Erratum : Local existence of a solution of a semi-linear wave equation in a neighborhood of initial characteristic hypersurfaces

Erratum : Existence locale d'une solution d'équation d'onde semi-linéaire dans un voisinage des hypersurfaces caractéristiques initiales

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I am confused to have made a mistake in lemma 2.3. This does change neither the results obtained in the article, nor the global process of the proof. But it implies the following corrections. The right lemma 2.3. is the following one.

Lemma 2.3 *If $m < m'$ and $k < k'$ then $\mathcal{H}_{m',k'} \hookrightarrow \mathcal{H}_{m,k}$ with compact embedding.*

and we need to add an inequality in lemma 2.4. which becomes

Lemma 2.4 *If $f \in \mathcal{H}_{m,k} \cap \mathcal{H}_{m,k'}$ with $k < k'$ then $\forall\, \gamma \in [0;1]$,*
$f \in \mathcal{H}_{m,\gamma k+(1-\gamma)k'}$ and $\| f \|_{\mathcal{H}_{m,\gamma k+(1-\gamma)k'}} \leq \| f \|^{\gamma}_{\mathcal{H}_{m,k}} \| f \|^{1-\gamma}_{\mathcal{H}_{m,k'}}$.
Similarly, if $f \in \mathcal{H}_{m,k} \cap \mathcal{H}_{m',k}$ with $m < m'$ then $\forall\, \gamma \in [0;1]$,
$f \in \mathcal{H}_{\gamma m+(1-\gamma)m',k}$ and $\| f \|_{\mathcal{H}_{\gamma m+(1-\gamma)m',k}} \leq \| f \|^{\gamma}_{\mathcal{H}_{m,k}} \| f \|^{1-\gamma}_{\mathcal{H}_{m',k}}$.
Furthermore, if $f \in \mathcal{H}_{m,k} \cap \mathcal{H}_{m',k'}$ with $m < m'$ and $k < k'$ then $\forall\, \gamma,\ \delta \in [0;1]$,
$f \in \mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}$ and $\| f \|_{\mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}} \leq \| f \|^{\gamma\delta}_{\mathcal{H}_{m,k}} \| f \|^{1-\gamma\delta}_{\mathcal{H}_{m',k'}}$.

(the proof of these lemmas state at the end of this erratum).

Then it occurs at page 72 for the convergence of $(\tilde{\varphi}_{\varepsilon'}(u))$: we have to work with $\mathcal{H}_{m'',0}$ ($m'' < m$) instead of $\mathcal{H}_{m,0}$ to get a compact embedding. So we replace page 72 l.3 untill l.12 by :
By compactness of embedding $\mathcal{H}_{m,2} \hookrightarrow \mathcal{H}_{m'',0}$ with $0 < m'' < m$ (see lemma 2.3), if $(\tilde{\varphi}_{\varepsilon'}(u))$ weakly converges to $\tilde{\varphi}(u)$ in $\mathcal{H}_{m,2}$, then $(\tilde{\varphi}_{\varepsilon'}(u))$ strongly converges to $\tilde{\varphi}(u)$ in $\mathcal{H}_{m'',0}$. By interpolation (see lemma (2.4)) , if $m'' < m' < m$ and $0 < k < 2$ we have

$$\begin{aligned} \| \tilde{\varphi}_{\varepsilon'}(u) - \tilde{\varphi}_{\varepsilon}(u) \|_{\mathcal{H}_{m',k}} &\leq \| \tilde{\varphi}_{\varepsilon'}(u) - \tilde{\varphi}_{\varepsilon}(u) \|^{\nu}_{\mathcal{H}_{m'',0}} \| \tilde{\varphi}_{\varepsilon'}(u) - \tilde{\varphi}_{\varepsilon}(u) \|^{1-\nu}_{\mathcal{H}_{m,2}} \\ &\leq \| \tilde{\varphi}_{\varepsilon'}(u) - \tilde{\varphi}_{\varepsilon}(u) \|^{\nu}_{\mathcal{H}_{m'',0}} \left(\| \tilde{\varphi}_{\varepsilon'}(u) \|_{\mathcal{H}_{m,2}} + \| \tilde{\varphi}_{\varepsilon}(u) \|_{\mathcal{H}_{m,2}}\right)^{1-\nu} \\ &\leq \| \tilde{\varphi}_{\varepsilon'}(u) - \tilde{\varphi}_{\varepsilon}(u) \|^{\nu}_{\mathcal{H}_{m'',0}} (2c)^{1-\nu} \end{aligned}$$

