Part I

Introduction
Outline

1 Motivation
   - Why Study Labor Economics?
   - Why Focusing on Unemployment?

2 Modern theories of Unemployment.
   - Search Frictions.
   - Other important specificities of labor markets.

3 Plan of the Course
   - Part II: Competitive Model
   - Part III: Search-Matching Model
   - Parts IV and V: Workers’ Motivation and Bargaining
One may easily argue that labor markets crucially determine macroeconomic outcomes:

1. The labor share is quite stable across developed countries and account for about two thirds of national income.
2. The vast majority of households draw most of their incomes from wage payments.
3. Differences in job creation between the US and Europe explain a significant share of the differences in output growth.
### Table 1. The Labor Share and Real Wages in 12 OECD Countries

<table>
<thead>
<tr>
<th></th>
<th>Labor share Levels</th>
<th>Changes 1970-90</th>
<th>Real wage growth 1970-90</th>
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</thead>
<tbody>
<tr>
<td>United States</td>
<td>69.7</td>
<td>68.3</td>
<td>66.5</td>
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<tr>
<td>Canada</td>
<td>66.9</td>
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<tr>
<td>Japan</td>
<td>57.5</td>
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<td>Germany</td>
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<td>France</td>
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<td>Netherlands</td>
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<td>Belgium</td>
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<td>Norway</td>
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<tr>
<td>Sweden</td>
<td>69.7</td>
<td>73.6</td>
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<tr>
<td>Finland</td>
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<td>69.6</td>
<td>72.3</td>
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<tr>
<td><strong>Mean</strong></td>
<td>66.2</td>
<td>68.4</td>
<td>65.1</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>3.6</td>
<td>3.3</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Note.— All variables in percentages. The labor share corresponds to the business sector, the real wage is the real compensation per employed person in the private sector. It includes an imputed labor remuneration for the self-employed on the basis of the average wage. Source: *OECD Economic Outlook* Statistics on Microcomputer Diskette.

<table>
<thead>
<tr>
<th>Table 1: Growth accounting for the European Union, United States and Japan, 1965-1997</th>
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<tbody>
<tr>
<td><strong>GDP growth (%)</strong></td>
</tr>
<tr>
<td>European Union</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Japan</td>
</tr>
</tbody>
</table>

| European Union | 4.8 | 4.3 | 42.6 | 42.1 | 4.7 | -9.8 | 52.6 | 67.7 |
| United States | 3.7 | - | 37.4 | - | 38.6 | 33.5 | 24.0 | 29.1 |
| Japan | 9.0 | - | 47.0 | - | 9.6 | 9.2 | 43.4 | 43.8 |

| European Union | 2.5 | 2.6 | 38.8 | 38.0 | 14.9 | -12.5 | 46.3 | 74.6 |
| United States | 2.5 | - | 37.3 | - | 52.8 | 56.2 | 9.9 | 6.5 |
| Japan | 3.6 | - | 54.1 | - | 16.7 | 12.2 | 29.2 | 33.7 |

| European Union | 2.2 | 2.3 | 34.2 | 28.1 | 16.5 | 18.3 | 49.3 | 53.6 |
| United States | 2.4 | - | 36.0 | - | 32.2 | 33.5 | 31.8 | 30.6 |
| Japan | 2.1 | - | 64.3 | - | 25.1 | 17.8 | 10.6 | 17.9 |

Notes: The European Union figures are simple averages calculated over the sample.
Why Focusing on Unemployment?

The persistence of unemployment:

1. is at odds with the market-clearing equilibrium condition commonly assumed in economics.
2. points to the need for a specific approach to Labor Economics.
3. is often regarded as the single most important problem facing European countries.
### Are Unemployment rates relevant statistics?

<table>
<thead>
<tr>
<th></th>
<th>Unemployment rate (%)</th>
<th>Participation rate (%)</th>
<th>Employment rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>6.7</td>
<td>73.8</td>
<td>68.9</td>
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<tr>
<td>Austria</td>
<td>4.0</td>
<td>70.7</td>
<td>67.8</td>
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<td>Belgium</td>
<td>6.2</td>
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<td>Canada</td>
<td>7.3</td>
<td>76.5</td>
<td>70.9</td>
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<tr>
<td>Denmark</td>
<td>4.2</td>
<td>79.2</td>
<td>75.9</td>
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<tr>
<td>Finland</td>
<td>9.2</td>
<td>74.6</td>
<td>67.7</td>
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<tr>
<td>France</td>
<td>8.8</td>
<td>68.0</td>
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<tr>
<td>Germany</td>
<td>8.0</td>
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<tr>
<td>Italy</td>
<td>9.6</td>
<td>60.7</td>
<td>54.9</td>
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<tr>
<td>Japan</td>
<td>5.2</td>
<td>72.6</td>
<td>68.8</td>
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<tr>
<td>Luxemburg</td>
<td>1.9</td>
<td>64.2</td>
<td>63.0</td>
</tr>
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<td>Netherlands</td>
<td>2.1</td>
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<td>Norway</td>
<td>3.5</td>
<td>80.3</td>
<td>77.5</td>
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<td>Portugal</td>
<td>4.3</td>
<td>71.8</td>
<td>68.7</td>
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<td>Spain</td>
<td>10.5</td>
<td>65.8</td>
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<td>Sweeden</td>
<td>5.1</td>
<td>79.3</td>
<td>75.3</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.5</td>
<td>81.2</td>
<td>79.1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>4.8</td>
<td>74.9</td>
<td>71.3</td>
</tr>
<tr>
<td>United States</td>
<td>4.8</td>
<td>76.8</td>
<td>73.1</td>
</tr>
<tr>
<td>European Union</td>
<td>7.4</td>
<td>69.2</td>
<td>64.1</td>
</tr>
<tr>
<td>Total OECD</td>
<td>6.4</td>
<td>69.8</td>
<td>65.3</td>
</tr>
</tbody>
</table>

**Table 1:** Rates of unemployment, participation, and employment in 19 OECD countries in 2001 (source: OECD data).
Unemployment rates over time

Figure 1: Unemployment rate in EU, US, Japan

Definitions and sources: Standardised unemployment rates, OECD Economic Outlook database. EU-11 is the euro area: the aggregate of current EU members excluding the UK, Denmark, Greece, and Sweden.
Real compensations over time

The positive correlation across countries between Unemployment and Real Compensation does not square with market adjustments to the equilibrium.
Unemployment cannot be reduced to an "excess supply of labor" since unfilled vacancies and searching workers **coexist**. To explain this phenomenon economists refer to frictions:

1. Due to the high degree of heterogeneity and differentiation of workers and jobs, matching is time and resources consuming for both job seekers and firms.

2. Screening is necessary due to the prevalence of informational asymmetries between employers and employees (employers do not observe all the characteristics of workers and vice-versa).

