Space-time structure in the Lorentzian type IIB matrix model

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ESI workshop “Matrix Models for Noncommutative Geometry and String Theory”
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Based on collaboration with Asato Tsuchiya (Shizuoka Univ.)

type IIB matrix model

Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996

a conjectured nonperturbative formulation of superstring theory

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

SO(9,1) symmetry

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices.

Wick rotation $\left(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10}\right)$

$\Rightarrow$ Euclidean matrix model SO(10) symmetry

Crucial properties of the type IIB matrix model

as a nonperturbative formulation of superstring theory

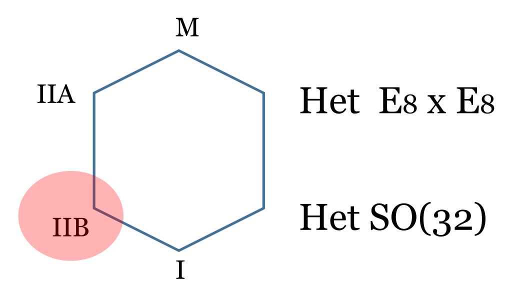
- The connection to perturbative formulations can be seen manifestly by considering type IIB superstring theory in 10d.

  worldsheet action,
  light-cone string field Hamiltonian, etc.

- A natural extension of the "one-matrix model", which is established as a nonperturbative formulation of non-critical strings

  regarding Feynman diagrams in matrix models as string worldsheets.

- It is expected to be a nonperturbative formulation of <u>the unique theory</u> underlying the web of string dualities.



  (Other types of superstring theory can be represented as perturbative vacua of the type IIB matrix model)

Lorenzian v.s. Euclidean

The reason why no one dared to study the Lorentzian model for many years:

$$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) = -2\,\underbrace{\mathrm{tr}\,(F_{0i})^2 + \mathrm{tr}\,(F_{ij})^2}_{\text{opposite sign}}$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

ill defined as it is !

Once one Euclideanizes it by $A_0 = -iA_{10}$,

$$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu})^2$$

positive definite!

The flat direction ($[A_\mu, A_\nu] \sim 0$) is lifted due to quantum effects.

(Aoki-Iso-Kawai-Kitazawa-Tada '99)

Euclidean model is well defined without any need for cutoffs.

Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

Our claim : the Lorentzian type IIB matrix model describes our Universe.

- The definition of the model is not straightforward.
- Monte Carlo studies suffer from the sign problem.
- We have to use the complex Langevin method.
- (3+1)-dim. expanding behavior emerges dynamically.
- The mechanism suggests a singular space-time structure.
- We discuss the emergence of a smooth space-time.
- Emergence of the Standard Model will be discussed by Asato Tsuchiya.

Plan of the talk

0. Introduction
1. Defining the Lorentzian type IIB matrix model
2. Complex Langevin method
3. Emergence of (3+1)-dim. expanding behavior
4. Emergence of a smooth space-time
5. Summary and discussions

1. Defining the Lorentzian type IIB matrix model

Partition function of the Lorentzian type IIB matrix model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]

partition function

$$Z = \int dA\, d\Psi\, e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$$\xi_0 \equiv -i\xi_2$$

(The worldsheet coordinates should also be Wick-rotated.)

Regularizing the Lorentzian model

- Unlike the Euclidean model,
  the Lorentzian model is NOT well defined as it is.

$$Z = \int dA\, d\Psi\, e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{iS_{\mathrm{b}}} \underbrace{\mathrm{Pf}\mathcal{M}(A)}_{\text{polynomial in } A}$$

pure phase factor

- One possibility is to deform the path slightly
  into the Euclidean direction.

(Yuhma Asano, private communication)

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}$$

$u = 1$ Euclidean → $u = 0$ Lorentzian

Path deformed theory well-defined for $0 < u \leq 1$

(Yuhma Asano, private communication)

$$e^{iS_\mathrm{b}(A)} = e^{-S(\tilde{A})}$$

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}$$

$$S(\tilde{A}) \sim -2e^{i\frac{\pi}{2}(1-u)}\mathrm{tr}\,[\tilde{A}_0, \tilde{A}_i]^2 - e^{-i\frac{\pi}{2}(1-u)}\mathrm{tr}\,[\tilde{A}_i, \tilde{A}_j]^2$$

positive real part for $0 < u \leq 1$

$\boxed{\mathrm{Re}\, S(\tilde{A}) \geq 0}$

Due to Cauchy's theorem,
it is expected that $\langle \mathcal{O}(\mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0, \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i)\rangle_u$
is independent of u.

If we define the Lorentzian model by taking the $u \to +0$ limit,

$$\langle \mathcal{O}(A_0, A_i)\rangle_{\mathrm{Lorentzian}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3}{8}\pi}A_0, \mathrm{e}^{i\frac{1}{8}\pi}A_i)\rangle_{\mathrm{Euclidean}}$$

Doesn't seem to be physical…

IR cutoffs as an alternative regularization

- First we generalize the model with two parameters (s, k).

$$Z = \int dA\, e^{-S(A)} \mathrm{Pf}\mathcal{M}(A)$$

$$S(A) = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2} \mathrm{tr}\, [A_0, A_i]^2 - \frac{1}{4} \mathrm{tr}\, [A_i, A_j]^2 \right\}$$

Pure imaginary action is hard to deal with numerically.
"*s*" deforms the model keeping Lorentz symmetry.

$$A_0 = e^{-ik\pi/2} \tilde{A}_0$$

Hermitian

$(s, k) = (0, 0)$ corresponds to the Lorentzian model.

"*k*" tilts the time direction in the imaginary direction.

- Introduce the IR cutoffs so that the extent in temporal and spatial directions become finite.

$$\frac{1}{N} \mathrm{tr}\, (\tilde{A}_0)^2 = \kappa L^2$$

$$\frac{1}{N} \mathrm{tr}\, (A_i)^2 = L^2$$

In what follows, we set $L = 1$ without loss of generality.