with $\nu = \gamma k/2$, where γ is such that $m' = \gamma m'' + (1-\gamma)m$.
From this we can deduce that $(\tilde{\varphi}_{\varepsilon'}(u))$ strongly converges to $\tilde{\varphi}(u)$ in $\mathcal{H}_{m',k}$.
In particular, if $\frac{n-1}{2} < m' < m$ and $k = 1$, by inclusion $\mathcal{H}_{m',1} \subset C^0$ (see lemma 2.2.) we

see that $(\tilde{\varphi}_{\varepsilon'}(u))$ strongly converges to $\tilde{\varphi}(u)$ in $C^0([0;R]\times\mathbb{T}^{n-1})$.

At the end of the page 72, we replace the word
(then strongly in $\mathcal{H}_{m-1,1}$ by compactness of the embedding $\mathcal{H}_{m,1}\hookrightarrow\mathcal{H}_{m-1,1}$)
by
(then strongly in $\mathcal{H}_{m-1,\frac{3}{4}}$ by compactness of the embedding $\mathcal{H}_{m,1}\hookrightarrow\mathcal{H}_{m-1,\frac{3}{4}}$).

At the page 73, we have to replace l.6 to l.12 by :
for all μ such that $m-1<\mu<m$ and $\frac{3}{4}<\kappa<1$, let σ defined by $\mu=\sigma(m-1)+(1-\sigma)m$, and σ' by $\kappa=\sigma'\frac{3}{4}+(1-\sigma')1$, we have

$$\begin{aligned}\| \frac{\partial}{\partial v}\tilde{\varphi}_{\varepsilon"}(u)-\frac{\partial}{\partial v}\tilde{\varphi}(u)\|_{\mathcal{H}_{\mu,\kappa}} &\leq \| \frac{\partial}{\partial v}\tilde{\varphi}_{\varepsilon"}(u)-\frac{\partial}{\partial v}\tilde{\varphi}(u)\|^{\sigma\sigma'}_{\mathcal{H}_{m-1,\frac{3}{4}}}\| \frac{\partial}{\partial v}\tilde{\varphi}_{\varepsilon"}(u)-\frac{\partial}{\partial v}\tilde{\varphi}(u)\|^{1-\sigma\sigma'}_{\mathcal{H}_{m,1}}\\ &\leq \| \frac{\partial}{\partial v}\tilde{\varphi}_{\varepsilon"}(u)-\frac{\partial}{\partial v}\tilde{\varphi}(u)\|^{\sigma\sigma'}_{\mathcal{H}_{m-1,\frac{3}{4}}}(2c)^{1-\sigma\sigma'}.\end{aligned}$$

Thus

$$\| \frac{\partial}{\partial v}\tilde{\varphi}_{\varepsilon"}(u)-\frac{\partial}{\partial v}\tilde{\varphi}(u)\|_{\mathcal{H}_{\mu,\kappa}}\to 0.$$

In particular, as $\mu>m-1>\frac{n-1}{2}$ and $\kappa>\frac{3}{4}$, the embedding $\mathcal{H}_{\mu,\kappa}\hookrightarrow C^0$ holds,

Always in page 73, the proof of lemma 6.1. still holds by taking $\mathcal{H}_{m'-|\alpha|,1}$ $(m'<m)$ instead of $\mathcal{H}_{m-|\alpha|,1}$.

