Partial deconfinement phases in gauged multi-matrix quantum mechanics.

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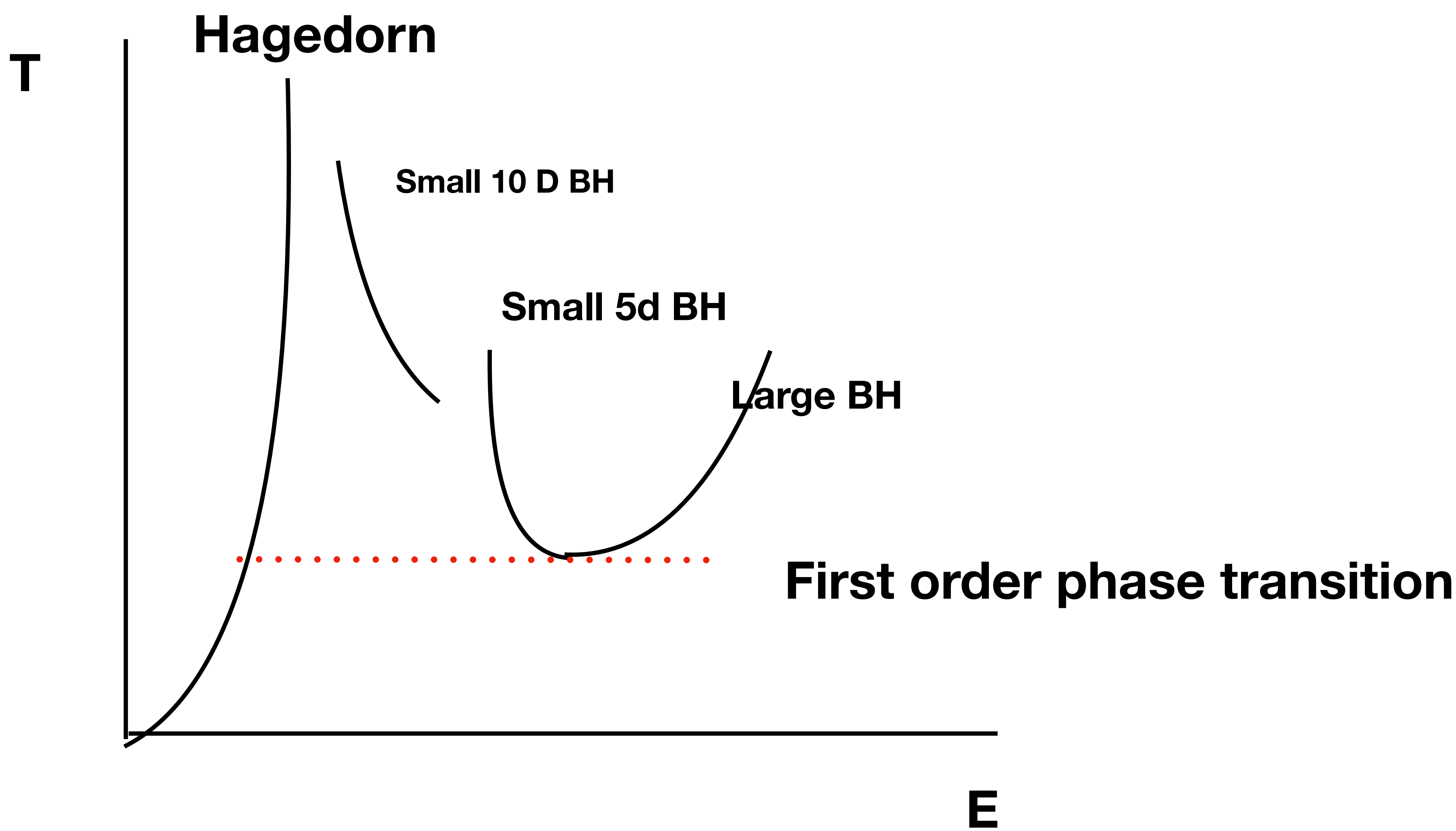
gauge/gravity

- Gauged matrix quantum mechanics are theories of "strings": planar diagrams.

- Most theories of strings are of quantum gravity type.

- Best known example is $AdS_5 \times S^5$ being dual to N=4 SYM.

