Mapping and its Observer*

Manfred Füllsack
University of Vienna†

Department of Philosophy
Universitätsstr. 7
A-1010 Vienna
Austria
manfred.fuellsack@univie.ac.at

Abstract

The paper discusses aspects of a project that strives to base an understanding of what economics call "productivity" on a complexity theoretic foundation. The core thesis of this project is that productivity can best be grasped by referring to two features commonly associated with knowledge - "non-reducibility in consumption" and "time preference". The paper in hand focuses on theoretical aspects concerning the ontological status of the *observer* in defining productivity alongside these features. Oriented on the Theory of Social Systems by Niklas Luhmann and methodologically drawing on Multi-Agent-Simulation, it investigates the thesis that the observer itself - circularly - can be conceptualized as a consequence of these two features.

1 Introduction

Economic theory has a long history of advancing different factors of production. The list stretches from land (physiocracy), trade (mercantalism), labor (Locke, Smith, Ricardo) and its exploitation (Rodbertus, Marx) up to capital (Clark), technical progress (Solow) and innovation (Coase). Lately economists focus on knowledge as the relevant factor. But what exactly is knowledge?

Again there is a long list of explanations and aspects. Among the more interesting I consider two aspects to be of special interest when trying to grasp what might correlate knowledge and productivity. These aspects are first the principle of "non-reducibility in consumption" ("jointness of supply", according to Barry and Hardin 1982), i.e. the fact that knowledge does not seem to get less when passed on; and second the principle of "time preference" which has been promoted as the knowledge of a future discounted value of current values. (Böhm-Bawerk, Fischer, Samuelson). On its own, this later principle might suffice to explain interest rates and other factors of economic growth. It does not - in my opinion - explain productivity in a philosophically satisfying way. To be more precise, on its own it does not explain how something manages to produce output at least as high, if not higher, than input, or in more physical terms, how entropy at least temporally can be suspended. However, time preference seems to point into a promising direction. Together with the principle of "non-reducibility in consumption" one can sense a condition of productivity in the possibility to distribute current expanses over vast numbers of future benefits - a condition that seems to reach deep enough to connect it to the physics of disequilibria far from thermo-dynamic equilibrium, i.e. to the physics of dissipative structures.

Let's introductorily illustrate this condition with the simple example of a hammer. Imagine a world without hammers, where nails (or their predecessors) have to be worked on with stones. With a stone you manage to drive in a nail in 10 minutes. A hammer would reduce this to one minute's work. However, to construct a hammer needs one hour of time. So if you need just one or two nails to be fixed, it does not pay to care for a hammer. It would not be productive - unless you sense that sometimes in future you might run into nails again.

Productivity in this example emerges distributed and only because there is recurrence ahead, the recurrence of future nails which makes it rational to spend time on the fabrication of a hammer. Productivity arises because you can distribute the costs of a one-time effort over a great number of instances by (re-)using "free copies" of the effort. Thus, in order to be productive, nature, or firms or whatever, have to find regularities over which to spread costs via cheap copies. And if these regularities are not regular enough you might look for regulators - as Ashby's "requisite variety" states - which are variable enough to counter the variations in regularity. These regulators then follow the same principle. They might be costly at first, but cheap through copying their effect in the long run.

This principle seems apt to shade light on funda-

*in: Trappl, Robert (ed.): Cybernetics and Systems 2010. Vienna: Austrian Society for Cybernetic Studies, p.243-148.

†The preparation and presentation of this paper was supported by the International Federation for Systems Research.

mental philosophical quests such as "Why is something, rather than nothing?" as well as on (higher order) biological or economical questions like "What entails growth?" A tentative answer might be: an operable ratio of costly pattern recognition and cheap copying. I will call a process that responds to this ratio *mapping*.