In this talk, we focus on the case $k = \dfrac{1+s}{2}$

$$A_0 = e^{-ik\pi/2}\tilde{A}_0$$

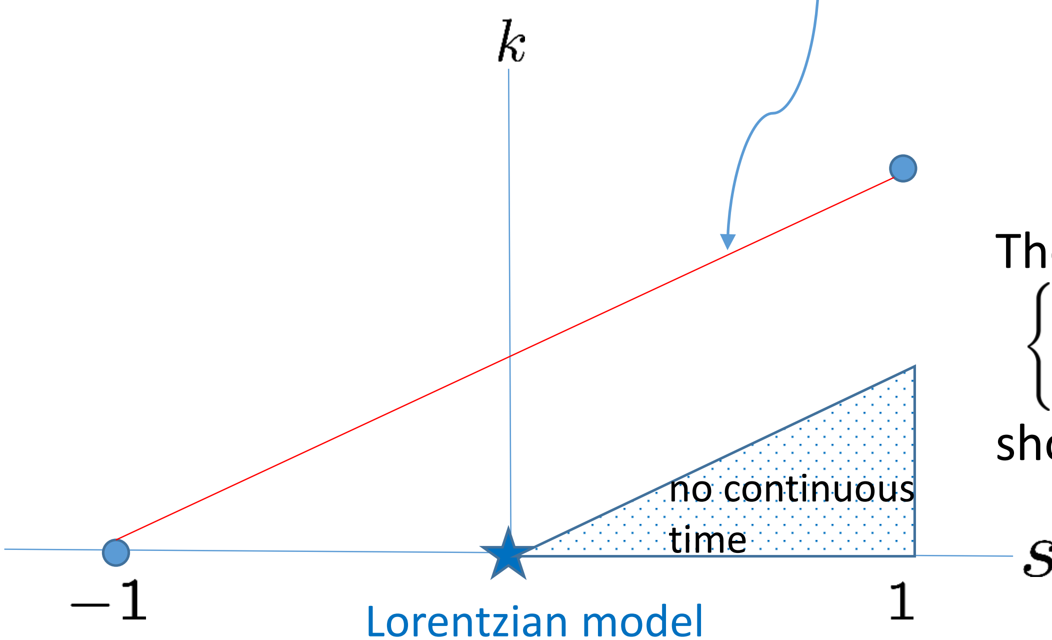
$$\begin{aligned} S &= N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\} \\ &= N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2}e^{-ik\pi}\mathrm{tr}\,[\tilde{A}_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\} \end{aligned}$$

IR cutoffs

$$\begin{aligned} \frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 &= \kappa \\ \frac{1}{N}\mathrm{tr}\,(A_i)^2 &= 1 \end{aligned}$$

The first term can be made real positive by choosing $e^{-i\frac{\pi}{2}(1-s)}e^{-ik\pi} = -1$

$\Rightarrow$ $k = \dfrac{1+s}{2}$ We focus on this case for the moment.



The limits

$$\begin{cases} 1) & N \to \infty \\ 2) & (s, k) \to (0, 0) \end{cases}$$

should be taken eventually.

2. Complex Langevin method

The complex Langevin method

Parisi ('83), Klauder ('83)

$$Z = \int dx \, w(x) \qquad x \in \mathbb{R}$$

complex

MC methods inapplicable due to sign problem !

Complexify the dynamical variables, and consider their (fictitious) time evolution :

$$z^{(\eta)}(t) = x^{(\eta)}(t) + i\, y^{(\eta)}(t)$$

defined by the complex Langevin equation

$$\frac{d}{dt} z^{(\eta)}(t) = \boxed{v(z^{(\eta)}(t))} + \eta(t)$$

Gaussian noise (real)

probability $\propto \mathrm{e}^{-\frac{1}{4}\int dt\, \eta(t)^2}$

$$\langle \mathcal{O} \rangle \stackrel{?}{=} \lim_{t\to\infty} \langle \boxed{\mathcal{O}(z^{(\eta)}(t))} \rangle_\eta$$

$$v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$$

Rem 1 : When w(x) is real positive, it reduces to one of the usual MC methods.

Rem 2 : The drift term $v(x) \equiv \frac{1}{w(x)} \frac{\partial w(x)}{\partial x}$ and the observables $\mathcal{O}(x)$ should be evaluated for complexified variables by analytic continuation.

Introducing the "time ordering"

$$A_0 = e^{-ik\pi/2}\tilde{A}_0$$

Hermitian

Using SU(N) symmetry $A_\mu \to U A_\mu U^\dagger$,
we diagonalize $\tilde{A}_0 = \mathrm{diag}(\tilde{\alpha}_1, \cdots, \tilde{\alpha}_N)$,
where $\tilde{\alpha}_1 < \tilde{\alpha}_2 < \cdots < \tilde{\alpha}_N$.

$$Z = \int d\tilde{A}_0\, dA_i\, e^{-S} = \int d\tilde{\alpha}\, dA_i\, \Delta(\tilde{\alpha})\, e^{-S}$$

$$\Delta(\tilde{\alpha}) = \prod_{a>b} (\tilde{\alpha}_a - \tilde{\alpha}_b)^2$$: van der Monde determinant

In order to respecting the time ordering
$\tilde{\alpha}_1 < \cdots < \tilde{\alpha}_N$ in the CLM,
we make the following change of variables:

$$\tilde{\alpha}_1 = 0\ , \quad \tilde{\alpha}_2 = e^{\tau_1}\ , \quad \tilde{\alpha}_3 = e^{\tau_1} + e^{\tau_2}\ , \quad \cdots\ , \quad \tilde{\alpha}_N = \sum_{a=1}^{N-1} e^{\tau_a}\ ,$$

and complexify τ_a $(a = 1, \cdots, N-1)$.

Complex Langevin equation

The constraints are replaced by some appropriate potential.

$$\frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 = \kappa$$
$$\frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$$

$$S_{\mathrm{pot}} = \frac{1}{p}\gamma_{\mathrm{s}}\left(\frac{1}{N}\mathrm{tr}\,(A_i)^2 - 1\right)^p + \frac{1}{p}\gamma_{\mathrm{t}}\left(\frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 - \kappa\right)^p$$

We use $p = 4$.