3. Due to coordination failures some open vacancies may end up with more than one applicant and some with none.

4. Workers face important mobility costs which hinder the reallocation process across sub-markets.

Even if Frictions were small, the actual size of the flows would generate significant Macro Effects.
Search and Matching Models

1. **Search** models capture the notion of frictional unemployment. Because it is resources consuming for workers and firms to find a job, unemployment naturally arises.

2. The **search-matching** model extends this mechanism to macro dimension by endogenizing Labor demand.
As opposed to machines, **efficient** work is not automatically extracted from workers. This raises the question of how to motivate workers?

The repartition of revenues occurs through a **bargaining** process between employees and employers.

Given their obvious social implications, legislations in the labor markets are numerous. As a result the labor markets are very different from 'Walrasian’ markets. Instead, their functioning is strongly determined by **social norms**.
Part II: Competitive Model

2. Labor Demand: Complementarity/Substitutability between Labor and Capital.
3. Limits of the competitive model: Counterfactual predictions about the business-cycle fluctuations of Employment and Wages.
4. Minimum Wage: Does it explain Unemployment?
### Part III: Search-Matching Model

Other Topics

Part IV: Efficiency Wage

1. Efficiency Wage Model of Shapiro and Stiglitz and the ‘Non-shiriking’ Rate of Unemployment.
2. Predictions about the business-cycle fluctuations of Employment and Wages.

Part V: Bargaining

1. Union Model of McDonald and Solow.
2. Predictions about the business-cycle fluctuations of Employment and Wages.
3. Game Theoretical Foundation of Nash-Bargaining.
References


Part II

Labor Supply and Labor Demand
Outline

4 Labor Supply
   - Intensive Margin
   - Extensive Margin
   - Effect of Non-wage Income
   - Effect of Wage
   - Empirical Estimation of Labor Supply

5 Labor Demand
   - Labor Demand in the Short Run
   - Cost Minimization
   - Profit Maximization
   - Empirical Estimation of Labor Demand

6 Minimum Wage

7 Business Cycle Fluctuations

8 References
For our purpose, we are mainly interested in deriving the **Aggregate** supply, i.e. the sum of individuals’ labor supply. Individual decisions can be decomposed as follows:

- **Extensive Margin**: Whether or not to participate in the labor market?
- **Intensive Margin**: For participants, how many hours of labor to supply?

As usual, we analyze the problem backward and so start by characterizing decisions at the Intensive margin.
The problem is largely similar to optimal bundle allocation in Consumer theory. The two goods are consumption $C$ and leisure $L$.

The wage is the opportunity cost of Leisure.

The determination of the optimal allocation is complicated by the fact that available income is a positive function of the price of leisure. This generates an additional Income Effect.
Optimization Problem

Definition

- $w$: Wage per unit of time.
- $R$: Non-wage Income.
- $L_0$: Time Endowment.

Optimization Problem

$$\max \{ C, L \} \ U(C, L)$$

s.t. $C \leq w(L_0 - L) + R \equiv R_0 - wL$
First Order Conditions

Lagrangian

\[ \mathcal{L} = U(C, L) + \lambda(R_0 - C - wL) \]

F.O.C.

\[ \frac{\partial U(C^*, L^*)}{\partial L} = w \frac{\partial U(C^*, L^*)}{\partial C} \]
Illustration

The diagram illustrates the relationship between consumption and leisure. The graph shows a curve representing the relationship, with consumption on the y-axis and leisure on the x-axis. Key points on the curve include:

- **R₀**: This point marks a significant level of consumption.
- **C^**: This point indicates a specific consumption level.
- **R**: This point represents a lower consumption level.

The curve suggests a diminishing marginal utility of leisure, as consumption decreases as leisure increases. The graph likely demonstrates how changes in leisure affect consumption, possibly due to factors such as wage levels or non-wage income.
**Optimization with Participation Constraint**

\[
\text{MAX}_{\{C,L\}} \ U(C, L)
\]

\[s.t. (i) \ C \leq w(L_0 - L) + R \equiv R_0 - wL\]

\[(ii) \ L \leq L_0\]

**Lagrangian**

\[
\mathcal{L} = U(C, L) + \lambda(R_0 - C - wL) + \mu(L_0 - L)
\]
F.O.C. with Participation constraint

\[ \frac{\partial U(C^*, L^*)}{\partial C} = \lambda \]

\[ \frac{\partial U(C^*, L^*)}{\partial L} = \lambda w + \mu \]

1. \( \mu = 0 \Rightarrow \textbf{Participation Constraint} \) does not bind. Then Solution is as before.

2. \( \mu > 0 \Rightarrow \textbf{Participation Constraint} \) binds. Then \( L^* = L_0 \) and \( C^* = R \).
Definition

The **Reservation wage** $w_r$ is the minimum wage such that the agent participates to the labor market. Hence:

$$w_r \equiv \frac{\partial U(R,L_0)}{\partial L} / \frac{\partial U(R,L_0)}{\partial C}$$
Leisure is a normal good

Persons with higher non-wage income work less. This substantiates the premise that leisure is a normal good.
Reservation wage and non-wage income

Proposition 1

If leisure is a normal good, the reservation wage is increasing in non-wage income.

Proof:

Claim: If leisure is a normal good, $U_{LC}U_C > U_{CC}U_L$.

Proof: Differentiate F.O.C. and budget constraint with respect to $R$

\[
C_R^* + wL_R^* = 1 \\
U_{LC}C_R^* + U_{LL}L_R^* = w(U_{CC}C_R^* + U_{CL}L_R^*)
\]

Combine the two equations and replace $w$ by $U_L/U_C$ to obtain

\[
L_R^* \left( \frac{2U_{CL}U_LU_C - U_{LL}(U_C)^2 - U_{CC}(U_L)^2}{U_C} \right) = U_{LC}U_C - U_{CC}U_L
\]

The term in brackets on the left hand side is positive because $U(C, L)$ is quasi-concave. Thus $L_R^* > 0$ iff $U_{LC}U_C - U_{CC}U_L$, which proves the Lemma. ■

Differentiating the reservation wage yields

\[
\frac{\partial w_r}{\partial R} = \frac{U_{LC}(R, L_0)U_C(R, L_0) - U_{CC}(R, L_0)U_L(R, L_0)}{(U_C(R, L_0))^2} > 0
\]

where the inequality follows from the Claim.
Proposition 2

If leisure is a normal good, an increase in wages has a positive **Substitution Effect** and negative **Income Effect** on labor supply. Thus the relationship between wages and hours worked is in general ambiguous.

**Proof:** Differentiate optimal labor supply with respect to \( w \)

\[
\frac{\partial L^*(w, R_0)}{\partial w} = L_w^*(w, R_0) + L_{R_0}^*(w, R_0) \frac{\partial R_0}{\partial w}
\]  

(2)

Since \( \partial R_0 / \partial w = L_0 \) and Leisure is a normal good we know that \( L_{R_0}^* \partial R_0 / \partial w > 0 \). This is the **Income Effect**.