Phase diagram in AdS



Hawking-Page = confinement/deconfinement (Witten)

For global AdS transition occurs only at infinite N

Want to analyze the phase diagram at fixed energy, below the first order phase transition in the dual CFT.

What is the dual to the small black hole phase?

Is it like a coexistence phase?

How to relate it to the Hagedorn phase transition? (proliferation of strings).

Outline

- A toy model: 2 matrices (long strings and Young tableaux)
- Submatrix deconfinment: what does it mean?
- Small black holes in AdS

Simplest gauge theory

1-matrix model quantum mechanics

$$H = tr(\dot{X}^2) + tr(V(X))$$

Invariant under U(N): take singlet sector.

$$V(X) = X^2$$

Solved by free fermions.

There is no phase transition

Next simplest model

$$H = Tr(\dot{X}^2) + Tr(\dot{Y}^2) + V(X,Y)$$

With X,Y in adjoint of U(N): a 2 matrix model.

$$V(X,Y) = Tr(X^2) + Tr(Y^2)$$

In free theory at large N, there is a confinement/deconfinement first order phase transition

The “order parameter” is the dependence with N of the entropy.

$$S_{conf} \simeq O(1)$$ **(low T)**

$$S_{deconf} \simeq O(N^2)$$ **(high T)**

To get the phase transition we need to study the density of states with the energy: counting states.

Write states in an oscillator basis:

$$(a^\dagger)^i_j = A$$

$$(b^\dagger)^i_j = B$$

All states are produced by matrix valued raising operators.

Gauge invariance requires upper indices be contracted with lower indices

$$tr(ABA\dots)$$

For example: traces and multitraces (strings — copied from AdS/CFT dictionary)

For single traces.

$$\ell = \#\text{Letters}$$

$$\#\ \text{States}_{1-string} \sim 2^{\ell}/\ell$$

The entropy is the log

$$S \simeq \ell \log 2$$

From first law

$$T = \frac{1}{dS/d\ell} = \frac{1}{\log 2}$$

Multi-traces only add subleading corrections to the entropy: same T.

Protocol

- Study at large energy but much less than the number of degrees of freedom of deconfined phase

$$1 << E << N^2$$

How do these excitations fit in the matrix?

Another counting of states

$$(a^\dagger)^{i_1}_{j_1}(a^\dagger)^{i_2}_{j_2}\ldots(a^\dagger)^{i_k}_{j_k}$$

Transforms as tensor of U(N) x U(N) (upper and lower indices)

Decompose into irreps: Young diagrams (symmetrize/amtisymmetrize)

Same diagram on upper and lower indices: bose statistics of a oscillator.

Same works with B: we count all states this way.

Take tensor product on upper (and lower indices) and decompose again.