Now we replace page 75 l.12 to l.16 by
So we can apply the lemma 6.2. on $\frac{\partial}{\partial v}\tilde{\varphi}(u+h,s,y)-\frac{\partial}{\partial v}\tilde{\varphi}(u,s,y)$, we obtain

$$\begin{aligned}&\| \int_0^v \frac{\partial}{\partial v}\tilde{\varphi}(u+h,\sigma,y)-\frac{\partial}{\partial v}\tilde{\varphi}(u,\sigma,y)d\sigma\|_{\mathcal{H}_{m',1}([0;R]\times\mathbb{T}^{n-1})}\\ &\qquad\leq(R^{\frac{3}{2}}+1)\| \frac{\partial}{\partial v}\tilde{\varphi}(u+h,\sigma,y)-\frac{\partial}{\partial v}\tilde{\varphi}(u,\sigma,y)\|_{\mathcal{H}_{m',0}([0;R]\times\mathbb{T}^{n-1})}. \qquad (6.5)\end{aligned}$$

Furthermore, we work with $\mathcal{H}_{\mu,0}$ instead of $\mathcal{H}_{\mu,1}$ (and $\mathcal{H}_{\mu+2,0}$ instead of $\mathcal{H}_{\mu+2,1}$) since page 75 l.17 untill page 76 l.14 (notice that lemma 6.4 is also true with $\mathcal{H}_{\mu,0}$).

It remains to give the proof of lemma 2.3.:
We want to show that if $m<m'$, $k<k'$ then the embedding $\mathcal{H}_{m',k'}\hookrightarrow\mathcal{H}_{m,k}$ is compact. We deal with the equivalent norm $\rceil f \upharpoonright$ defined above and we will denote it also $\| f\|_{\mathcal{H}_{m,k}}$. As $(1+|\overline{\alpha}|)^{2m}\leq(1+|\overline{\alpha}|)^{2m'}$ and $(1+|\alpha_0|)^{2k}\leq(1+|\alpha_0|)^{2k'}$ it is clear that $\| ... \|_{\mathcal{H}m,k}\leq\| ... \|_{\mathcal{H}m',k'}$. Set $i:\mathcal{H}_{m',k'}\hookrightarrow\mathcal{H}_{m,k}$, i is a compact operator if it changes a bounded set in a relatively compact set. Let (f_n) a bounded sequence of $\mathcal{H}_{m',k'}$. We have seen that $\mathcal{H}_{m',k'}$

is reflexive so we can extract a subsequence $(f_{n'})$ of (f_n) which weakly converges to f in $\mathcal{H}_{m',k'}$, and $\| f \|_{\mathcal{H}_{m',k'}} \leq \liminf \| f_{n'} \|_{\mathcal{H}_{m',k'}} \leq M$. We consider $\| f_{n'} - f \|^2_{\mathcal{H}_{m,k}}$ and cut the sum on $\alpha \in \mathbb{Z}^n$ in two parts, namely I and II, as it follows

$$\| f_{n'} - f \|^2_{\mathcal{H}_{m,k}} \quad = \quad I + II$$

with

$$\begin{aligned} I &= \sum_{|\alpha| \leq A} | < \psi_\alpha, f_{n'} - f > |^2 (1 + |\alpha_0|)^{2k} (1 + |\overline{\alpha}|)^{2m} \\ II &= \sum_{|\alpha| > A} | < \psi_\alpha, f_{n'} - f > |^2 \frac{(1 + |\alpha_0|)^{2k'}}{(1 + |\alpha_0|)^{2(k'-k)}} \frac{(1 + |\overline{\alpha}|)^{2m'}}{(1 + |\overline{\alpha}|)^{2(m'-m)}} \end{aligned}$$