Now, differentiate as before the F.O.C. and budget constraint with respect to \( w \)

\[
C_w^* + L^* + wL_w^* = 0
\]

\[
U_{LC} C_w^* + U_{LL} L_w^* = U_L + w(U_{CC} C_w^* + U_{CL} L_w^*)
\]

Rearranging finally yields

\[
L_w^* \left( \frac{2U_{CL} U_L U_C - U_{LL}(U_C)^2 - U_{CC}(U_L)^2}{U_C} \right) = -(U_C)^2 - L^*(U_{LC} U_C - U_{CC} U_L)
\]

Hence \( L_w^* < 0 \). This is the **Substitution Effect**.
Proposition 3

If leisure is a normal good, the labor supply is an increasing function of wages near the reservation wage.

Proof: Replace $L^*_R_0$ using equation (1) into equation (2) to obtain

$$\frac{\partial L^*(w, R_0)}{\partial w} \left( \frac{2U_{CL}U_LU_C - U_{LL}(U_C)^2 - U_{CC}(U_L)^2}{U_C} \right) = -(U_C)^2 + (L_0 - L^*)(U_{LC}U_C - U_{CC}U_L)$$

When $w \to w_r$, $L^* \to L_0$ and so $\lim_{w \to w_r} \frac{\partial L^*(w, R_0)}{\partial w} < 0$. 
Self-selection and Estimation

Estimates the following regression

\[
\ln h_i = \alpha_w \ln w_i + \alpha_R \ln R_i + X\theta_i + \varepsilon_i
\]

where \( \theta_i \) is an \([n,1]\) matrix collecting the individual’s \( i \) characteristics (e.g. age, education, gender...) and \( X \) is an \([1,n]\) matrix with the coefficients attached to each characteristic.

**Self-selection:** Restricting the sample to participants or setting \( h_i = 0 \) for non-participants, limits attention to individuals with a higher propensity to work \( \Rightarrow \) Biased Estimates.

**Solution:** One should estimate the full-model with Participation constraint. To do so, a parametric assumption on the utility function (e.g. Cobb-Douglas) is needed.

**Main difficulty:** By definition wages offered to non-participant are not observed \( \Rightarrow \) need to infer them.
Main Empirical Findings

- Elasticity of labor supply is small.
- Elasticity of labor supply by women is higher than that of men. Estimates for married women lie between 0 and 1, whereas estimates for married men lie between 0 and 0.2.
- Most variations in Aggregate labor supply are due to adjustments at the extensive margin, i.e. individuals entering and/or exiting the labor market.
Optimization in the short run

By short run, we mean that the only flexible factor is labor. To simplify matters, we assume that the inverse demand function is iso-elastic so that $P(Y) = Y^\eta$ where

$$0 > 1/\eta = Y'(P)P/Y(P) > -1$$

is the elasticity of output demand.

### Optimization Problem

$$\text{MAX}_{\{L\}} \Pi(L) = P(Y)Y - wL$$

s.t. $Y = F(L)$

### F.O.C.

$$F'(L) = \left( \frac{1}{1+\eta} \right) \frac{w}{P}$$
Differentiating the F.O.C. with respect to $w$ yields

$$\frac{\partial L}{\partial w} = \left(\frac{1}{1 + \eta}\right) \frac{1}{F'^2 P' + PF''} < 0$$

Hence the short run labor demand is a negative function of labor cost. In terms of elasticity

$$\eta_{L,w} \equiv \frac{\partial L}{\partial w} \frac{w}{L} = \frac{1}{LP' F' P} + \frac{F'' L}{F'} = \frac{1}{\eta L F' F' + F'' L}$$

Hence, the higher the market power (i.e. the smaller $\eta$), the less elastic the short run labor demand.
Technology of Production

We now characterize optimal labor demand in the long-run. We restrict our attention to the following Technology:

**Production Technology**

- 2 Factors of Production: Labor, $L$, and Capital, $K$.
- Production Function $F(K, L) : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is strictly concave, so that $F_K(\cdot) > 0$, $F_L(\cdot) > 0$, $F_{KK}(\cdot) < 0$ and $F_{LL}(\cdot) < 0$.
- $F(K, L)$ is homogenous of degree $\theta$, so that $F(\lambda K, \lambda L) = \lambda^\theta F(K, L)$ for all $\lambda > 0$. 
Conditional Demand

We first consider the problem of a firm that wants to minimize the cost of producing a given quantity of output $Y$. Let $r$ denote the interest factor, i.e. $1+$interest rate.

**Optimization Problem**

\[
\text{MIN}_{\{L, K\}} \; wL + rK
\]

s.t. $F(K, L) \geq Y$

**F.O.C.**

\[
\frac{\partial F(K, L)}{\partial L} = \left( \frac{w}{r} \right) \frac{\partial F(K, L)}{\partial K}; \text{ where } F(K, L) = Y.
\]
Cost Function

Replacing these optimal conditional demands allows us to define the minimum cost as a function of input prices and output produced.

**Properties of Cost Function** $C(w, r, Y)$

**P1** $C(w, r, Y)$ is increasing in $w$ and $r$ and homogenous of degree 1 in $(w, r)$.

**P2** $C(w, r, Y)$ is concave in $w$ and $r$.

**P3** It satisfies Shephard’s Lemma: $\bar{L} = C_w(w, r, Y)$ and $\bar{K} = C_r(w, r, Y)$.

**P4** $C(w, r, Y)$ is homogenous of degree $1/\theta$ in $Y$, so that $C(w, r, Y) = C(w, r, 1)Y^{(1/\theta)}$.

See pp. 234-238, in CZ, Chapter 4, for Proof.
Properties of conditional demand

Deriving conditional demand for Labor in Shephard’s Lemma (P3) w.r.t \( w \) shows that it is decreasing in \( w \)

\[
\frac{\partial L}{\partial w} = C_{ww} \leq 0
\]

Deriving conditional demand for Labor in Shephard’s Lemma (P3) w.r.t \( r \) shows that

\[
\frac{\partial L}{\partial r} = \frac{\partial K}{\partial w} = C_{wr}
\]

Thus the effect on conditional labor demand of a rise of one euro in the price of capital is similar to the effect on conditional capital demand of a rise of one euro in the price of labor.
We now examine the full problem where the entrepreneur determines the optimal level of output.