However, there is an aside to this story - which here shall be put in the center. The hammer introduces not only a time gain, but also a new perspective. From a point of view *with* a hammer the process of nailing looks different than from a point of view *without*. What has been productive without - let's say to drive in 5 nails with a stone - now looks pretty inefficient. The hammer introduces, so to say, a new observational viewpoint that re-defines productivity. In other words, the hammer reminds us of the *involvement of an observer in the observed process* which Heinz von Foerster[1] did not tire to emphasis and which Niklas Luhmann [e.g. 1984: 406] marked as an indispensible principle of modern (i.e. functionally differentiated) society. So if we want to suggest a consistent theory of productivity, the emergence of this observer - being itself a "productive" conception - has to be explainable by the same principle.

This feat entails circularity and therewith a paradox. We have to explain an operable ratio of future benefits to current costs in what we call mapping because we, the observers, being ourselves a product of this ratio, observe it that way. In order to cope with this paradox, we can't do other than "invisibilize" it temporarily [Luhmann 1984: 77]. Therefore, in the following text we will act as in daily life. We will ignore the paradox at first, in order to gain a stabile footing (a "scientific system", as Luhmann would call it) from where we can re-introduce it ("re-entry") and therewith observe the observer and its emergence.

2 Mapping

I therefore start out quite conventionally by outlining a simple process of mapping. For this I implement a population of computer-generated agents in an environment with variable resources supply. The supply changes with "seasons" and the agents have to run their metabolism from it. The agents' metabolism is randomly tuned to higher and lower degrees of consumption. In other words, there are agents with high consumption in spring, middle in summer, low in fall and zero in winter, there will be others with high consumption in different times and again others with again differently tuned metabolisms. Having high consumption in winter however, when supply is zero, and low in fall, when supply is highest, is unfavorable. It impairs the fitness of the agent, therewith diminishing her reproductive success. The outcome of this little Genetic Algorithm is straight forward. Allowing for a one per cent mutation rate, the simulation, within a couple of steps, generates a population of well adapted agents whose metabolism is high when supply of resources is high, and whose metabolism is low when supply of resources is low.

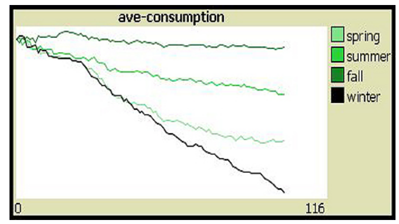

Figure 1: Evolution of metabolisms over 100 steps, in respect to a supply of resources in spring of 30, in summer of 60, in fall of 200 and in winter of 0, modeled with Netlogo.

A "blind teleology" (Depew, Dawkins) seems to take effect here. The agents map the regularities of their environment, i.e. the changing of seasons. They evolutionary synchronize their consumption with their environmental conditions. While the process of mapping itself is quite costly - in terms of the agents dying in the course of the adaptation process - its reproduction is not. Once established, the information is transmitted to the next generation nearly for free.

3 Meta-mapping

The principle repeats on all levels. In a variation of the model I assumed agents that by adapting and therewith surviving alter the environment to which they adapt. In other words, agents generate "self-made" (or "artificial") environments. One might think of greenhouse-gases emitting agents that by becoming numerous change the climate of their world and subsequently the seasonal supply of resources. To make the process graphically more distinct, I choose a rather abrupt influence starting after 70 steps and turning the seasonal order upside down.

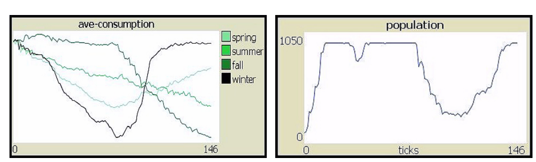

Figure 2: Meta-adaptation. Agents adapt to seasons, and after 70 steps to their own influence on their environment.

Hardly surprising, the population adapts again. And again both evolutions are costly in the beginning (see the downward hump in the population plot), but once established, are processed nearly for free.

