Remarks, and *Corrigendum*, to

P.T. Chruściel and O. Lengard

Solutions of wave equations in the radiating regime

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I am very grateful to Szymon Łęski for pointing out the following:

1. The sketch of the proof of Theorem 4.9 given in the paper suggests that the supplementary hypothesis

$$\psi_1 \in \mathscr{C}^0_\infty(\Omega_{x_0,T}) \tag{*}$$

is needed to be able to use Lemma A.5 to estimate the non-linearity in $\mathscr{C}^{(mq-p)\delta}_\infty$. Here the nonlinearity is viewed as a function of $x^{q\delta}f$, where $f = (\psi_1, x\psi_2, x\varphi)$. (Note that by hypothesis we have $x\psi_2 \in \mathscr{C}^{1+\beta'}_\infty \subset \mathscr{C}^0_\infty$, similarly for $x\varphi$.) The property (*) does actually follow from the equations, and does not need to be assumed, the argument goes as follows: First, because of the non-linearity, Equation (3.46b) reads now

$$\partial_x\psi = c_2 + x^{-p\delta}H_\psi(x^\mu, x^{q\delta}f)\ , \tag{3.46b'}$$

where $f = (\psi_1, x\psi_2, x\varphi)$. We have $x^{-p\delta}H_\psi(x^\mu, x^{q\delta}f) = O(x^{(mq-p)\delta})$ which is integrable in x, and so is c_2, hence $\psi \in L^\infty$. Next, set $y^a = (v^A, \tau)$, and write $p = (p_1, p_2, p_3) = (x^{q\delta}\psi_1, x^{q\delta+1}\psi_2, x^{q\delta+1}\varphi)$. We have the equation

$$\begin{aligned}\partial_x\partial_a\psi &= \partial_a c_2 + x^{-p\delta}\Big(\partial_a H_\psi\Big)(x^\mu, x^{q\delta}f)\\ &\quad + x^{-p\delta}\Big(\partial_{p_2} H_\psi\Big)(x^\mu, p)x^{q\delta+1}\partial_a\psi_2\\ &\quad + x^{-p\delta}\Big(\partial_{p_3} H_\psi\Big)(x^\mu, p)x^{q\delta+1}\partial_a\varphi\\ &\quad + x^{-p\delta}\Big(\partial_{p_1} H_\psi\Big)(x^\mu, p)x^{q\delta}\partial_a\psi_1\ .\end{aligned}$$

It should be clear that each term in the sum above has a zero of order m (or higher) in the variables $(f, \partial_a f)$. We know by hypothesis that $(\psi_1, x\psi_2, x\varphi, x\partial_a\psi_2, x\partial_a\varphi) \in L^\infty$, which allows us to analyse all terms above, *except the last one*, as we did for (3.46b'). This allows us to write the following system of equations for $\partial_a\psi_1$:

$$\partial_x\partial_a\psi_1 - \underbrace{x^{-p\delta}\Big(\partial_{p_1} H_\psi\Big)(x^\mu, p)x^{q\delta}}_{=O(x^{(mq-p)\delta})}\partial_a\psi_1 \quad = \quad c_2'\ ,$$

with c_2' integrable in x. Proposition B.3 gives $\partial_a\psi_1 \in L^\infty$. Continuing in this way with $\partial_a\partial_b\psi_1$, *etc*, one obtains (*).

2. Statement of theorem 4.10: point 1 and equation (4.66) should be changed to

   There exists $\tau_+ > 0$ such that f exists on Ω_{x_0,τ_+}, with

$$\|\widetilde{f}\|_{L^\infty(\Omega_{x_0,\tau_+})} < \infty\ . \tag{4.66}$$

3. All occurrences of $\tau+2x$ should be replaced by $x+2\tau$.

Furthermore, I would like to thank Roger Tagne Wafe and Julien Loizelet for drawing my attention to the following list of misprints:

1. First line of proof of Propostion 3.1: change M_{x_2,x_1-t} to M_{x_2,x_1-2t}
2. In the proof of Proposition 3.1 there are four occurrences of $2\alpha+1$ which should be replaced by $|2\alpha+1|$ (p. 11 and 12)
3. There should be a minus sign in front of the left-hand-side of the equation immediately after (3.29) (last equation on p. 12)
4. p.14, in the proof of Proposition 3.2, the reference to equation (3.29) should be a reference to the second unnumbered equation on p. 13
5. p. 17, there is a spurious x factor in the fourth line of the proof of Propostion 3.3, so change

   One can rewrite Equations (3.37) as $x\partial_\tau(\varphi,\psi)=$...

   to

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6. There are some terms missing in equation (3.51), which should read

$$\begin{aligned}\partial_\tau\varphi+(E^\tau_-)^{-1}\left\{(B_{11}+B_-)\varphi+\ell\psi\right\} &= (E^\tau_-)^{-1}(E^i_-\partial_i\varphi-\ell^A\partial_A\psi-B_{12}\psi+a)\\ \partial_x\psi-(E^x_+)^{-1}\left\{\ell^\dagger\varphi-(B_{22}+B_+)\psi\right\} &= (E^\tau_-)^{-1}((\ell^A)^\dagger\partial_A\varphi+E^\tau_+\partial_\tau\psi+E^A_+\partial_A\psi-B_{21}\varphi+b)\ ,\end{aligned}$$

7. There are several signs wrong in equations (4.13)-(4.14), which should read

$$\begin{array}{llllr} e_-(\phi_+) & -D_{e_A}\psi_A & +\frac{n-1}{2(x+\tau+1/2)}\phi_+ & = & \frac{n-1}{2(x+\tau+1/2)}\phi_--a_+\ ,\\ -e_A(\phi_+) & +e_+(\psi_A) & -\frac{1}{(x+\tau+1/2)}\psi_A & = & b_A\ ,\\ e_-(\phi_A) & -e_A(\phi_-) & +\frac{1}{(x+\tau+1/2)}\phi_A & = & a_A\ ,\\ -D_{e_A}\phi_A & +e_+(\phi_-) & -\frac{n-1}{2(x+\tau+1/2)}\phi_- & = & -\frac{n-1}{2(x+\tau+1/2)}\phi_+-b_-\ ,\end{array}$$

8. Equation (4.17) should read

$$a_+\equiv b_-\equiv G\equiv\Omega^{-\frac{n+3}{2}}H(x^\mu,\Omega^{\frac{n-1}{2}}\tilde f)\ .$$

9. There are several occurrences of $\mathscr{H}^\alpha$ which should be $\mathscr{H}^\alpha_k$'s in equations (4.26) and (4.27); moreover in (4.27) the decay exponent on $\|\phi_-(s)\|^2_{\mathscr{H}_k^{\alpha'-1/2}}$ should be $\|\phi_-(s)\|^2_{\mathscr{H}_k^{\alpha'}}$, so (4.27) should read

$$\begin{aligned}\hat E_{\alpha'}(t) \ \le\ & C\left\{\hat E_{\alpha'}(0)e^{Ct}+\int_0^t e^{C(t-s)}\left(\|a_+(s)\|^2_{\mathscr{H}_k^{\alpha'}}\right.\right.\\ &\left.\left.+\|\phi_-(s)\|^2_{\mathscr{H}_k^{\alpha'}}+\sum_A\|b_A(s)\|^2_{\mathscr{H}_k^{\alpha'-1/2}}\right)ds\right\}\ .\end{aligned}$$

10. The displayed equation on 5-th line on p. 31 has a sign wrong in the exponent, and should read

$$e_+[(x+2\tau)\phi_-] \quad \leq \quad \hat{C}x^\alpha \ ,$$