**Optimization Problem**

\[
\text{MAX}_{\{Y\}} \Pi(w, r, Y) \equiv P(Y)Y - C(w, r, Y)
\]

**F.O.C.**

\[
P(Y) = \frac{1}{1+\eta} C_Y(w, r, Y)
\]

S.O.C. is satisfied iff \[ \frac{1}{1+\eta} > \theta \]

The firm sets his price by applying a mark-up \[ 1/(1 + \eta) \] to its marginal cost \( C_Y(\cdot) \).
Long-run elasticity of labor demand

By Shephard’s Lemma (P3), we have $L^* = C_w(w, r, Y^*)$ where a star superscript indicates long-run optima. Differentiating this relation w.r.t. $w$ yields

$$\frac{\partial L^*}{\partial w} = C_{ww} + C_{wY} \frac{\partial Y^*}{\partial w}$$

- **The substitution effect:** $C_{ww}$ is negative and equal to the derivative of the conditional labor demand.
- **The scale effect:** From the S.O.C. it is easy to show that it is negative because $C_{wY}$ and $\frac{\partial Y^*}{\partial w}$ must be of opposite sign.
Estimation Results

Estimation procedures either assume a specific Production Function (e.g. Cobb-Douglas, CES...) or even more directly postulate a particular Cost Function. For obvious technical reasons, most estimates available are for the conditional elasticity of labor demand.

Main findings

- Estimates of the elasticity of conditional aggregate labor demand w.r.t. \( w \) are negative, between \([-0.75, -0.15]\) and centered on \(-0.3\).
- The scale effect increases (as expected) the absolute value of the elasticity to values around 1.
- Unskilled labor is more easily substitutable for capital than skilled labor.
Minimum wage and unemployment

The usual way to introduce unemployment in the competitive model is to impose a price floor, i.e. a minimum wage. Empirical evidence, however, does not substantiate the importance of this explanation:

- Over the last 30 years, there has been little change in minimum wage relative to average earnings. Thus it is hard to argue that, everything else equal, it has played an important role in the rise of European unemployment during this period.

- Empirical estimates document a positive and significant correlation between the minimum wage and unemployment among workers below 24 years of age. But for older workers the effect turns out to be non-significant.

- A ”natural experiment” study by Card and Kruger (1994) on the fast-food industry in the US even exhibits a positive effect on employment of the minimum wage.
No obvious correlation between level of minimum wage and relative proportion of job seekers among low-skilled workers. If anything, the relative proportion is higher in Anglo-saxon countries.
Business Cycle Facts

The following properties of Business Cycles fluctuations are well documented:

- Hours worked and Employment are Pro-cyclical.
- Unemployment is Counter-cyclical.
- Real Wages are acyclical or mildly Pro-cyclical.
Trend GDP Hours. Source: Prescott.
Correlation Hours GDP US. Source: Prescott.

Deviations from Trend of U.S. GDP and Hours.
Correlation Wage GDP US.

Pro-Cyclical in the 1970’s.
Almost acyclical from the 1980’s.
Predictions of competitive model

Reconciling these empirical regularities with theory is one of the main challenge facing Macroeconomists:

- The cyclical component of real wage excludes Supply shock.
- But Demand shocks with a nearly vertical labor supply imply that we should observe strong Procyclicality in real wages and near Acyclicality in labor input. That is the exact opposite of what is documented in the data.
To address this issue, we basically need to derive an aggregate labor supply with a higher elasticity than the individual labor supply. At least two approaches have been proposed:

1. Effect on Aggregation of the prevalence of adjustments at the extensive margin.

2. Imperfection in labor markets and deviation from the competitive setting.

In this course, we will focus on the second type of explanation.
References

**Labor Supply**
CZ, Chapter 1.

**Labor Demand**
CZ, Chapter 4.

**Minimum Wage**

**Business Cycles**
Part III

Search-Matching Model
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9 Job Search
- The basic model
- Worker Turnover
- Comparative statics
- Diamond’s critique

10 The Search-matching model
- The Matching Process
- Wage Setting
- The Equilibrium
- Efficiency

11 Appendix: Asset Equations
- Poisson processes
- Asset Equation

12 References
The basic model

The model describes the behavior of a worker searching for a job in a world with imperfect information.

The basic model is based on the following hypotheses:

1. Workers are risk neutral.
2. The environment is stationary.
3. The intensity of search cannot be adjusted by the worker.
4. Workers cannot search for alternative job while working.
For the moment, we assume that:

- Time is discrete.
- Worker discount the future at rate $1/(1+r)$.
- When a firm meets a job seeker, it offers him a constant wage over the duration of the job.
- Jobs last forever.
- At the end of each period:
  1. **Employed workers** receive their wages.
  2. **Unemployed workers** receive benefits $z$ and sample a wage offer from the exogenous distribution $H(w)$. 
A job seeker can reject the wage offer and continue to search. Accordingly, the **Bellman equations** for employed and unemployed workers read

**Discounted Utilities**

Let $E$ denote the discounted utility of an employed worker

$$ E(w) = \frac{1}{1 + r} \left( w + E(w) \right) $$

and $U$ the discounted utility of a job seeker

$$ U = \frac{1}{1 + r} \left( z + \int_0^\infty \max\{U, E(w)\} \, dH(w) \right) \quad (3) $$
Job seekers have to solve an **optimal stopping problem**. Since $E(w) = w/r$ is strictly increasing, there exists a unique reservation wage $w_r$ such that job seekers turn down every job offers if $w < w_r$ and accept every job offers if $w \geq w_r$.

At the reservation wage, workers are indifferent between working and being unemployed so that: $E(w_r) = U$. Replacing $U = w_r/r$ and $E(w) = w/r$ into (3) yields

$$w_r = \frac{1}{1+r} \left( rz + \int_0^\infty \max\{w, w_r\} dH(w) \right)$$

$$= z + \frac{1}{r} \left( \int_{w_r}^\infty (w - w_r) dH(w) \right)$$

$$= z + \frac{1}{r} \left( \int_{w_r}^\infty [1 - H(w)] dw \right)$$
Continuous time

As shown in Appendix (1), similar solutions can be derived considering the worker’s asset equations in continuous time.

Discounted Utilities in Continuous time

\[
\begin{align*}
    rE(w) &= w \\
    rU &= z + \lambda \left( \int_{0}^{\infty} \max\{0, E(w) - U\} dH(w) \right)
\end{align*}
\]

where job seekers contact firms at rate \( \lambda \).