We might interpret such meta- or second-order-mappings as a sort of self-provision of regularities that can trigger new mapping processes. The system, so

[1] Cf. for example von Foerster's [1993: 143] ingenious example of numbers which obviously are not to be mapped to some order in a conventional mathematical sense. The set is 8, 5, 4, 9, 1, 7, 6, 3, 2, 0 and its "mapping-order" is the alphabetic sequence of the beginning letters of its English terms, i.e. "eight", "five", "four", "nine", "one" ... Obviously this order is not (at least not at first) to be perceived as such from a mathematical viewpoint. In other words, this order is - like any other order - *context*- or *observer*-dependent.

to say, in exploiting discovered regularities, generates new regularities to which it - or another system - might adapt on a next level. And this new adaptation again generates regularities to which again mapping is triggered, and so on. Similar to a Russian Matrioshka, one could imagine an potentially endless sequence of dissipative structures building one on the other by exploiting the regularities generated by the regularity exploiting process of the *n-1*-level. Mapping, in other words, gets complex by mapping and therewith seems to generate a remarkable heterogeneous and versatile world.

4 Emergence and the Observer

By mapping self-made regularities the system, so to speak, turns back on its own consequences. From a certain perspective we might say, the system starts mapping itself. In operating on its own operations it performs sort of a "re-entry" [Spencer-Brown 1969] and therewith - if gaining enough complexity - might close off against external factors. The system becomes an "autopoietically" closed system which "self-referentially" generates its own operating conditions [Luhmann 1984: 60f]. Only therewith it becomes a system in a strict sense of the word, i.e. a system that maintains a difference to its environment and therewith can be discerned from it - *by an observer*.

As mentioned at the beginning, there are good reasons to consider the observer a basic order concept without which no mapping and no autopoietically closure and thus no system would ever be. From this perspective, it would be the observer who introduces the order or structure or regularity on which mapping can take off and proceed, and systems can emerge and differentiate further. Thus, in order to understand mapping we should try to answer the question where the observer comes from? This might be equivalent to the question, how mapping gets started? (or: where productivity originates?)

At first let's take one more look at the implications of the observer. There is an ongoing debate in complexity theory about the epistemological status of forms or structures such as the Glider in John Convay's *Game of Life* (GOL) (Dennett, Abbott). In the sense of mapping as outlined in the beginning of this paper (though without the evolutionary aspect) the Glider might be interpreted as a distinct constellation of GOL-cells which gains and maintains its form on the base of regularities generated by the rules of GOL. In this respect the Glider is a map of these regularities and as such again provides regularities on which, for example, Paul Rendell [2002] could emulate his GOL-Turing Machine. Structures or forms like the Glider[2] are considered *emergent* entities or orders. In regard to the micro-specifications on an *n*-order-level - here for example the rules of GOL - they can be seen as macro-structures on an *n+1*-order-level. Starting with a famous remark by Aristotle, it has often been mentioned that this difference of order marks a "gap", a "jump", so to speak, (or a "fulguration" with Lorenz) of one level of order to another. Emergence, as the becoming of a whole that is more than the sum of its parts, implies such a "gap".

That this "gap" is not necessarily a "mystery gap" [Epstein 2006: 37] as often implied in classical emergentism, has to do with the role of the *observer*. In the example of the Glider, the information needed for generating a seemingly constant form from a sheer constellation of cells (as it obviously is one on the *n*-order-level of GOL) is provided by the observer of GOL. This observer *constructs* the Glider (which without it is not). It does this in regard to its own capacities to deal with complexities, for example to deal with the constellations resulting from a GOL-run. It scours these complexities for regularities, for repeating patterns which by their salience in regard to other, more evasive constellations, can (with the on-board means of the observer) be discerned and therewith deployed as a map - a map whose foremost characteristic therefore is not similarity, but salience, or in the terminology of Luhmann: difference.

The observer, so to speak, by giving them *form*, adds the in*form*ation to emergent orders[3] that is needed to - by themselves - make them seem to arise mysteriously. The observer in this sense is Adam Smith's "invisible hand", it is the "conductor" that synchronizes fireflies, fiddler crabs and pacemaker cells, and it is the originator of traffic jams, efficient markets and stock market disasters. No doubt, we should try to understand its origin.