The effective action reads:

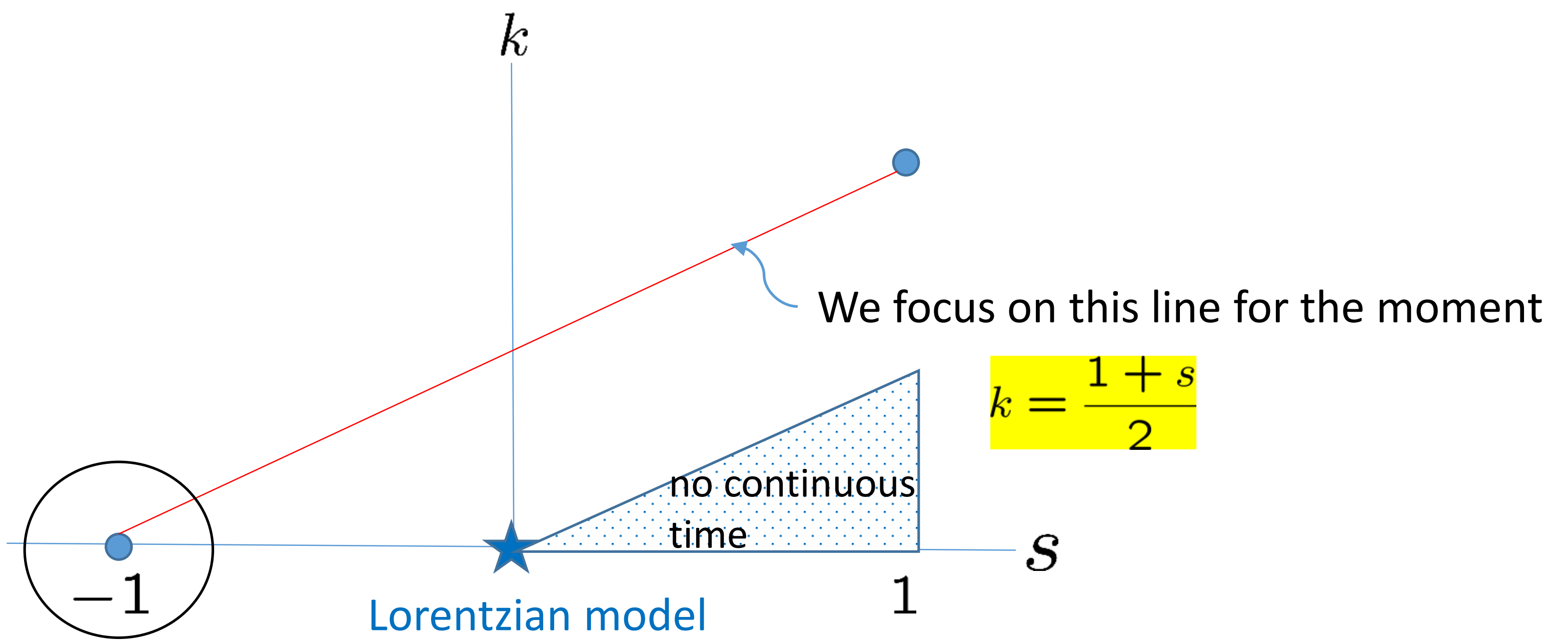
$$\begin{aligned} S_{\mathrm{eff}} \;=\; & N\beta\, e^{-i\frac{\pi}{2}(1-s)}\left\{\frac{1}{2}e^{-ik\pi}\mathrm{tr}\,[\tilde{A}_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2\right\} \\ & + \frac{1}{p}\gamma_{\mathrm{s}}\left(\frac{1}{N}\mathrm{tr}\,(A_i)^2 - 1\right)^p + \frac{1}{p}\gamma_{\mathrm{t}}\left(\frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 - \kappa\right)^p \\ & - \log\Delta(\tilde{\alpha}) - \sum_{a=1}^{N-1}\tau_a \end{aligned}$$

Treat τ_a as complex variables,
and A_i as general complex matrices,
and solve the complex Langevin equation:

$$\left\{\begin{aligned} \frac{d\tau_a}{dt} &= -\frac{\partial S_{\mathrm{eff}}}{\partial \tau_a} + \eta_a \\ \frac{d(A_i)_{ab}}{dt} &= -\frac{\partial S_{\mathrm{eff}}}{\partial (A_i)_{ba}} + (\eta_i)_{ab} \end{aligned}\right.$$

3. Emergence of (3+1)-dimensional expanding behavior

Results at (s,k)=(-1,0) in the 6D bosonic model



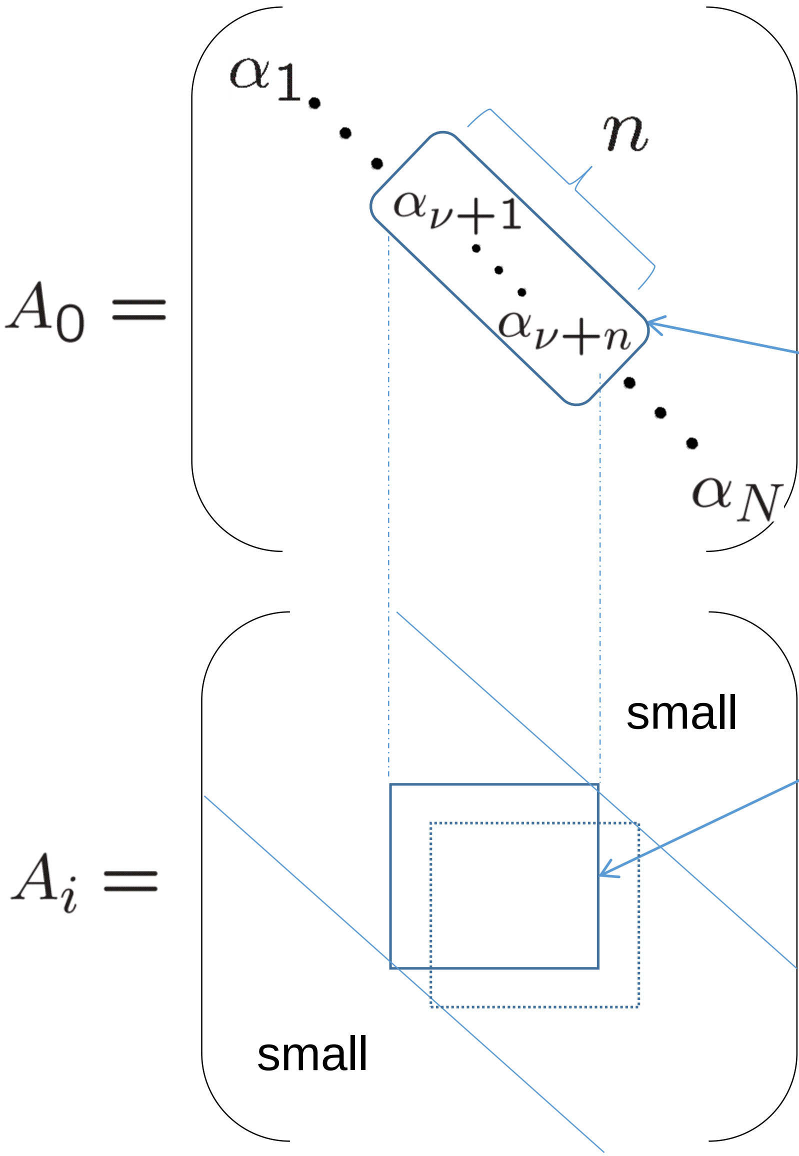
$A_0 = \tilde{A}_0$ k=0; no tilt in the time direction (real time).