$$Y_A \otimes Y_B \simeq \oplus Y_{A+B}$$

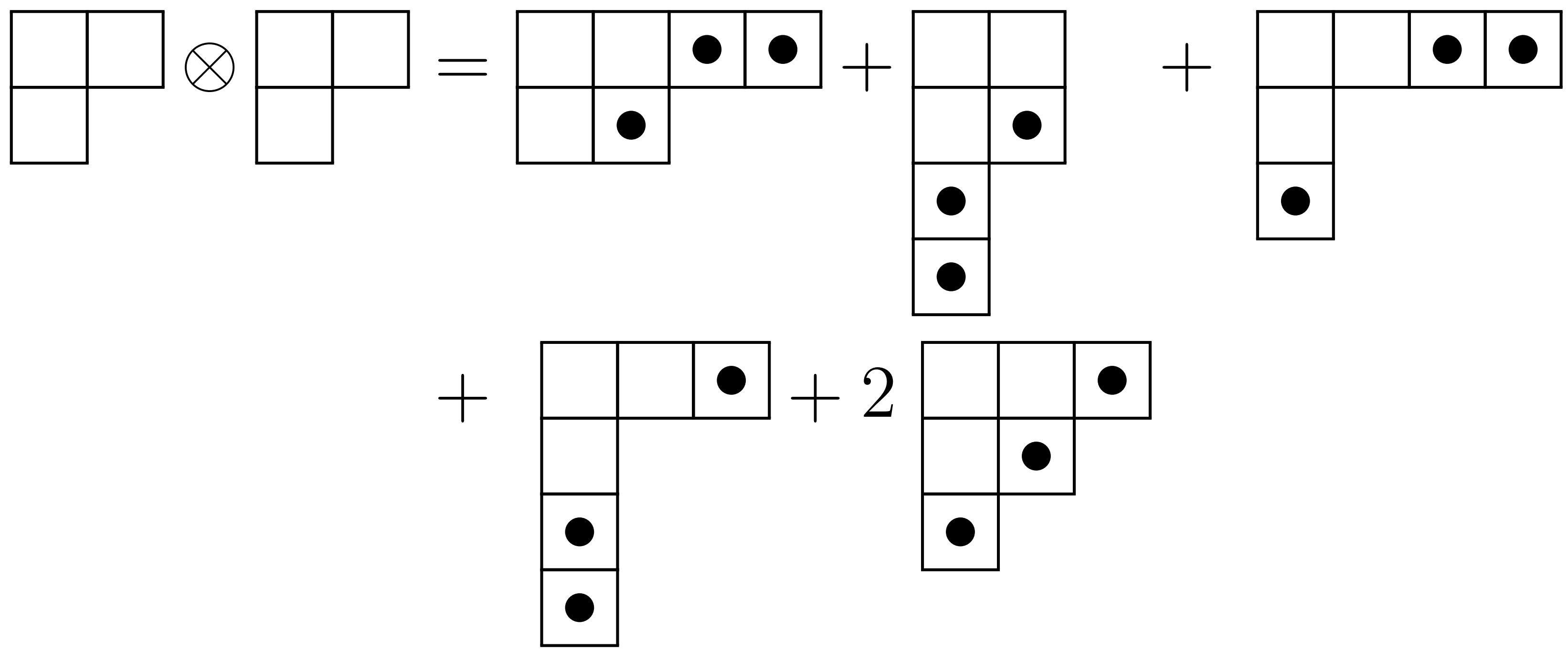
This still counts all states: but there might be multiplicities in products.

For fixed energy E, we need E boxes

$$E = \ell$$

To get a singlet: upper index boxes need to have same shape as lower index boxes.

For example, we multiply the A,B diagrams and decompose



We want to count the number of rows. For a typical tableaux we expect that

$$n_{rows} \simeq n_{cols} \simeq O(\sqrt{\ell})$$

In the matrix model of a single matrix the number of columns or rows can be interpreted as a count of D-branes.

They are called Giant gravitons and dual giants.

Want to interpret these as the rank of the matrix that is excited.

Technical fact: Young diagram describes highest weight state of irreducible.

Unbroken gauge group of highest weight state is

$$U(N - n_{rows})$$

Same entropy

$$S \simeq \ell \log(2) \simeq \alpha n_{rows}^2$$

Interpret the right hand side as a deconfined ensemble for matrices of size

$$n_{rows} \times n_{rows}$$

at fixed temperature (determined by the prefactor)

Submatrix deconfinement

Two conditions:

$S \simeq n^2$ **deconfinement**

$U(N-n)$ is unexcited$=$ confined

Coexistence

First order phase transitions allow for a coexistence phase. This is usually separated in volume (different spatial regions with different phases).

Here, deconfined phase and confined phase "coexist": they are separated in eigenvalue space on the matrix.

Corollary

- Long String ensemble is equivalent to excited D-branes
- Gives example of smooth transition from a string to a black hole (Susskind-Horowitz-Polchinski)

AdS black holes

Dual order parameter for deconfinement is
the topology change.

Small AdS black holes have same topology as large AdS black holes
(Euclidean, or presence of horizons)
They should be deconfined!

Exist in micro canonical ensemble: they can not be
deconfining the whole gauge group (much less entropy for same T)

10 D BH should also deconfine, but they also break
the SO(6) symmetry to SO(5) (localized in a point on the sphere)

Field theory dual?

Conjecture

- Small black holes are deconfined on a submatrix (Asplund+Berenstein) — based on a model that does not get the dynamics correctly

- This assumption leads to a reasonable model of thermodynamics (Hanada+Maltz)— No explanation of the R-symmetry breaking pattern

- Can’t control directly
- However, can “boost” black hole: give it R-charge
- States that preserve one half of SUSY have a lot of R-charge: can be used to control the dynamics.