In the remainder of the lecture notes, we favor this interpretation.
Job destruction

The basic model cannot account for the fact that job separations occur quite frequently (e.g. average job duration in US is close to 3 years). Thus we extend the model by considering that jobs are destroyed at the rate $\delta$. Then the asset equations are

$$rE(w) = w + \delta(U - E(w))$$

$$rU = z + \lambda \left( \int_{0}^{\infty} \max\{0, E(w) - U\} dH(w) \right)$$

The asset equation for $E$ adds the flow income, $w$, to the capital loss, $U-E$, times the rate of job separation, $\delta$. Notice that the gain derived from employment is

$$E(w) - U = \frac{w - rU}{r + \delta}$$
Replacing the worker surplus in (1) and the definition of the reservation wage \( rU = W_r \) yields

\[
\begin{align*}
    w_r &= z + \frac{\lambda}{r+\delta} \int_{w_r}^{\infty} (w - w_r) dH(w) \\
    &= z + \frac{\lambda}{r+\delta} \int_{w_r}^{\infty} [1 - H(w)] dw
\end{align*}
\]

It is easy to verify that the reservation wage
- is unique.
- maximizes the intertemporal utility of a job-seeker.
Interpretation of optimal reservation wage

The optimal reservation wage is higher than the flow income of job seekers $(w_r > z)$ because it also includes the option to search

$$w_r = \underbrace{z}_{\text{Flow income}} + \frac{\lambda}{r + \delta} \int_{w_r}^{\infty} (w - w_r) dH(w)$$

Intuitively, by accepting a given job offer, the job seeker renounces to the possibility of finding a better job. Thus, he ”kills” his option to search. The job seeker takes into account this cost and so ask for a premium over his current flow income. Notice that this effect is less important if workers can also search while being on-the-job.
Comparative statics on the optimal reservation wage

$w_r \uparrow$ when

- $z \uparrow$ because the opportunity cost of employment increases.
- $\lambda \uparrow$ because the option to search increases.

$w_r \downarrow$ when

- $r \uparrow$ because workers value less the future which decreases the option to search.
- $\delta \uparrow$ because the average job spell decreases which in turn decreases the option to search.
Comparative statics on unemployment duration

The average unemployment duration is

\[ T_u = \frac{1}{\lambda[1 - H(w_r)]} \]

Accordingly, \( T_u \) is an increasing function of \( z \). So:

- higher unemployment benefits raise the average unemployment duration.
- \( \lambda \) has an ambiguous effect because it increases the rate of arrival of job offers but also augments the number of rejected job offers (\( w_r \uparrow \)). In practice, the former effect dominates so that \( \partial T_u / \partial \lambda < 0 \).
The rate of unemployment evolves as follows

\[
\dot{u} = \delta (1 - u) - \lambda [1 - H(w_r)] u
\]

Flows into Unemployment \hspace{1cm} Flows out of Unemployment

The equilibrium rate of unemployment is given by

\[
\frac{\delta}{\delta + \lambda [1 - H(w_r)]}
\]

The comparative statics for unemployment are therefore similar but with opposite signs to the ones for unemployment duration.
Diamond’s critique

Diamond (1971) raised the following critique: since workers accept all wage offers above the reservation wage, firms have no incentive to offer a wage $w > w_r$. Hence the wage distribution should be concentrated at $w_r$. But then the option to search has no value, and so the equilibrium wage is equal to $z$. This in turn raises the question of why workers are searching in the first place?

Diamond’s critique highlights that the partial model cannot be extended to an equilibrium setting without further assumptions.
On way to answer this critique is to vary the model’s interpretation by thinking of job seekers

- as fishermen looking for lakes. Then $H$ would be the distribution of fish across lakes and thus can be taken as exogenous.

- as searching across islands. On each island there are many firms with a constant returns to scale technology using only labor. The productivity of a randomly selected island is distributed according to $F$. The labor market on each island is competitive, so that workers are paid their productivity. Again the wage distribution is exogenous.
From the partial model to the equilibrium model

There are other possible answers (with arguably more economic content) to Diamond’s critique:

- **Worker heterogeneity**: under certain conditions, the wage distribution will correspond to the reservation wages of the different categories. This explanation, however, does not explain wage dispersion among similar workers.

- **On the job search**: then firms have an incentive to increase their wage offers above the reservation wage in order to retain their workers and also to attract workers which are already employed.

- **Match uncertainty**: if the productivity of the job is due to the quality of the firm-worker match and if workers have some bargaining power, then the wage distribution will reflect the productivity distribution of jobs.
Motivation

Although the basic search model can be recast as an equilibrium model, it has the main drawback of not explaining the determinants of job creation, since the contact rate $\lambda$ is taken as given.

To devise a full equilibrium framework, we need to take into account the behavior of firms. We have to specify the

- **Matching process**: How workers and firms meet? This will be done through the introduction of an aggregate matching function.

- **Wage setting**: How wages are determined? This will be done by considering that wages are set through Nash-bargaining.
The search intensity of firms can be quantified using the number of vacancies posted in the economy. The coexistence of unemployed persons and vacant jobs measures frictional unemployment. The \textbf{Beveridge Curve} illustrates the negative relation between the unemployment rate $u$ and the vacancy rate $v$ (the ratio of the number of vacant jobs to the size of the labor force).
Beveridge Curve

Austria: 60-99

France: 60-99
The relation between vacancies and unemployment can be captured introducing an aggregate matching function. The matching function $m(v, u)$ gives the number of jobs created as a function of the number of vacancies and unemployed persons. Obviously, $m(\cdot)$ is nonnegative and increasing in both arguments. It is also commonly assumed to be concave and such that $m(0, u) = m(v, 0) = 0$ for all $(u, v)$. In order to rule out size effects, it is standard to assume that the matching function has **constant returns to scale**, so that

$$
\frac{m(v, u)}{v} = m(1, u/v) = q(\theta),
$$

where the tightness parameter $\theta$ is the vacancy-unemployment ratio ($v/u$) and $q'(\theta) < 0$. 
Foundation of matching function

One can think of the matching function as a production function which, instead of mapping labor and capital into output, maps job seekers and vacancies into matches.

The easiest justification for this reduced-form specification, is to refer to *coordination frictions*. Assume that:

- $u$ unemployed workers know the location of the $v$ vacancies and send each period one application per period.
- If a vacancy receives more than one application it selects an applicant at random and forms a match.
- The remaining applicants return to the unemployment pool and apply again in the next period.
Coordination frictions

Each vacancy receives the application of a worker with probability $1/v$. Since there are $u$ applicants, the vacancy receives no applications with probability $(1 - 1/v)^u$. Thus the number of matches per period is

$$m(v, u) = v[1 - (1 - 1/v)^u]$$

As $u$ and $v$ become large, $(1 - 1/v)^u \rightarrow e^{-(u/v)}$ so that

$$m(v, u) = v[1 - e^{-(u/v)}] \text{ or } q(\theta) = 1 - e^{(-1/\theta)}$$

This is a CRS matching function with standard properties.
Let \( p \) denote the productivity of the firm-worker match. Obviously the flow profit of the firm \( \pi \equiv p - w \). Treating the productivity as a constant, we can define the value for the firm of the job as \( J(w) \). Let \( V \) denote the value of an open vacancy.