$$S = N\beta \left\{ -\frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 + \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

favors A_j close to diagonal

favors maximal non-commutativity between A_j

How to extract time-evolution



$$\alpha_1 < \cdots < \alpha_N$$

definition of time "t"

$$t = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

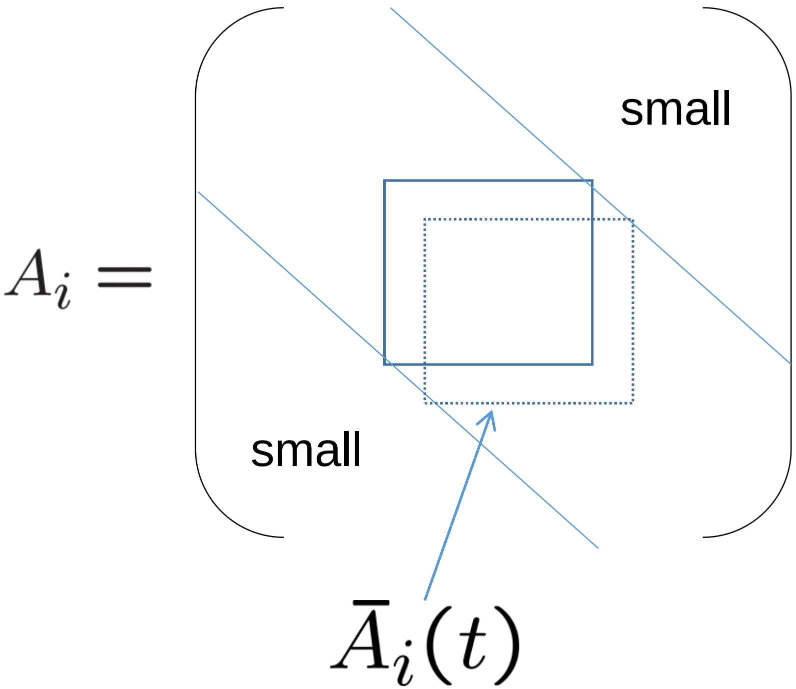
non-trivial dynamical property

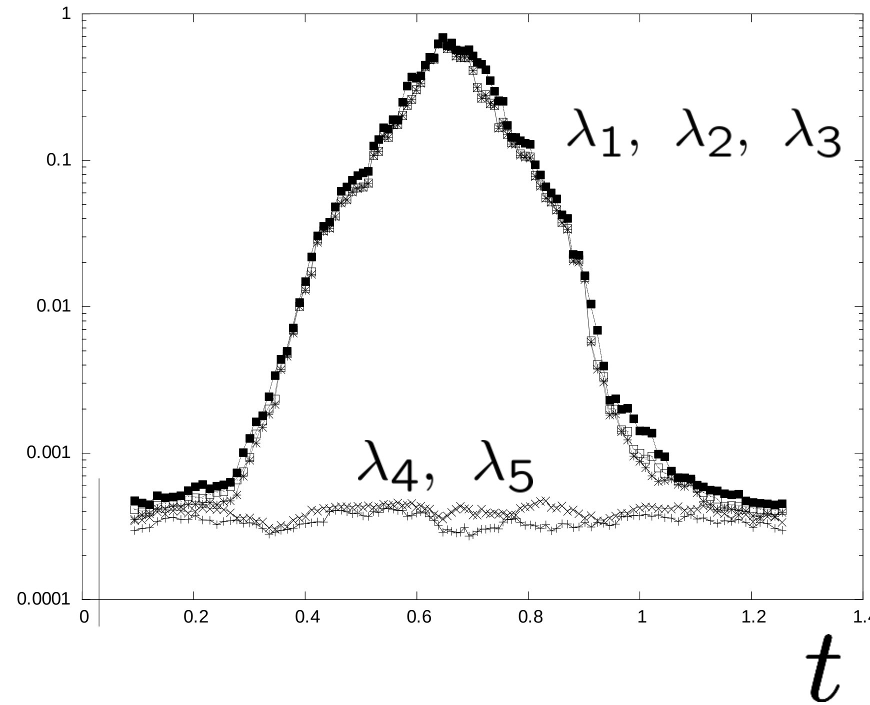
Emergence of (3+1)-dim. expanding behavior

$$N = 128\ , \quad \kappa = 0.13\ , \quad \beta = 2\ , \quad (s,k) = (-1,0)\ , \quad n = 16$$

eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$

$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^\dagger(t))$$





SSB : SO(5) → SO(3) occurs at some point in time.

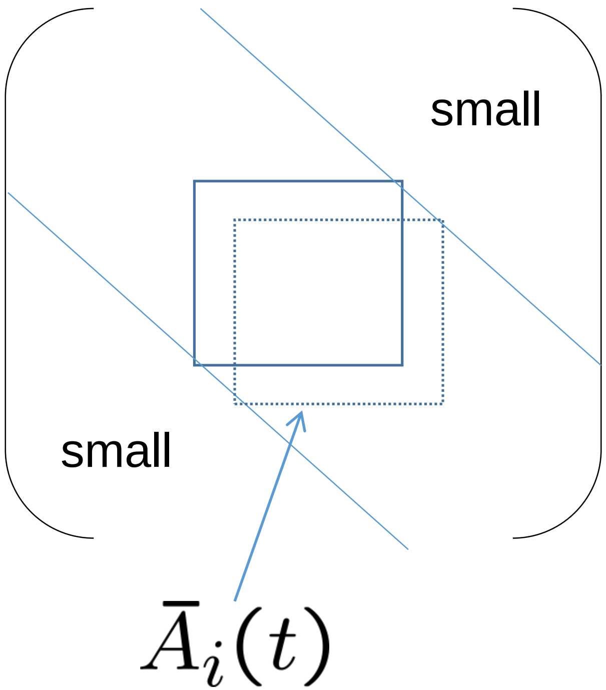
The mechanism of the SSB

$$S = N\beta \left\{ -\frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 + \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