What a mostly half-BPS state looks like

$$Tr(ZZZZZZZZZY\,ZZZZZZZZX\,ZZZZZZ\ldots)$$

R-charge = number of Z

Energy of these states are controlled by integrability (Minahan, Zarembo, Beisert …)

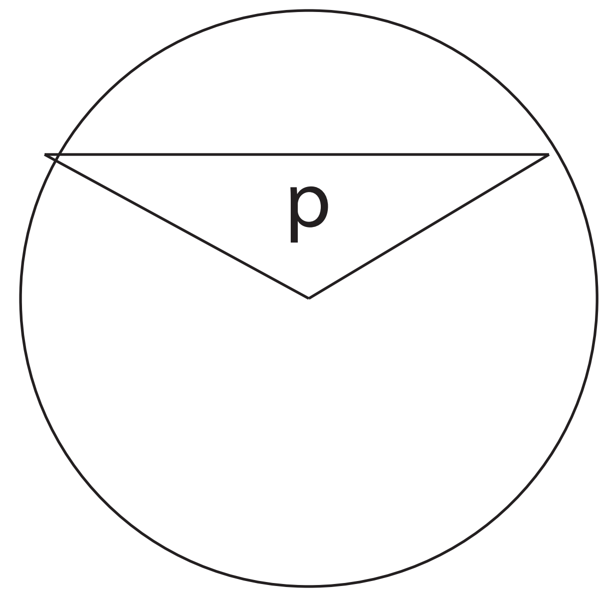
$$E - R = \sum \sqrt{1 + \lambda \sin^2(p/2)}$$

Dispersion relation of magnons: at strong coupling, p needs to be small, at weak coupling no constraints.

Geometry of giant magnon

$$\sqrt{1 + \lambda \sin^2 p/2}$$

Distance squared between ends of arc

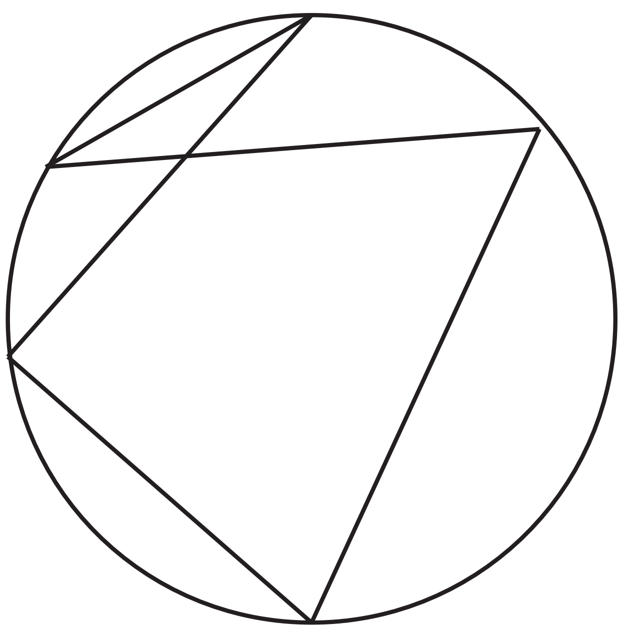


p is an angle on the sphere: D.B. Correa, Vazquez;
Hoffman-Maldacena: giant magnons

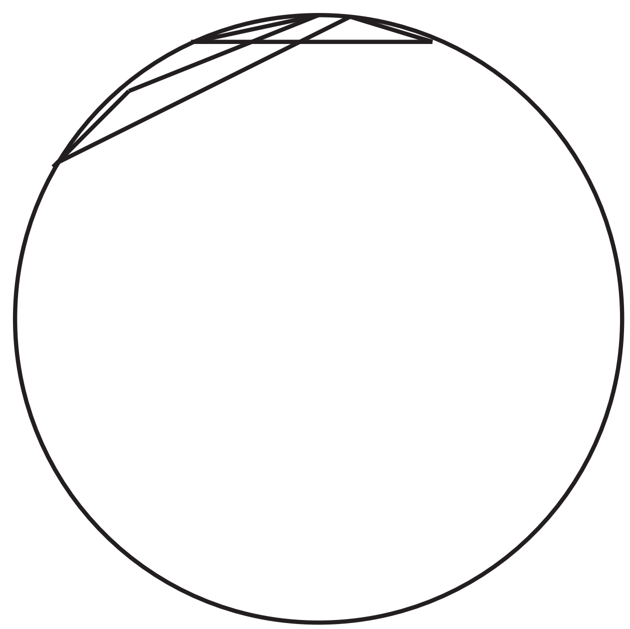
Picture of typical magnon changes as we change coupling

Fix energy: at weak coupling no cost for long magnons, at strong coupling localization on sphere.

