r)\rangle\Big|^2 \;=$$

$$\frac{|N_{HL}|^2}{2}\Big\{\big|\langle b_1|B_H(t_l)\rangle_l\big|^2\big|\langle b_2|B_L(t_r)\rangle_r\big|^2 \;+\; \big|\langle b_1|B_L(t_l)\rangle_l\big|^2\big|\langle b_2|B_H(t_r)\rangle_r\big|^2$$

$$-\;2\;\underbrace{(1-\zeta)}_{\text{modification}}\;\Re e\Big[\langle b_1|B_H(t_l)\rangle_l^*\langle b_2|B_L(t_r)\rangle_r^*\langle b_1|B_L(t_l)\rangle_l\langle b_2|B_H(t_r)\rangle_r\Big]\Big\}$$

**Decoherence parameter ζ**

$$0 \;\le\; \zeta \;\le\; 1$$

Bertlmann-Grimus 1997

pure QM — total decoherence, LRT

# Asymmetry of Events

**Aim**: to determine range of $\zeta$ by experimental data

How **?** consider as particles
like-beauty events $(B^0, B^0)$ and $(\bar{B}^0, \bar{B}^0)$
unlike-beauty events $(B^0, \bar{B}^0)$ and $(\bar{B}^0, B^0)$

**Probabilities**

$$P_\zeta(B^0, t_l; B^0, t_r) = \frac{|p|^2}{|q|^2}\frac{1}{8}\left(P^{QM}_{\text{like}}(t_l, t_r) + 2\zeta\, e^{-\Gamma(t_l+t_r)}\cos\Delta m\Delta t\right)$$

$$P_\zeta(B^0, t_l; \bar{B}^0, t_r) = \frac{1}{8}\left(P^{QM}_{\text{unlike}}(t_l, t_r) - 2\zeta\, e^{-\Gamma(t_l+t_r)}\cos\Delta m\Delta t\right)$$

**Asymmetry** directly sensitive to interference term

$$A(t_l, t_r) = \frac{P_{\text{unlike}}(t_l, t_r) - P_{\text{like}}(t_l, t_r)}{P_{\text{unlike}}(t_l, t_r) + P_{\text{like}}(t_l, t_r)}$$

$$A_\zeta(t_l, t_r) = (1-\zeta)\, A^{\text{QM}}(t_l, t_r) \quad \text{with} \quad A^{\text{QM}}(t_l, t_r) = \frac{\cos\Delta m\,\Delta t}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t)}$$

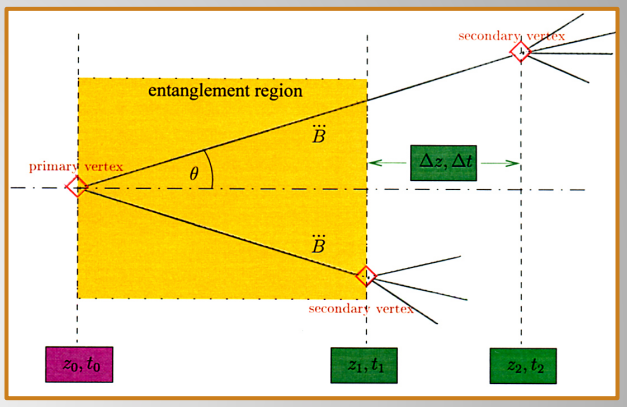
$A_{exper}$ → $\zeta_{exper}$

# Experimental Results

Decoherence parameter measures quantitatively deviations from pure QM

Problem:
Exact vertex determination from tracks of decay products is difficult task !

Lorentz boost $\beta\gamma = 0.425$
$\Delta z \approx \Delta t \cdot \beta\gamma c$




| | | |
|---|---|---|
| $\zeta_{\text{Go-Bay}}$ | $= 0.029 \pm 0.057$ | BELLE |
| $\zeta_{\text{Richter}}$ | $= -0.045 \pm 0.155$ | BELLE |
| $\zeta_{\text{B-G}}$ | $= -0.06 \pm 0.10$ | ARGUS, CLEO |

**Comparison with data from strangeness system $K^oK^o$**

| | |
|---|---|
| $\zeta_{\text{BGH}} = 0.13 \pm 0.16$ | Bertlmann – Grimus – Hiesmayr, from CPLEAR data |
| $\zeta_{\text{KLOE}} = 0.003 \pm 0.018_{\text{stat}} \pm 0.006_{\text{sys}}$ | di Domenico etal, from *CP* suppressed decays |

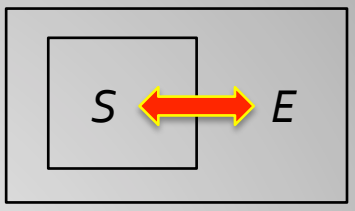
## Conclusion

$B^oB^o$ and $K^oK^o$ systems are close to QM, $\zeta = 0$, and far from total decoherence, $\zeta = 1$,
**massive systems are entangled at macroscopic scale — LRT excluded**
0.1 mm for $B^oB^o$ 9 cm for $K^oK^o$

