Remarks, and *Corrigendum*, to

P.T. Chruściel, S. Łęski
Polyhomogeneous solutions of wave equations without corner conditions
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I am very grateful to (mainly) Roger Tagne Wafe, but also to Julien Loizelet, for pointing out the following:

1. The minus sign in front of the integral in (3.21) should be a plus.

2. The term $y\partial_y\mathring{\psi}$ at the r.h.s. of (3.24a) should be $x\partial_x\mathring{\psi}$.

3. In Remark 3.10, the condition "$\psi_2 \in x^{-1+\delta}\mathscr{A}^{\delta}_{\{x=0\}}$" should be replaced by "$\psi_2|_{x=y} \in x^{-1+\delta}\mathscr{A}^{\delta}_{\{x=0\}}$".

4. The spaces $\mathscr{C}^{\alpha}_{k}$ in the last two equations on p. 111 should be $\mathscr{C}^{\alpha}_{\{x=0\},k}$, so that (3.48) and the next equation should actually read
$$-\partial_x e_a \quad \leq \quad x^{\alpha}\|c\|_{\mathscr{C}^{\alpha}_{\{x=0\},k}} + x^{\epsilon}\|b\psi\|_{\mathscr{C}^{-\epsilon}_{\{x=0\},k}} \; . \tag{3.48}$$
$$\|\psi\|_{L^\infty([0,y_1])} \leq \|\psi|_{x=y}\|_{L^\infty} + \frac{\mathring{y}^{\alpha+1}}{|\alpha+1|}\|c\|_{\mathscr{C}^{\alpha}_{\{x=0\},k}} + \frac{y_1^{1-\epsilon}}{1-\epsilon}\|b\|_{\mathscr{C}^{-\epsilon}_{\{x=0\},k}}\|\psi\|_{L^\infty([0,y_1])}$$
(an absolute value of $|\alpha+1|$ has also been fixed above).

5. Let us call (3.45a) the second undisplayed equation after (3.45) on page 111
$$\partial_x(y\partial_y\psi) - \underbrace{x^{-p\delta}\Big(\partial_{w_1}H_\psi\Big)(x^\mu,w)x^{q\delta}}_{=O(x^{(mq-p)\delta-\epsilon})}y\partial_y\psi_1 = O(x^{-1-\epsilon}) \; . \tag{3.45a}$$
Then, on p. 112, the first sentence after the end of the proof of Lemma 3.12 should read "From (3.45a) and ..."

6. p. 116, the last sentence before the Acknowledgements should read "...hence the epsilons in Lemma 3.13."

7. The index j which is summed over in equation (A.5), p. 119, is not useful.

8. p. 120: a file manipulation error lead to a disappearence of primes over several m's, which makes the argument starting in the line before (A.7) until the end of the page completely absurd. The correct version reads:
Let $m' > m$, we then have
$$r'_m - \sum_{i=N(m)+1}^{N(m')}\sum_{\ell=0}^{N_i} f_{i\ell}\, x^{n_i}\ln^\ell x = r'_{m'} \in \mathscr{C}^{0}_{\{x=0\},\infty} \cap \mathscr{C}^{m'}_{\{x=0\},0} \; , \tag{0.1}$$

with each term in the sum being $O(x^m)$ (otherwise $r'_{m'}$ wouldn't be $O(x^{m'})$). Recall the usual interpolation inequality [?], for $0 < k < \ell$,

$$\|f\|_{C_k} \le C(k,\ell)\|f\|_{C_0}^{1-\frac{k}{\ell}}\|f\|_{C_\ell}^{\frac{k}{\ell}}\ ;$$

its weighted equivalent reads (compare the proof of [?, Lemma A.4])

$$\|r'_{m'}\|_{\mathscr{C}^{(1-\frac{k}{\ell})m'}_{\{x=0\},k}} \le C'(k,\ell)\|r'_{m'}\|^{1-\frac{k}{\ell}}_{\mathscr{C}^{m'}_{\{x=0\},0}}\|r'_{m'}\|^{\frac{k}{\ell}}_{\mathscr{C}^{0}_{\{x=0\},\ell}}\ .$$

Given $k \in \mathbb{N}$ we choose $\ell = 2k$, $m' = 2m$, leading to

$$r'_m - \sum_{i=N(m)+1}^{N(m')}\sum_{\ell=0}^{N_i} f_{i\ell}\ x^{n_i}\ln^\ell x \in \mathscr{C}^m_{\{x=0\},k} \quad\Longrightarrow\quad r'_m \in \mathscr{C}^m_{\{x=0\},k}\ .$$

Since k is arbitrary, we find that

$$r'_m \in \mathscr{C}^m_{\{x=0\},\infty}\ ,$$

and our claim follows. $\square$

9. Bigger extreme brackets in (A.13) and (A.14) would make the equations clearer.

10. A "min" in point (1) of Proposition A.6 should be a "max", so that this point should read:

    If $f \in \mathscr{T}^{\alpha,(\beta;k)}_{\{0\le x\le y\},\infty}$ then $\partial_x f \in \mathscr{T}^{\alpha-1,(\beta;k)}_{\{0\le x\le y\},\infty}$ and $\partial_y f \in \mathscr{T}^{\alpha,(\beta-1;\max(k-1,0))}_{\{0\le x\le y\},\infty}$

11. Note that the space $\mathscr{C}^\alpha_{\infty|0}$ coincides with $\mathscr{C}^\alpha_\infty$, so one might as well use the latter in the statement of Theorem B.9, p. 138.

12. There are spurious y-derivatives in the last term of (A.32), the correct equation being

$$\begin{aligned}\partial^\ell_x\partial^\beta_v\int_x^y f(x,v^A,s)ds &= \int_x^y \partial^\ell_x\partial^\beta_v f(x,v^A,s)\,ds \\ &\quad -\sum_{i=0}^{\ell-1} C_{\ell,i}\partial_x^{\ell-1-i}\partial^\beta_v f(x,v^A,s)\Big|_{s=x}\ , \qquad \text{(A.32)}\end{aligned}$$

13. p. 139, a factor 1/2 is missing in the second before last equation, which should therefore read

$$\psi(x,v^A,\tau) = \psi(x+2\tau,v^A,0) + \frac{1}{2}I_1(c_2)\ .$$