Weak coupling



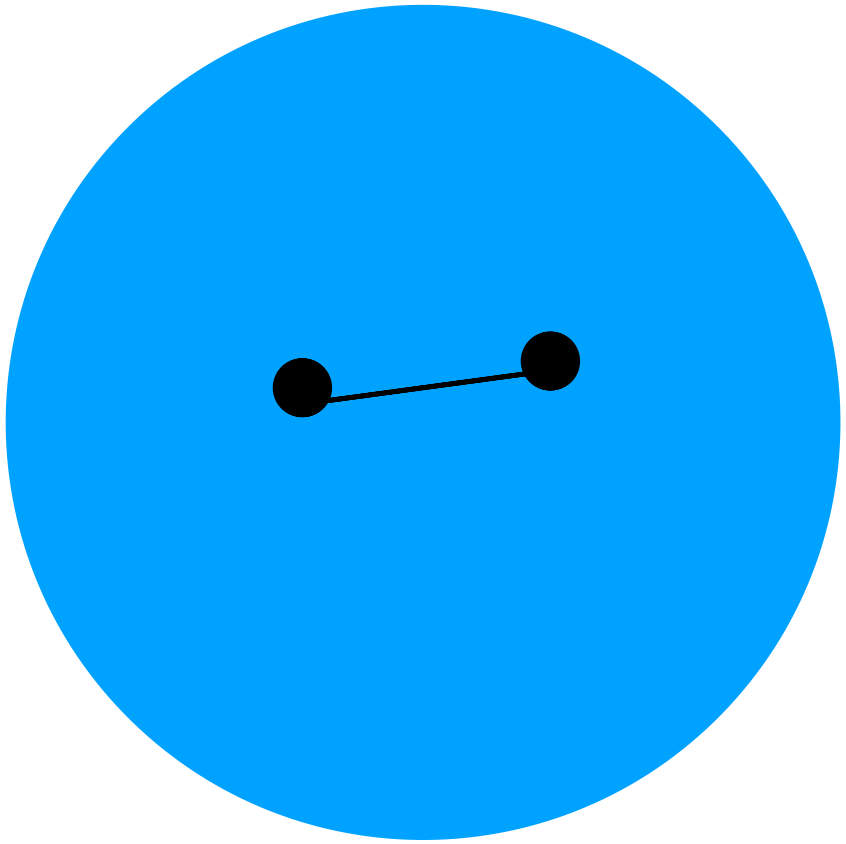
Strong coupling: localized



Similar story with D-branes (giants and dual giants)

Spectrum of strings stretching between giants and dual giants is known
D. B., Correa, Dzienkowski, Vazquez: many papers

Combinatorial techniques:Balasubramanian, D.B, Feng, Huang;
De Mello Koch, Bekker, Smolic x 2, Stephanou, Ramgoolam, …



Also controlled by segments on disk with similar dispersion relation.

More entropy when giants coincide (less energy per string, so more strings).

More entropy when R-charge divided into more giants: the configuration localizes and moves to the edge of LLM disk

$$S_{M,near} \propto M^{1/2}$$

Seems that strings and D-branes are counting same states (short sticks)

R-charge of state breaks explicitly SO(6) -> SO(4) x SO(2)

Here I showed that the SO(2) is spontaneously broken:
can argue that SO(4) unbroken.

Evidence for SO(6)->SO(5) breaking in the absence of R-charge.

Conclusion

- There is evidence for a "coexistence phase" of small black holes: partial deconfinement on a submatrix.
- A computable example where Long Strings = black holes (as excited D-branes) (Susskind-Horowitz-Polchinski) in toy model.
- Can account for SO(6)-> SO(5) symmetry breaking in small black hole duals: it is a property of strong coupling + suggestion of long strings = D-brane black holes.