5 Distinction and Indication

According to Luhmann [1997: 69f, and Georg Spencer-Brown 1969] observation is just an operation of distinction and indication.[4] In order to grasp the abstractness of this concept, we might recall the activities of the demon of James Clerk Maxwell who by sorting equally distributed molecules in a gas and indicating the fast and thus warm half of them seemed to generate energy from nothing. Maxwell's demon generated a difference out of a formerly undifferentiated mass, a difference that makes a difference, i.e. information according to Bateson. This demon is our observer.

Leonid Bunimovich [2001] recently replaced the simple double-chamber of Maxwell's demon with a

[2] Other basic examples are the distinct patterns in Tom Schelling's famous segregation experiments, or the synchronization of moving oscillators, fireflies for example, etc.

[3] This is meant literally: it is information in the Shannon sense with which the possibility-space of interpretations of the patterns on the n-order-level of GOL is reduced to an amount at which the specific form of the glider becomes probably. The information is gained by induction in the way of, for instance, Fred Attneave's [1954] experiments with visual perception.

[4] The reason for conceptualizing observation in this formal way lies in the possibility to base a calculus on it from which - by deploying the same principle over and over - the emergence of systems can be *observed* that by becoming sufficiently complex turn their operations on themselves and become *self-observing*. Cf. Luhmann 1984: 63f.

mushroom-shaped container whose specific geometry traps particles in different phase-spaces from which they do not seem to escape anymore. This Bunimovich-mushroom therewith acts like the demon with the difference that - at first glance - it seems to differentiate particles for free.

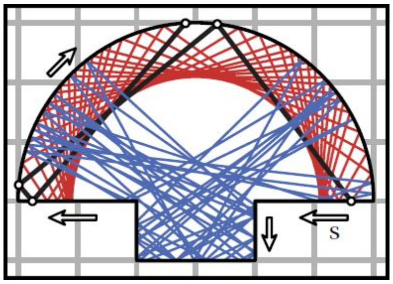

Figure 3: Bunimovich-Mushroom. Black and gray lines indicate different trajectories.

On second thought however, the question arises from where the form of the mushroom comes from. Without doubt, it is a rather complex structure on its own and can't be presupposed to any differentiating and thus observing process for free. The same goes for differentiating feed-backs as the ones in *Polya*-urns or in *Benard*-convections. They seem to generate *cosmos* from *chaos* by simple first random moves - choosing the first ball from the *Polya*-urn - and therewith producing order of the one or the other kind, i.e. a regularity which waits to be mapped. However, they are all ridden with prerequisites on their own. The urn, the balls, the heat, the fluid etc. indicate that there is no order without order, no mapping without mapping, no pattern without a pattern. The question about the origin of the observer runs up on the basic problem of "firstness".

6 Firstness?

Or is the speech of "firstness", the quest for an origin, for a final cause, misleading from the beginning?[5] Should we better conceive a process in which order and its observer emerge *concurrently*? Or to be more exact, in which they emerge "warped concurrently", that is "synchronously a-synchronously", "*gleichzeitig ungleichzeitig*" as Luhmann and others put it.

If we consider this possibility, we should once more point out the abstractness of the observer. It might suffice to imagine a highly volatile structure which by generating a pattern for a short moment of time, i.e. a basic kind of density for instance, of regularity, provides in*form*ation to another structure with which the probability of this other structure to emerge or to "exist" (or to operate or to whatever) rises a tiny little bit. If we understand information with Shannon as "reduction of uncertainty" we might assume that without this information provided by the first structure the possibility space of the second would be too large to keep going. Its probability to manage an operable ratio of output to input would be too low. The clou of the observer-conception thereby is that this provision of information works *reciprocally*. By however temporally flashing into existence the two (and many more) structures provide each other mutually with just enough information to reduce their "uncertainty" (here more exactly: their improbability) to a level at which they might at the least gain time to wait for a third information to further raise their probability of being - and therewith generate a structure, a regularity, which, though still highly volatile, may be used by other structures to persist and thus to go on mapping. In other words, the observer is not a *first*, but a *concurrently*emerging structure, a structure, however, without which no mapping would have sufficient probability to happen, without which, in other words, no structures - however dissipative - would be.