favors A_j close to diagonal

favors maximal non-commutativity between A_j

$A_i =$



maximize $\mathrm{NC} = -\mathrm{tr}\,[\bar{A}_i(t), \bar{A}_j(t)]^2$

for $\mathrm{tr}\,(\bar{A}_i(t))^2 = \mathrm{const.}$

$$\bar{A}_i(t) \quad \propto \quad \sigma_i \qquad \text{for } i = 1, 2, 3$$

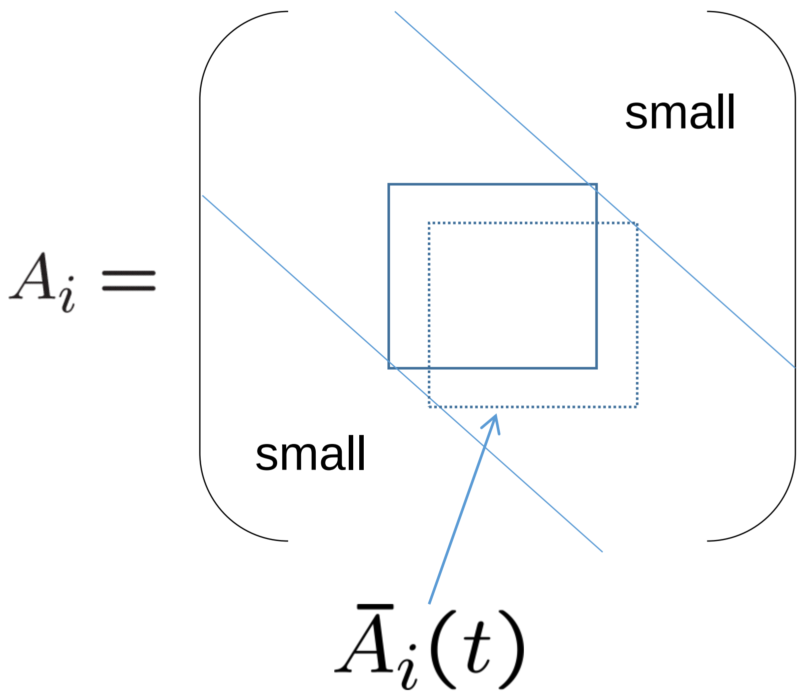
$$\bar{A}_i(t) \quad = \quad 0 \qquad \text{for } i \geq 4$$

up to SO(5) rotation

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601

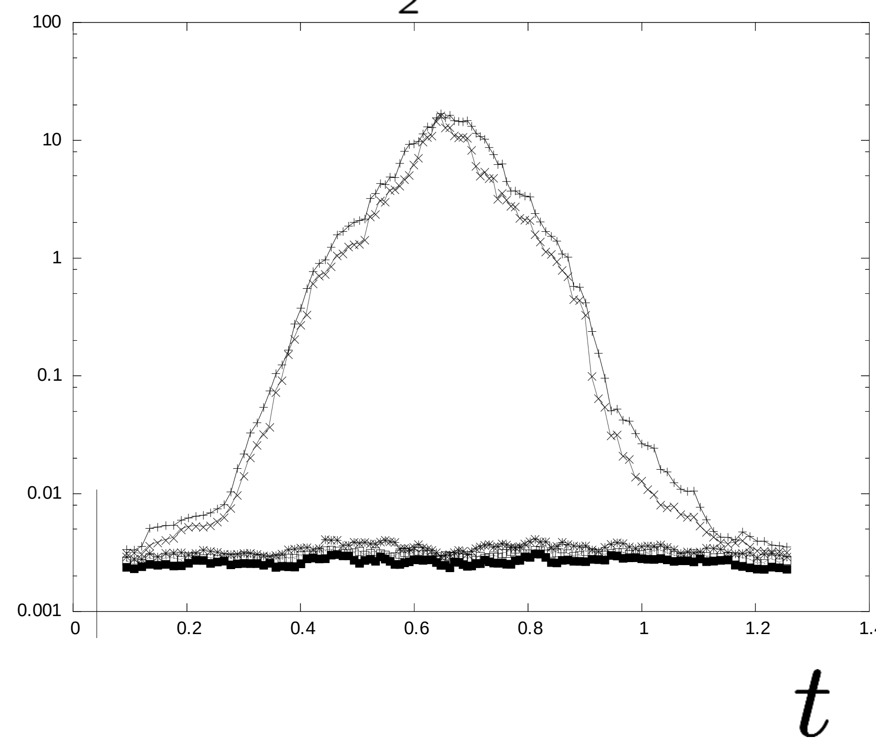
Confirmation of the mechanism

$$N = 128\ , \quad \kappa = 0.13\ , \quad \beta = 2\ , \quad (s,k) = (-1,0)\ , \quad n = 16$$



