

Remarks, and *Corrigendum*, to

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Solutions of wave equations in the radiating regime

Bull. Soc. Math. de France **133** (2005), 1–72, math.AP/0202015.

I am very grateful to Szymon Łęski for pointing out the following:

1. The sketch of the proof of Theorem 4.9 given in the paper suggests that the supplementary hypothesis

$$\psi_1 \in \mathcal{C}_\infty^0(\Omega_{x_0, T}) \quad (*)$$

is needed to be able to use Lemma A.5 to estimate the non-linearity in $\mathcal{C}_\infty^{(mq-p)\delta}$. Here the nonlinearity is viewed as a function of $x^{q\delta} f$, where $f = (\psi_1, x\psi_2, x\varphi)$. (Note that by hypothesis we have $x\psi_2 \in \mathcal{C}_\infty^{1+\beta'} \subset \mathcal{C}_\infty^0$, similarly for $x\varphi$.) The property (*) does actually follow from the equations, and does not need to be assumed, the argument goes as follows: First, because of the non-linearity, Equation (3.46b) reads now

$$\partial_x \psi = c_2 + x^{-p\delta} H_\psi(x^\mu, x^{q\delta} f), \quad (3.46b')$$

where $f = (\psi_1, x\psi_2, x\varphi)$. We have $x^{-p\delta} H_\psi(x^\mu, x^{q\delta} f) = O(x^{(mq-p)\delta})$ which is integrable in x , and so is c_2 , hence $\psi \in L^\infty$. Next, set $y^a = (v^A, \tau)$, and write $p = (p_1, p_2, p_3) = (x^{q\delta}\psi_1, x^{q\delta+1}\psi_2, x^{q\delta+1}\varphi)$. We have the equation

$$\begin{aligned} \partial_x \partial_a \psi &= \partial_a c_2 + x^{-p\delta} \left(\partial_a H_\psi \right) (x^\mu, x^{q\delta} f) \\ &\quad + x^{-p\delta} \left(\partial_{p_2} H_\psi \right) (x^\mu, p) x^{q\delta+1} \partial_a \psi_2 \\ &\quad + x^{-p\delta} \left(\partial_{p_3} H_\psi \right) (x^\mu, p) x^{q\delta+1} \partial_a \varphi \\ &\quad + x^{-p\delta} \left(\partial_{p_1} H_\psi \right) (x^\mu, p) x^{q\delta} \partial_a \psi_1. \end{aligned}$$

It should be clear that each term in the sum above has a zero of order m (or higher) in the variables $(f, \partial_a f)$. We know by hypothesis that $(\psi_1, x\psi_2, x\varphi, x\partial_a \psi_2, x\partial_a \varphi) \in L^\infty$, which allows us to analyse all terms above, *except the last one*, as we did for (3.46b'). This allows us to write the following system of equations for $\partial_a \psi_1$:

$$\partial_x \partial_a \psi_1 - \underbrace{x^{-p\delta} \left(\partial_{p_1} H_\psi \right) (x^\mu, p) x^{q\delta}}_{=O(x^{(mq-p)\delta})} \partial_a \psi_1 = c'_2,$$

with c'_2 integrable in x . Proposition B.3 gives $\partial_a \psi_1 \in L^\infty$. Continuing in this way with $\partial_a \partial_b \psi_1$, *etc*, one obtains (*).

2. Statement of theorem 4.10: point 1 and equation (4.66) should be changed to

There exists $\tau_+ > 0$ such that f exists on Ω_{x_0, τ_+} , with

$$\|\tilde{f}\|_{L^\infty(\Omega_{x_0, \tau_+})} < \infty. \quad (4.66)$$

3. All occurrences of $\tau + 2x$ should be replaced by $x + 2\tau$.

Furthermore, I would like to thank Roger Tagne Wafe and Julien Loizelet for drawing my attention to the following list of misprints:

1. First line of proof of Propostion 3.1: change M_{x_2, x_1-t} to M_{x_2, x_1-2t}
2. In the proof of Proposition 3.1 there are four occurrences of $2\alpha + 1$ which should be replaced by $|2\alpha + 1|$ (p. 11 and 12)
3. There should be a minus sign in front of the left-hand-side of the equation immediately after (3.29) (last equation on p. 12)
4. p.14, in the proof of Proposition 3.2, the reference to equation (3.29) should be a reference to the second unnumbered equation on p. 13
5. p. 17, there is a spurious x factor in the fourth line of the proof of Propostion 3.3, so change

One can rewrite Equations (3.37) as $x\partial_\tau(\varphi, \psi) = \dots$

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6. There are some terms missing in equation (3.51), which should read

$$\begin{aligned} \partial_\tau \varphi + (E_-^\tau)^{-1} \{ (B_{11} + B_-) \varphi + \ell \psi \} &= (E_-^\tau)^{-1} (E_-^i \partial_i \varphi - \ell^A \partial_A \psi - B_{12} \psi + a) \\ \partial_x \psi - (E_+^x)^{-1} \{ \ell^\dagger \varphi - (B_{22} + B_+) \psi \} &= (E_-^\tau)^{-1} ((\ell^A)^\dagger \partial_A \varphi + E_+^\tau \partial_\tau \psi + E_+^A \partial_A \psi - B_{21} \varphi + b), \end{aligned}$$

7. There are several signs wrong in equations (4.13)-(4.14), which should read

$$\begin{aligned} e_-(\phi_+) - D_{e_A} \psi_A + \frac{n-1}{2(x+\tau+1/2)} \phi_+ &= \frac{n-1}{2(x+\tau+1/2)} \phi_- - a_+, \\ -e_A(\phi_+) + e_+(\psi_A) - \frac{1}{(x+\tau+1/2)} \psi_A &= b_A, \\ e_-(\phi_A) - e_A(\phi_-) + \frac{1}{(x+\tau+1/2)} \phi_A &= a_A, \\ -D_{e_A} \phi_A + e_+(\phi_-) - \frac{n-1}{2(x+\tau+1/2)} \phi_- &= -\frac{n-1}{2(x+\tau+1/2)} \phi_+ - b_-, \end{aligned}$$

8. Equation (4.17) should read

$$a_+ \equiv b_- \equiv G \equiv \Omega^{-\frac{n+3}{2}} H(x^\mu, \Omega^{\frac{n-1}{2}} \tilde{f}).$$

9. There are several occurrences of \mathcal{H}^α which should be $\mathcal{H}_k^{\alpha'}$'s in equations (4.26) and (4.27); moreover in (4.27) the decay exponent on $\|\phi_-(s)\|_{\mathcal{H}_k^{\alpha'-1/2}}^2$ should be $\|\phi_-(s)\|_{\mathcal{H}_k^{\alpha'}}$, so (4.27) should read

$$\begin{aligned} \hat{E}_{\alpha'}(t) \leq C \left\{ \hat{E}_{\alpha'}(0) e^{Ct} + \int_0^t e^{C(t-s)} \left(\|a_+(s)\|_{\mathcal{H}_k^{\alpha'}}^2 \right. \right. \\ \left. \left. + \|\phi_-(s)\|_{\mathcal{H}_k^{\alpha'}}^2 + \sum_A \|b_A(s)\|_{\mathcal{H}_k^{\alpha'-1/2}}^2 \right) ds \right\}. \end{aligned}$$

10. The displayed equation on 5-th line on p. 31 has a sign wrong in the exponent, and should read

$$e_+[(x + 2\tau)\phi_-] \leq \hat{C}x^\alpha ,$$