[5] Consider the remark of Jacques Derrida according to which no "first" would be a "first" without a "second" and thus the "second" as well might - in a conventional context of thinking - be considered the "first". Consider also concepts like "entanglement" in quantum mechanics. Cf: Lloyd 2006: 117.

7 The Topology of the Possible

A couple of scientific findings are backing this thesis. The most famous perhaps being the auto-catalytic Hyper-cycle of Schuster and Eigen [1979] in which polynucleotide-synthesis generates enzymes as catalysts providing other polynucleotide molecules with information that enhances their stability, or in other words, that raises the probability of their continuance. The length of polynucleotides necessary to form DNA and RNA as structures of life would not be feasible without this vice versa or reciprocal - and this means *circular* - provision of probability.

A somehow more detailed picture of this process and therewith some idea of how the mutual provision of information works might provide a short look at the experiments of Walter Fontana on the "topology of the possible" [e.g. 2005]. Fontana distinguishes the *genotype* and the *phenotype* of RNA structures as "sequences" (of molecular building blocks such as A,U,G,C) from "shapes", with the "shape" being the specific form a RNA molecule takes on as a consequence of different binding powers of its building blocks. The crucial aspect is that sequences, i.e. genotypes, can be copied (replicated), whereas the shape can not. It is generated through the folding process of the sequence. Evolutionary, the actual target of selection is the shape. By standing the fitness test, it enables the genotype to persist. The genotype on the other hand is target to mutation, therewith generating new shapes, i.e. phenotypes.

The temporary stability of the shape, needed for this mutual provision of *evolvability*, is gained by the fact that mutation can change particular positions in the genotype but does not always immediately alter the shape. Several mutations at various sequence positions do not affect the shape. They are called "neutral mutations". Some of these "neutral mutations", though not altering the shape on their own, make it dependent on mutations of neighboring, maybe otherwise neutral pairs.[6] This principle is called "epistasis"[7] and seems to be responsible for a somehow "jumping" (not steady) co-evolution of geno- and phenotypes, or more general for a co-evolution of structures or forms providing each other with information needed to become something from nothing.[8]

The decisive aspect here seems to be concurrency, or more exactly *warped concurrency*. With this we seem to be back at our starting point, the thesis that the probability needed for such concurrency to arise can only be gained by spreading its "costs" over a large enough sample, that is by kind of collecting reams of tiny probability fluctuations (which by themselves would not even come close to be called probability) in order to gain one probability level that is high enough for a co-evolution to take off.

8 Concurrency

In order to gain understanding of size and ratios of such samples I (2010) experimented with respective concurrency in a variation of Joshua Epstein's *Demographic Prisoner Dilemma* [1998, 2006] which sort of spatializes the famous investigations of Rapoport [1965], Axelrod [1984] and co. In Epstein's variant of the game, agents move on a grid playing a round of prisoners-dilemma (PD) with every (Von-Neumann-) encounter. Other than in Axelrod's tournament the pay-offs here are allowed to be negative but follow the defining rule of $t > r > p > s$. Agents accumulate wealth from this pay-offs. If wealth declines below zero they die, if wealth climbs to 10 they reproduce. Offspring inherit part of the wealth of their parent and, with a certain mutation rate, the strategy of whether to cooperate or to defect. Even with high mutation rate cooperation has a strong footing in Epstein's setting.

In my version agents start out with a mutation rate of 100 per cent. That is, they don't inherit the strategy of their parent. And they are exposed to a rougher environment than in Epstein's world. (typical pay-offs are $t = 9$, $r = 2$, $p = -5$, $s = -7$) With this setting cooperation has little chance, and since defectors can't survive on their own, in most cases the population dies out within a couple of steps.