The function $f \mapsto < \psi_\alpha, f >$ is a continuous linear form on $\mathcal{H}_{m',k'}$, hence $< \psi_\alpha, f_{n'} > \to < \psi_\alpha, f >$ i.e. $< \psi_\alpha, f_{n'} - f > \to 0$. It implies that for all $\varepsilon_1 > 0$ there exists $\eta > 0$ such that for all $n' > \eta$, $\sum_{|\alpha| \leq A} | < \psi_\alpha, f_{n'} - f > |^2 < \varepsilon_1^2$. So

$$I \quad \leq \quad \varepsilon_1^2 (1 + A)^{2k+2m}.$$

We treat now the second term II. We notice that

$$\begin{aligned} \frac{1}{(1 + |\alpha_0|)^{2(k'-k)}} \frac{1}{(1 + |\overline{\alpha}|)^{2(m'-m)}} &\leq \frac{1}{\left[(1 + |\alpha_0|)(1 + |\overline{\alpha}|) \right]^{2\min(m'-m, k'-k)}} \\ &\leq \frac{1}{(1 + |\alpha|)^{2\min(m'-m, k'-k)}} \ . \end{aligned}$$

Thus

$$\begin{aligned} II &\leq \frac{1}{(1 + A)^{2\min(m'-m, k'-k)}} \| f_{n'} - f \|^2_{\mathcal{H}_{m',k'}} \\ &\leq \frac{1}{(1 + A)^{2\min(m'-m, k'-k)}} \left(\| f_{n'} \|_{\mathcal{H}_{m',k'}} + \| f \|_{\mathcal{H}_{m',k'}} \right)^2 \\ &\leq \frac{4M^2}{(1 + A)^{2\min(m'-m, k'-k)}}. \end{aligned}$$

Therefore for all $\varepsilon > 0$, we choose A tall enough to get $\frac{4M^2}{(1+A)^{2\min(m'-m, k'-k)}} \leq \frac{\varepsilon^2}{2}$. Then we set $\varepsilon_1 = \frac{\varepsilon}{\sqrt{2}(1+A)^{2k+2m}}$. So there exists η in $\mathbb{N}$ such that for all $n' \geq \eta$,

$$\begin{aligned} \| f_{n'} - f \|^2_{\mathcal{H}_{m,k}} &\leq \varepsilon_1^2 (1 + A)^{2k+2m} + \frac{4M^2}{(1 + A)^{2\min(m'-m, k'-k)}} \\ &\leq \frac{\varepsilon^2}{2} + \frac{\varepsilon^2}{2} = \varepsilon^2. \end{aligned}$$

We obtain that $(f_{n'})$ converges to f in $\mathcal{H}_{m,k}$. It means that $i(f_n)$ is a compact set *a fortiori* a relatively compact set. △

For the proof of the third inequality of lemma 2.4., it suffices to add after the previous version of the proof of this lemma:
Now we suppose that $f \in \mathcal{H}_{m,k} \cap \mathcal{H}_{m',k'}$ with $m < m'$, $k < k'$. Let $\gamma,\ \delta \in [0;1]$, it is clear that $\mathcal{H}_{m',k'} \subset \mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}$, so f is in $\mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}$. By using the inequalities above, we get that

$$\begin{aligned} \| f \|_{\mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}} &\leq \| f \|^{\gamma}_{\mathcal{H}_{m,\delta k+(1-\delta)k'}} \| f \|^{1-\gamma}_{\mathcal{H}_{m',\delta k+(1-\delta)k'}} \\ &\leq \| f \|^{\gamma\delta}_{\mathcal{H}_{m,k}} \| f \|^{\gamma(1-\delta)}_{\mathcal{H}_{m,k'}} \| f \|^{(1-\gamma)\delta}_{\mathcal{H}_{m',k}} \| f \|^{(1-\gamma)(1-\delta)}_{\mathcal{H}_{m',k'}} . \end{aligned}$$

As the norm $\mathcal{H}_{m,k'}$ and $\mathcal{H}_{m',k}$ of f can be bounded by the norm $\mathcal{H}_{m',k'}$ of f, we finally obtain that

$$\| f \|_{\mathcal{H}_{\gamma m+(1-\gamma)m',\delta k+(1-\delta)k'}} \leq \| f \|^{\gamma\delta}_{\mathcal{H}_{m,k}} \| f \|^{1-\gamma\delta}_{\mathcal{H}_{m',k'}} .$$

△