eigenvalues of $Q = \sum_{i=1}^{5} \left\{ X_i(t) \right\}^2$

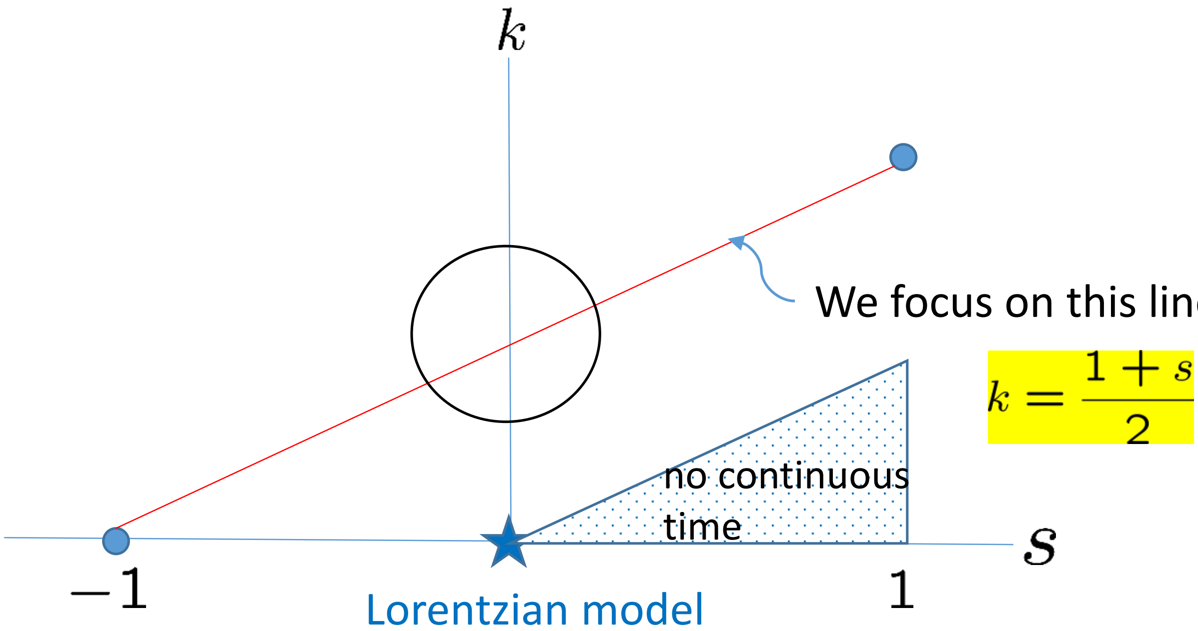
$$X_i(t) = \frac{1}{2}(\bar{A}_i(t) + \bar{A}_i^{\dagger}(t))$$



Only 2 eigenvalues of Q become large.

4. Emergence of a smooth space-time

Exploring the phase diagram near $s = 0$



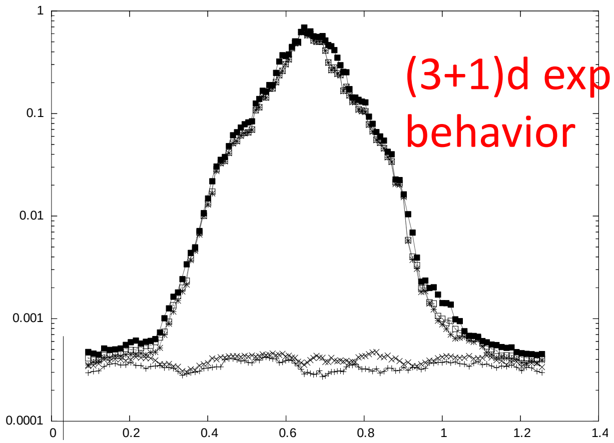
$$S = -N\beta \left\{ \frac{1}{2} \mathrm{tr}\, [\tilde{A}_0, A_i]^2 + \frac{1}{4} e^{-i\frac{\pi}{2}(1-s)} \mathrm{tr}\, [A_i, A_j]^2 \right\}$$

Real part changes sign at $s = 0$.

Can we obtain (3+1)-dim. expanding behavior
with a smooth space-time structure ?

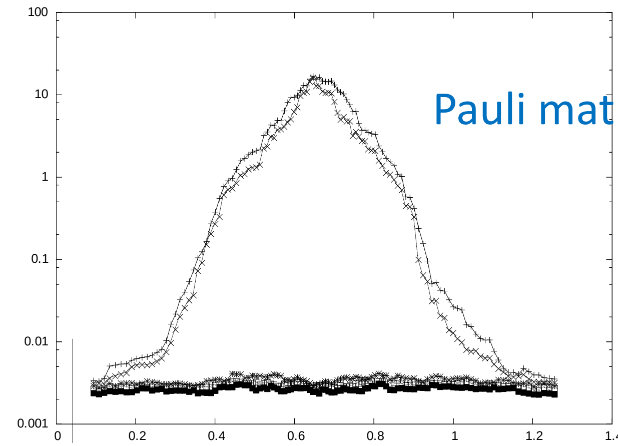
$s = -1$ v.s. $s \sim 0$

eigenvalues of $T_{ij}(t) = \frac{1}{n}\text{tr}\left\{X_i(t)X_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$