However, agents are not considered to be finally adapted already. They react to the regularities of their environment by inhibiting a strategy. They simply increase the probability of whether to cooperate or to defect in dependence of how often they are confronted with other cooperators or defectors. Therewith, other than in Epstein's version, not only the number and social setting of cooperators and defectors induces cooperation, but cooperation (or defection) works back on the agents as well. Agents' behavior, so to speak, *co-evolves* with the population.

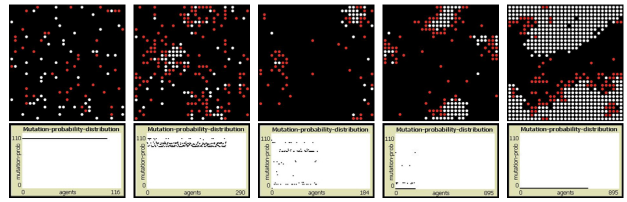

Figure 4: The taking off of a cooperators population (white) surrounded by defectors (black) in a Demographic PD with "learning" in (from left to right) 0, 15, 100, 200 and 500 steps. The plotters beneath show the development of the "uncertainty" about whether to cooperate or to defect. Each agent is represented by a black dot. In steps 0 uncertainty lies at 100 per cent, in step 500 at 0 per cent (both a black line).

In my trials, population took off in about 1 out of 10 cases, at times fluctuating in typical Lotka-Volterra-oscillations, but with cooperation eventually gaining a stabile footing.[9]

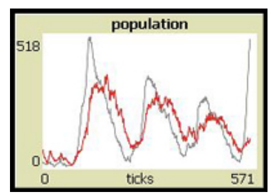

Figure 5: Population development over 500 steps with typical Lotka-Volterra-oscillations.

In the theory of multi-agent simulations, such feedbacks have been described as "second order emergence" (Gilbert 1995, 2002). Cristiano Castelfranchi (1998) speaks of "immergence" which works on a "cognitive" level of the individual agent.

In this context, what seems essential, is the reciprocity of the provision of probability (or information) among the two structures. On the one hand we might (analytically) distinguish emergent regularities of cooperation in some parts of the grid and of defection in others. By themselves however, these regularities are not sufficiently regular to enable the population to last. There is no chance for the regularities to be efficiently mapped. On the other hand, we have agents evolving regularities. But by themselves they as well are not able to inhibit their strategies effectively enough to enable the system to get going. Only when by chance a suitable constellation of adaptive agents *and* advantageous neighborhoods turns up, the process can take off. Regularities can be mutually deployed in order to map them and therewith produce

[6] If C becomes G the shape will not change, but will make it reactable to a change from A to U which otherwise (without the C-G-change) would also not alter the shape.

[7] Epstein and Axtell [1996] describe similar in regard to the changing of culture of *sugarscape*-agents by being influenced only in regard to one position of their cultural-code at a time.

[8] See also Jantsch 1980 for example, or investigations of networks of Steven Strogatz [2003]. Stuart Kauffman's recent "investigations" also seem to focus on this aspect.

[9] The simulation can be tested under http:// homepage.univie.ac.at/manfred.fuellsack/applets/coop.htm

new regularities - as for example those of the Lotka-volterra-oscillations - which then again might trigger some mapping on a next level. Referring to the abstract definition of the observer by Luhmann (i.e. distinction and indication), we might say the process *mutually observes* itself and only therewith gains enough probability to take off. May be in some sense we might even speak of *self-observation* which becomes possible because the system manages to distribute the costs of expensive efforts to find regularities - here of cooperation - over a vast enough population deploying cheap copies.

References

[Attneave, 1954] Attneave, Fred. *Informational aspects of visual perception. The Psychological Review 61*, p. 183-193, 1954.

[Axelrod, 1990] Axelrod, Robert. *The Evolution of Cooperation.* New York. Basic Books, 1990.