**generalized Nash-bargaining solution**

\[
w \in \arg \max \{(E(w) - U)^\beta (J(w) - V)^{1-\beta}\}
\]

Taking the natural logarithm and maximizing the Nash-bargaining problem w.r.t. \( w \) yields

\[
\beta [J(w) - V]E'(w) = -(1 - \beta)[E(w) - U]J'(w)
\]
Foundations of Nash-bargaining

John Nash used an axiomatic approach to prove that, when \( \beta = 1/2 \), the solution described before is the only one satisfying:

1. Invariance to affine transformations.
2. Pareto optimality.
3. Independence from Irrelevant Alternatives.
4. Symmetry.

Relaxing the symmetry axiom, yields the generalized Nash-bargaining solution for any \( \beta \in (0, 1) \).
Non-cooperative Foundations of Nash-bargaining

The axiomatic approach proposed by John Nash is a cooperative solution of the bargaining game. Thus it does not provide insights on how the bargaining process actually unfolds.

A game-theoretical description of the bargaining process has been proposed by Ariel Rubinstein. Each agents makes successive offers and counter-offers. The surplus is shared only after an agreement has been reached. Since agents discount the future they have an incentive to agree as soon as possible. Thus the bargaining power is given by the ratio of their discount factors. The subgame perfect equilibrium converges to the Nash solution when the time interval between offers and counteroffers becomes small.
Analytical solution of Nash-bargaining

The asset value of the job for the firm and worker satisfy

\[ rJ(w) = p - w + \delta(V - J(w)) \]

\[ rE(w) = w + \delta(U - E(w)) \]

We assume free entry so that the expected value of posting a vacancy is zero, i.e. \( V = 0 \). In next sub-section we will show that free-entry is satisfied in equilibrium, but for the moment we take it as given. Using equation (5) yields

\[ w = \beta p + (1 - \beta)rU \] (6)
Set-up

We now combine matching and bargaining to endogenize vacancy posting decisions. We assume that:

- Firms and workers are both infinitely lived and risk neutral
- Workers cannot search on-the-job.
- Each firm employs only one worker.
- All jobs have the same output $p$ which remains constant during the whole lifetime of the job.
Let $c$ denote the flow cost of vacancy posting. Then

$$rJ(w) = p - w + \delta(V - J(w))$$

$$rV = -c + q(\theta)(J(w) - V)$$

since $q(\theta)$ is the rate at which a vacancy is filled by a worker. **Free entry** is satisfied iff $V = 0$. Thus, in equilibrium, it must be the case that

$$J(w) = \frac{p - w}{r + \delta} = \frac{c}{q(\theta)}$$

(7)
Value of unemployment

Let \( z \) denote the flow value of non-market activity (unemployment benefit, household production...). Then

\[
    rE(w) = w + \delta(U - E(w))
\]

\[
    rU = z + \theta q(\theta)(E(w) - U)
\]

since \( \theta q(\theta) \) is the rate at which a job seeker finds a job (why?). Combining the two equations, we find that the surplus from being employed is

\[
    E(w) - U = \frac{w - z}{r + \delta + \theta q(\theta)}
\]
To derive the wage curve, we replace (7) into the asset equation for $U$

$$rU = z + \theta q(\theta)(E(w) - U)$$

$$= z + \theta q(\theta) \left( \frac{\beta}{1-\beta} \right) J(w) = z + \left( \frac{\beta}{1-\beta} \right) c\theta$$

Reinserting this expression into (6) yields

$$w = \beta p + (1 - \beta)z + \beta c\theta$$
The equilibrium

The values of the three endogenous variables $u$, $w$ and $\theta$ follow from the system of equations:

**Equilibrium conditions**

\[(BC) : u = \frac{\delta}{\delta + \theta q(\theta)}\]

\[(JC) : \frac{p - w}{r + \delta} = \frac{c}{q(\theta)}\]

\[(W) : w = \beta p + (1 - \beta)z + \beta c\theta\]
The equilibrium
We now discuss under which conditions the allocation is efficient. We define the social optimum as the allocation which maximizes the sum of agents’ utility; i.e. output net of the disutility of work and search costs. We simplify the problem by setting the discount rate $r$ to zero. Then the problem of the social planner is static.

**Planner’s problem**

$$\max_{\{\theta, u\}} \Omega \equiv p(1-u) + zu - cv = p + u(z - p - c\theta)$$

subject to

$$u = \frac{\delta}{\delta + \theta q(\theta)}$$
The Hosios condition

Reinserting the constraint into the objective function reduces the dimension of the optimization problem

$$\text{Max}_{\{\theta\}} p + \frac{\delta}{\delta + \theta q(\theta)} (z - p - c\theta)$$

The F.O.C. reads

$$\frac{(1 - \eta(\theta))(p - z)}{\delta + \theta q(\theta)\eta(\theta)} = \frac{c}{q(\theta)}$$

where $\eta(\theta) = -q'(\theta)(\theta/q(\theta))$ is the elasticity of the matching function.

Hosios condition

The decentralized and optimal allocation coincide when $\beta = \eta(\theta)$. 
Policy implications

Search frictions implies that there exists an \textit{optimal} positive level of unemployment.

In general, there are no reasons for the Hosios condition to be satisfied. This implies that there is room for optimal policy intervention when the workers’ bargaining power is too high or too low.

For example, when $\beta < \eta(\theta)$, the introduction of a biding minimum wage can restore efficiency.
The first step for deriving asset equations consists in defining the stochastic process that changes the state of the agents. For example, consider an unemployed worker. Suppose that on average she receives 3 job offers per year. Given this expected value, what is the probability that she will receive 5 offers in one year? Can we also compute the probability that she will receive a job offer in one month?

It is shown in this section how these two statistics can be easily computed using the Poisson distribution. In order to rigorously define Poisson processes, we need to introduce first Binomial distributions.
Binomial distribution

The binomial distribution is a discrete probability distribution which describes the number of successes in a sequence of $n$ independent yes/no experiments, each of which yielding success with probability $p$.

If the random variable $X$ follows the binomial distribution with parameters $n$ and $p$, we write $X \sim B(n, p)$. The probability of getting exactly $k$ successes is given by