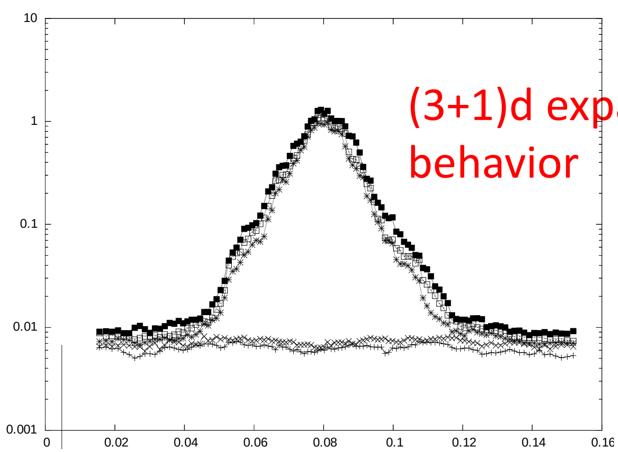
(3+1)d expanding behavior

$s = -1$



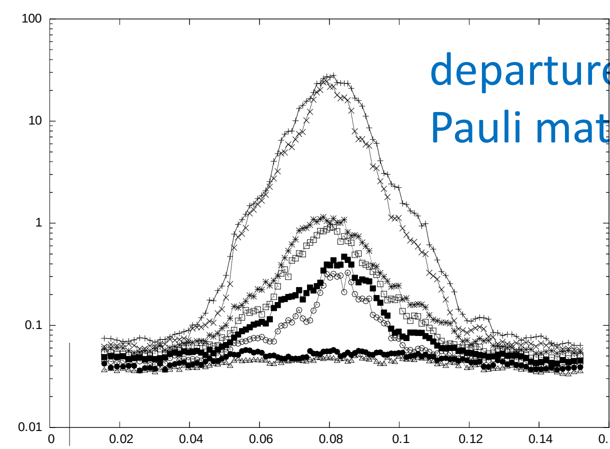
Pauli matrices

$N = 128$, $\kappa = 0.13$, $\beta = 2$, $(s, k) = (-1, 0)$, $n = 16$



(3+1)d expanding behavior

$s \sim 0$



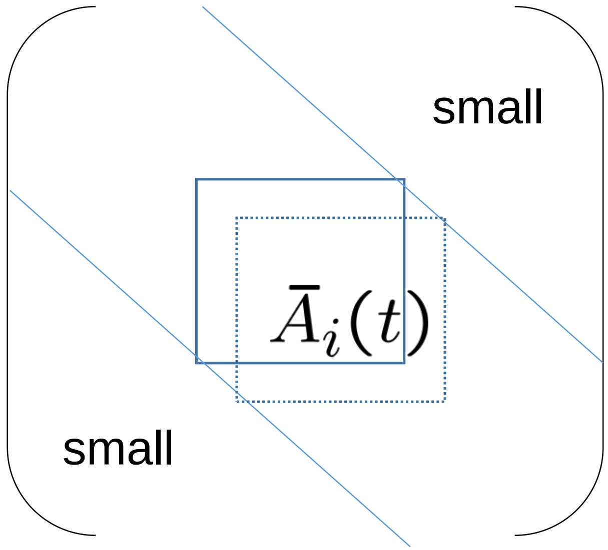
departure from Pauli matrices

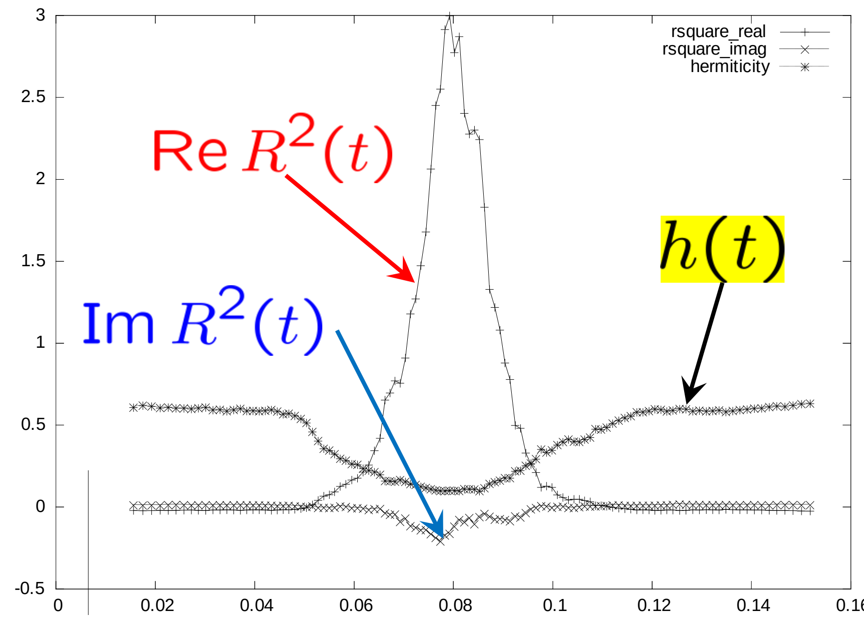
$N = 128$, $\kappa = 0.0037$, $\beta = 32$, $(s, k) = (0.0076, 0.5038)$, $n = 16$

Hermiticity of the spatial matrices

$N = 128\ , \quad \kappa = 0.0037\ , \quad \beta = 32\ , \quad (s, k) = (0.0076, 0.5038)\ , \quad n = 16$

$A_i =$





$$R^2(t) = \frac{1}{n}\mathrm{tr}\,(\bar{A}(t)^2)$$

$$h(t) = \frac{-\mathrm{tr}\,(\bar{A}_i(t) - \bar{A}_i(t)^\dagger)^2}{4\mathrm{tr}\,(\bar{A}_i(t)^\dagger \bar{A}_i(t))}$$

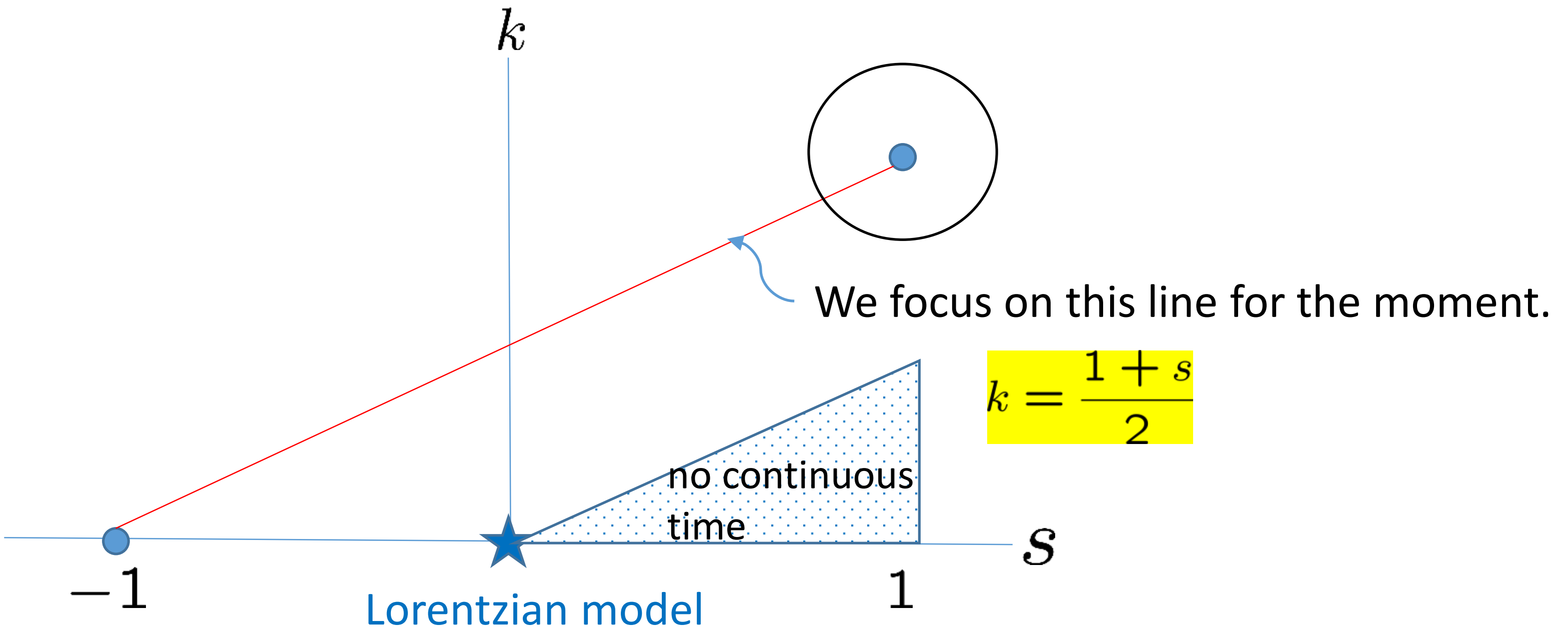
$$0 \le h(t) \le 1$$

Hermitian anti-Hermitian

Spatial matrices become close to Hermitian near the peak of $\mathrm{Re}\, R^2(t)$.