# Decoherence – Open Quantum System

System *S* interacts with environment *E*

mixing of states → decoherence




**Quantum master equation**

$$\frac{d\rho}{dt} = -iH\rho + i\rho H^\dagger - D[\rho]$$

density matrix of system

$\rho = \Sigma_i p_i |\Psi_i\rangle\langle\Psi_i|$ with $0 \le p_i \le 1$

**Dissipator** — projectors to eigenstates of *H*

$$D[\rho] = \lambda\,(P_i\rho P_j + P_j\rho P_i) \quad \text{with} \quad P_j = |e_j\rangle\langle e_j| \quad (j = 1, 2)$$

λ … decoherence parameter

Bertlmann-Grimus 1998

eigenstates

$$|e_1\rangle = |B_H\rangle_l \otimes |B_L\rangle_r \qquad |e_2\rangle = |B_L\rangle_l \otimes |B_H\rangle_r$$

start with entangled Bell state

$$\rho^- = |\psi^-\rangle\langle\psi^-| \quad \text{where} \quad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle - |e_2\rangle\right)$$

# Decoherence Parameter Relation

time dependence given by master equation

**Time dependent density matrix**

$$\rho(t) = \frac{1}{2} e^{-\Gamma t}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t}\left(|e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|\right)\right)$$

decoherence ⟹ mixed state

**Asymmetry** of unlike–like events

$$A^{\lambda}(t_l, t_r) = e^{-\lambda \min(t_l, t_r)} A^{QM}(t_l, t_r)$$

$(1-\zeta)$ in $\zeta$ – formalism

**Parameter relation**

$$\zeta(t_l, t_r) = 1 - e^{-\lambda \min(t_l, t_r)}$$

# Entanglement Loss – Decoherence

**von Neumann entropy**

$$S\left(\rho(t)\right) \;=\; -\,\mathrm{Tr}\left\{\,\rho(t)\,\log_2\rho(t)\,\right\}$$

measures degree of uncertainty in quantum state

**Entanglement of formation** — measure for entanglement

$$E(\rho) \;=\; \min \sum_i p_i\, S(\rho_i^l)$$

$0 \le E \le 1$

**Concurrence *C***

$$E(\rho) \;\equiv\; \mathcal{E}\left(C(\rho)\right) \;=\; H\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-C^2}\right)$$

$0 \le C \le 1$

with binary entropy function

$$H(x) \;=\; -\,x\log_2 x \;-\; (1-x)\log_2(1-x)$$

Bertlmann-Durstberger-Hiesmayr 2003

calculate concurrence

$$C\left(\rho(t)\right) \;=\; e^{-\lambda t}$$

**Entanglement loss**

$$1 - C\left(\rho(t)\right) \;=\; \zeta(t)$$
$$1 - E\left(\rho(t)\right) \;\approx\; 1/\ln 2 \cdot \zeta(t)$$

# Part IV

## Outlook

# To do's

**Experimentalists in collaboration with theorists !**

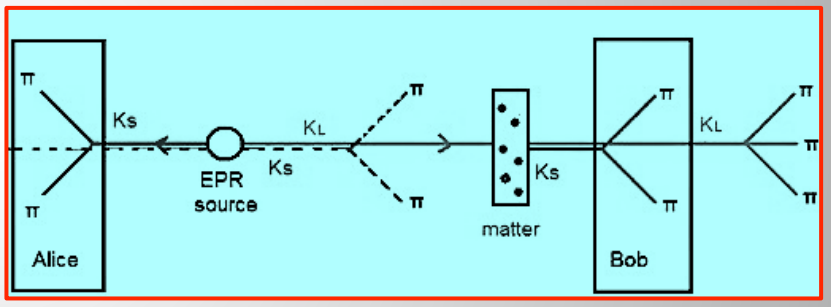
- Direct test of Bell inequalities, time variation — active measurements !

- Bell inequality with regenerator in kaon beam

  $K_S$ , $K_L$ measurements, dependent on regeneration parameter

  Bramon-Escribano-Garbarino 2006

  Hatice Tataroglu 2009




- Testing local realism in cascade decays $\eta_c \rightarrow VV \rightarrow PP\,PP$ in $\tau$ – charm factory

  test of Clauser-Horne inequality in polarization correlation of entangled *VV*

  Li & Qiao 2009, proposal for *BES-III* at *BEPC II* in Beijing

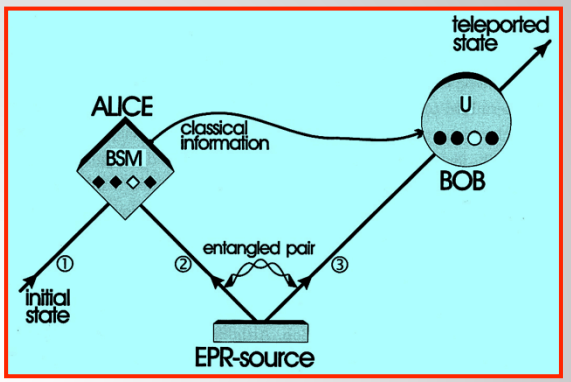
- Test of local realism by *CP* violation of kaons

  Genovese 2005

# To do's

- High energy quantum teleportation with kaons
  incoming kaon collides with one kaon of an entangled pair
  Yu Shi 2006

**utopic ??**




- Environmental decoherence effects — test of collapse models
  damping of oscillatory behaviour of mesons
  Bassi-Hiesmayr etal. 2013

- Entanglement in neutrino mixing and oscillation
  Blasone-Illuminati etal. 2008

- .....

# The End