$$\Pr\{X = k\} = \binom{n}{k} p^k (1 - p)^{n-k}$$

for $k = 1, 2, \ldots, n$ and where

$$\binom{n}{k} = \frac{n!}{k!(n - k)!}$$

is the binomial coefficient.
Pascal’s triangle

The Binomial distribution can be described visually using Pascal’s triangle

```
    1
   1 1
  1 2 1
 1 3 3 1
1 4 6 4 1
...```

Binomial and Poisson distribution

If $X \sim B(n, p)$, then the expected value of $X$ is

$$E[X] = np$$

and the variance is

$$\text{var}(X) = np(1 - p)$$

Poisson distribution

The Poisson distribution arises when $n$ is much larger than the expected number of successful outcomes. In other terms, Poisson processes are limits of binomial processes where $n \to \infty$ and $p \to 0$ while $np$ remains constant.
Let’s go back to the example considered at the beginning of this section. We know that on average an unemployed worker receives $\lambda = 3$ job offers per year. Given this expected value, how can we model the actual distribution of offer arrivals in a second? One possible model is to break up each year into tiny intervals of size $\delta > 0$ years, so there are a very large number, $n = 1/\delta$, of intervals in one year. Then we declare that in each interval, a job offer arrives with probability $p = \lambda\delta$ (this gives the right expected number of arrivals). So it is as if the worker were drawing a sequence of $n$ independent yes/no experiments, each of which yielding success with probability $\lambda\delta$. Note that this is not quite the same as counting the number of arrivals, since more than one job offer may arrive in a given interval. But if the interval is small enough, this is so unlikely that we can ignore the possibility. Hence the approximation holds in the limit where: $\delta \to 0 \Rightarrow n = 1/\delta \to \infty$. 
Poisson random variable

Under this model, the number $X$ of job offers which actually arrive in a year has a binomial distribution:

$$\Pr \{X = k\} = \binom{1/\delta}{k} (\lambda \delta)^k (1 - \lambda \delta)^{1/\delta - k}$$  \hspace{1cm} (8)

Now, let $\delta$ become infinitesimally small (while holding $k$ fixed) and make use of three approximations:

$$\binom{1/\delta}{k} \approx \frac{(1/\delta)^k}{k!}$$

$$(1 - \lambda \delta)^{1/\delta} \approx e^{-\lambda}$$

$$1 - \delta k \approx 1$$
Plugging the last equations in (8) yields

\[
\Pr \{X = k\} = \binom{1/\delta}{k} (\lambda \delta)^k (1 - \lambda \delta)^{1/\delta - k}
\]

\[
= \binom{1/\delta}{k} (\lambda \delta)^k (1 - \lambda \delta)^{(1 - \delta k)/\delta}
\]

\[
\approx \frac{(1/\delta)^k}{k!} (\lambda \delta)^k (1 - \lambda \delta)^{1/\delta}
\]

\[
= \frac{\lambda^k}{k!} (1 - \lambda \delta)^{1/\delta}
\]

\[
\approx \frac{\lambda^k}{k!} e^{-\lambda}
\]

(9)
Poisson random variable

What happens if we want to answer a question that involves an interval of \( t > 0 \) units of time rather than 1 unit of time? Then we have a new random variable \( X_t \) that is Poisson distributed with mean \( \lambda t \). The probability of seeing exactly \( k \) counts in \( t \) units of time is

\[
\Pr \{ X_t = k \} = \frac{(\lambda t)^k}{k!} e^{-\lambda t}
\]

The probability distribution (9) is known as the Poisson distribution. When system events appear according to a Poisson density, the system is called a Poisson process.
Properties of the Poisson distribution

As a sanity check on our distribution, the probability values (9) had better sum to 1. Using the Taylor expansion for $e^\lambda$, we can verify that they do:

$$\sum_{k \in \mathbb{N}} Pr\{X = k\} = \sum_{k \in \mathbb{N}} e^{-\lambda} \frac{\lambda^k}{k!} = e^{-\lambda} \sum_{k \in \mathbb{N}} \frac{\lambda^k}{k!} = e^{-\lambda} e^\lambda = 1$$

A further sanity check is that the expected number of arrivals in a year is indeed $\lambda$, namely

$$E[X] = \lambda$$

Similarly, the binomial distribution $B(n, p)$ has variance $np(1 - p)$. Since the Poisson distribution is the limit of $B(1/\delta, \lambda\delta)$ as $\delta$ vanishes, it ought to have variance

$$Var[X] = (1/\delta)(\lambda\delta)(1 - \lambda\delta) = \lambda(1 - \lambda\delta) \approx \lambda$$

The final approximation holds since $1 - \lambda\delta \approx 1$ for vanishing $\delta$. 
Pr \{X = k\} for \(k = 1, 2, \ldots, 10\). when \(E[X] = 3\)
Now we consider the waiting time $W$, which is the time one must wait to see the next count. The following two events are equivalent: $\{W > t\} = \{X_t = 0\}$. Both conditions say that no counts arrive in the first $t$ units of time. Hence

$$\Pr \{W > t\} = \Pr \{X_t = 0\} = e^{-\lambda t} \quad (10)$$

Furthermore, since $\Pr \{W > t\} = 1 - \Pr \{W \leq t\}$, the cumulative distribution of $W$ is $F(t) = \Pr \{W \leq t\} = 1 - e^{-\lambda t}$.

Finally, the density function $f(t)$ of $W$ can be found by differentiating $F(t)$ with respect to $t$. We obtain

$$f(t) = F'(t) = \lambda e^{-\lambda t}, \text{ for } t > 0.$$  

This is the density function of the so-called exponential distribution with rate $\lambda$. It has mean $E[W] = 1/\lambda$. Intuitively, it means that the worker waits $1/3 = 4$ months before receiving an offer.
Waiting time and exponential distribution

Notice that a Taylor approximation of \( \Pr\{W \leq t\} \) around \( t = 0 \) yields \( \Pr\{W \leq t\} \to \lambda t \). This is why people often refer to \( \lambda \) as the “instantaneous rate of arrival”.

An important property of the exponential distribution is that it is ”memoryless.” This means that, for \( 0 < s < t \),

\[
\Pr\{W > t \mid W > s\} = \Pr\{W > t - s\}.
\]

Given that there have been no counts up to time \( s \), the conditional probability that there are no counts up to the later time \( t \) is just the probability of seeing no counts in a period of length \( t - s \).

For example, if a machine breaks down according to a Poisson process, it doesn’t matter if it is 10 years old or just new. Surprisingly, the exponential law has remarkable explanatory power for the breakdown probability of many devices, one example being light bulbs.
Asset Equation

We want to derive the expected income of an unemployed worker. We begin by setting up the Bellman equation as though we were solving a discrete time problem. Let $\varepsilon$ denote the length of a “time period”.

While unemployed, the worker gets unemployment benefits equal to $b$ at every instant. He consumes all his income. This payoff must be discounted back to the beginning of the “period,” which is now, and that is the role played by the “instantaneous” discount factor

$$\frac{1}{1 + r\varepsilon}$$

Therefore

$$\frac{u(b)\varepsilon}{1 + r\varepsilon}$$

is the discounted value of the current period return over the short interval $\varepsilon$. 
Now, we assume that the worker randomly meet firms and then changes state from unemployed to employed. This event is modeled using a Poisson process: so in any small interval of time $\varepsilon$, the worker meets at most one potential employer. From (10), we know that the likelihood of meeting an employer during the time interval $\varepsilon$ is equal to $e^{-\lambda \varepsilon}$.