[Barry, Hardin, 1982] Barry, Brian / Hardin, Russell (Eds.). *Rational man and irrational society?* Beverly Hills, CA. Sage, 1982.

[Bunimovich, 2001] Bunimovich, Leonid A. *Mushrooms and other billiards with divided phase space. Chaos 11*, p. 802-808, 2001.

[Castelfranchi, 1998] Castelfranchi, Cristiano. *Simulating with Cognitive Agents: The Importance of Cognitive Emergence* in Sichman, Jaime S. / Conte, Rosaria / Gilbert, Nigel (eds.), *Multi-Agent Systems and Agent-Based Simulation.* p. 26-44, Berlin/Heidelberg. Springer, 1998.

[Conant, Ashby, 1970] Conant, Roger C. / Ashby, Ross, W. *Every Good Regulator of a System must be a Model of that System. International Journal of Systems Sciences, vol. 1, No. 2*, p. 89-97, 1970.

[Eigen, Schuster, 1979] Eigen, Manfred / Schuster, Peter *The Hypercycle.* Berlin, 1979.

[Epstein, Axtell, 1996] Epstein, Joshua M. / Axtell, Robert *Growing Artificial Societies. Social Science From the Bottom Up.* Cambridge, MA. MIT Press, 1996.

[Epstein, 1998] Epstein, Joshua M. *Zones of Cooperation in Demographic Prisoner's Dilemma. Complexity 4 (2)*, p. 36-48, 1998.

[Epstein, 2006] Epstein, Joshua M. (2006): *Generative Social Science: Studies in Agent-Based Computational Modeling.* Princeton. Princeton UP, 2006.

[Foerster, 1993] Foerster, Heinz von. *Was ist Gedaechtnis, dass es Rueckschau und Vorschau ermoeglicht?* In Foerster, H. *Wissen und Gewissen. Versuch einer Bruecke.* S. 299-336, Frankfurt/M. Suhrkamp, 1993.

[Fontana, 2005] Fontana, Walter. *The Topology of the Possible* In Wimmer, A. / Koessler, R. (eds) *Understanding Change: Models, Methodologies and Metaphors.* Palgrave Macmillan, 2005.

[Fuellsack, 2010] Füllsack, Manfred. *Gleichzeitige Ungleichzeitigkeit. Emergenz und die Parallelitaet aggregierter Dynamiken. conceptus 93*, 2010 (forthcoming).

[Gilbert, 1995] Gilbert, Nigel. *Emergence in Social Simulation* in Gilbert, Nigel / Conte, Rosaria (eds.) *Artificial Societies: The Computer Simulation of Social Life.* p. 144-156, London. UCL Press, 1995.

[Jantsch, 1980] Jantsch, Erich. *The Self-Organizing Universe: Scientific and Human Implications of the Emerging Paradigm of Evolution.* New York. Pergamon Press, 1980.

[Lloyd, 2006] Lloyd, Seth. *Programming the Universe. A Quantum Computer Scientist takes on the Cosmos.* New York. Vintage Books, 2006.

[Luhmann, 1984] Luhmann, Niklas. *Soziale Systeme. Grundriss einer allgemeinen Theorie.* Frankfurt/M. Suhrkamp, 1984.

[Luhmann, 1997] Luhmann, Niklas *Die Gesellschaft der Gesellschaft.* Frankfurt/M. Suhrkamp, 1997.

[Rapoport, Chammah, 1965] Rapoport, Anatol / Chammah, Albert M. *Prisoner's Dilemma.* AnnArbor. University of Michigan Press, 1965.

[Rendell, 2002] Rendell, Paul. *Turing Universality in the Game of Life* In Adamatzky, Andrew (ed.) *Collision-Based Computing.* New York. Springer, 2002.

[Spencer-Brown, 1969] Spencer-Brown, Georg. *Laws of Form.* London. Allen and Unwin, 1969.

[Strogatz, 2003] Strogatz, Steven. *Sync. How Order emerges from Chaos in the Universe, Nature and Daily Life.* New York. Hyperion, 2003.