Let $U$ and $E$ denote the expected lifetime utility of an unemployed and employed worker

$$U(\varepsilon) = \left(\frac{1}{1 + r\varepsilon}\right) \left(u(b)\varepsilon + \left(1 - e^{-\lambda \varepsilon}\right) E(\varepsilon) + e^{-\lambda \varepsilon} U(\varepsilon)\right)$$

$$\Rightarrow r\varepsilon U(\varepsilon) = u(b)\varepsilon + \left(1 - e^{-\lambda \varepsilon}\right) (E(\varepsilon) - U(\varepsilon))$$

$$\Rightarrow rU(\varepsilon) = u(b) + \left(\frac{1 - e^{-\lambda \varepsilon}}{\varepsilon}\right) (E(\varepsilon) - U(\varepsilon)) \quad (11)$$
Now consider the limit of (11) when $\varepsilon \to 0$. The numerator and denominator of $\left(\frac{1-e^{-\lambda \varepsilon}}{\varepsilon}\right)$ tend to zero. Hence we can use l’Hospital’s rule to obtain the limit

$$\lim_{\varepsilon \to 0} \frac{1-e^{-\lambda \varepsilon}}{\varepsilon} = \lim_{\varepsilon \to 0} \left(\frac{\partial (1-e^{-\lambda \varepsilon})}{\partial \varepsilon}\right) = \lim_{\varepsilon \to 0} \frac{\lambda e^{-\lambda \varepsilon}}{\varepsilon} = \lambda$$  \hspace{1cm} (12)

Substituting (12) into (11) yields

$$rU \text{ \underline{flow return}} = u(b) \text{ \underline{flow income}} + \lambda (E - U) \text{ \underline{capital gain}}$$  \hspace{1cm} (13)

where $U \equiv \lim_{\varepsilon \to 0} U(\varepsilon)$ and $E \equiv \lim_{\varepsilon \to 0} E(\varepsilon)$. 
There is a clear analogy with the pricing of financial assets (which yield periodic dividends and whose value may change over time), if we interpret the left-hand side of (13) as the flow return (opportunity cost) that a risk neutral investor demands if she invests an amount in a risk free asset with return $r$. The right-hand side of the equation contains the two components of the flow return on the alternative activity “unemployment”: the expected dividend derived from consumption, and the expected change in the asset value of the activity or capital gain due to the switch from unemployment to employment. This interpretation justifies the term “asset equations” for expressions like (13).
CZ, Chapter 3 and 9.

P, Chapter 1.

Part IV

Efficiency Wage
Outline

13 No-shirking condition

14 Employers

15 Equilibrium

16 References
Unemployment as a discipline device

The efficiency wage model of Shapiro and Stiglitz (1984) shows that if employers cannot perfectly and costlessly monitor the efforts of their employees, the economy will exhibit involuntary unemployment in equilibrium.

The intuition is straightforward. The only risk induced by shirking is to be fired:

- If there is no unemployment, shirkers immediately find a new job and thus pay no penalty. Moreover, when monitoring is imperfect, shirkers are also employed for some periods. Hence shirkers have the same lifetime income than non-shirkers but a higher utility since they provide no effort.

- If there is unemployment, workers have an incentive not to shirk. For if they are fired, they will incur some losses by remaining unemployed for some time.
The worker’s utility $U(w, e)$ depends positively on the wage $w$ and negatively on the effort $e$ so that $U_w(w, e) > 0$ and $U_e(w, e) < 0$. For simplicity, we assume that the utility function is separable and that workers are risk neutral so that: $U(w, e) = w - e$.

Again, for simplicity, we consider that efforts can take one of only two values $\{0, e\}$ where $e > 0$. 
We assume that:

- Workers discount the future at rate $r$.
- Jobs are destroyed at the exogenous rate $\delta$.
- Shirkers are detected at rate $d$.

Notice that when $d \to \infty$, detection is immediate. In other words, monitoring is perfect and the equilibrium converges to the one in a standard economy without moral hazard problem.
Let $E^S$ denote the asset value of a shirker and $E^{NS}$ that of a non-shirker. Decomposing the asset values yields

$$rE^S = w + (\delta + d)(U - E^S)$$

$$rE^{NS} = w - e + \delta(U - E^{NS})$$

No-Shirking Condition

$$E^{NS} \geq E^S \Rightarrow w \geq rU + \left(\frac{r + \delta + d}{d}\right)e \equiv \tilde{w} \quad (14)$$
To interpret the NSC, notice that it is equivalent to \( d(E^S - U) \geq e \). Thus when \( E^S = U \), so that there is no penalty associated with being unemployed, everyone will shirk.

\[ \tilde{w} \uparrow \text{ when} \]

- \( e \uparrow \), since workers require more compensation for their efforts.
- \( U \uparrow \), since the cost of being fired is lower.
- \( d \downarrow \), since the likelihood of being detected shirking decreases.
- \( \delta, r \uparrow \), since the expected returns from the job are more heavily discounted.
There are $M$ identical firms, each with a production function $Q_i = f(L_i)$ where $L_i$ is efficient labor.

Without loss of generality, we assume that workers that do not shirk provide one unit of efficient labor and 0 unit if they shirk.

Naturally, firms offer the lowest wage consistent with the NSC condition (14), so that $f'(L_i) = \tilde{w}$. This yields an aggregate labor demand $F'(L) = \tilde{w}$.

Finally we assume that $F'(N) = e$ to ensure that full employment is efficient.
Equilibrium wage

The asset value for $U$ is

$$rU = z + \lambda(E - U)$$

where $\lambda$ is the job finding rate and $E = E^{NS}$ since workers do not shirk in equilibrium.

Combining this equation with the asset value for $E^{NS}$, we obtain

$$w = e + z + (r + \lambda + \delta)(E - U)$$

As explained before, the NSC condition is equivalent to $d(E^{S} - U) \geq e$. In equilibrium, $E^{S} = E^{NS} = E$, so we can replace this equality into the previous equation to obtain

$$w = e + z + \frac{r + \lambda + \delta}{d} e$$

(15)
As explained in Part III, the equilibrium rate of unemployment sets the flows in and out of the unemployment pool equal. Hence, 
\[ u = \frac{\lambda}{\delta + \lambda}. \]
Replacing the equilibrium wage into the No-shirking wage (15) yields
\[ \tilde{\omega} = e + z + \frac{e}{d} \left( \frac{\delta}{u} + r \right) \]
Hence when \( u \to 0 \), \( \tilde{\omega} \) diverges to infinity. This shows that it becomes virtually impossible to motivate workers as unemployment disappears.
Equilibrium employment

\[ w^* + \frac{e}{d}(\delta + \tau) \]

NO SHIRKING REGION

\[ F'(L) \quad \text{NSC} \]
Solving the social planner problem shows that the optimum occurs at the point where the NSC curve intersects the Average product of labor locus.

When returns to labor are decreasing, this implies that wages are too low and unemployment too high in the decentralized economy. This is because the private cost $w$ is superior to the social cost $e$.

Hence, there is room for beneficial policy intervention. Namely, wages should be subsidized by taxing away all pure profits.