Classical solution seems to be dominating in this region.

Approaching from $s = 1$



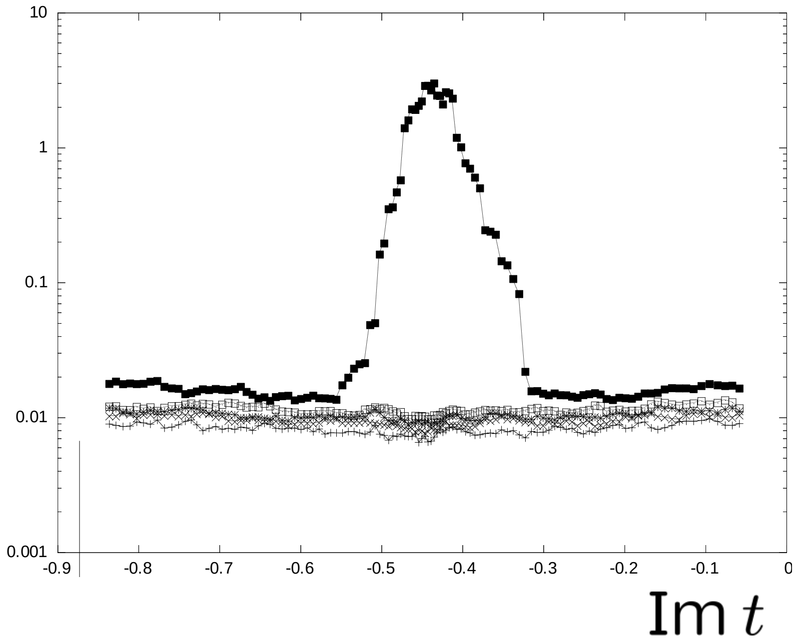
$$S = -N\beta \left\{ \frac{1}{2} \mathrm{tr}\, [\tilde{A}_0, A_i]^2 + \frac{1}{4} e^{-i\frac{\pi}{2}(1-s)} \mathrm{tr}\, [A_i, A_j]^2 \right\}$$

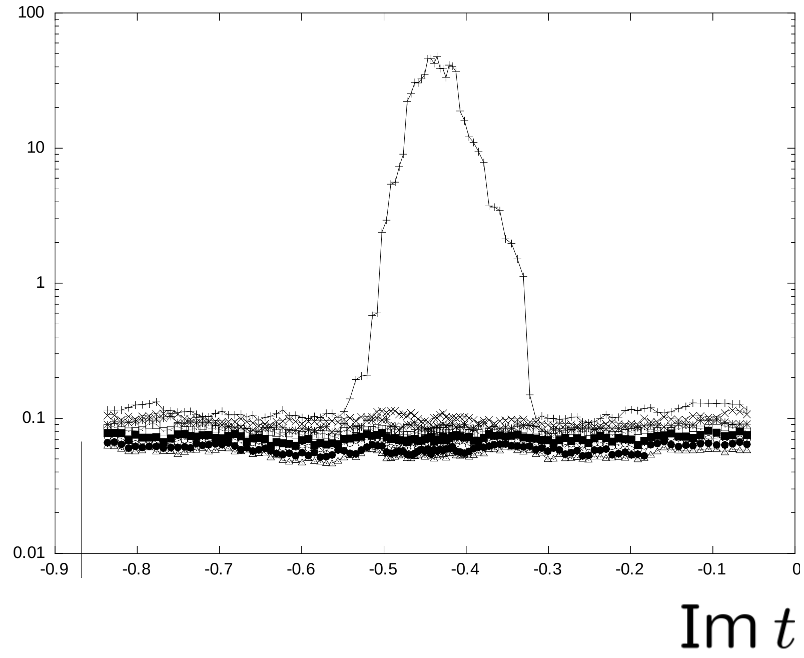
Real part changes sign at $s = 0$.

$s = 1$ corresponds to the Euclidean model with the constraints: $\frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 = \kappa$, $\frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$

Eulidean model with the constraints yields (1+1)d expanding behavior

eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$



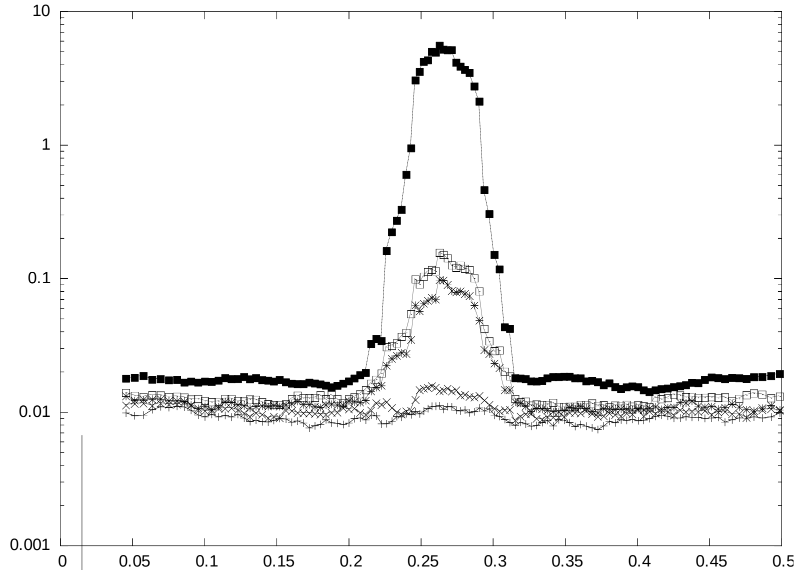


$N = 128$, $\kappa = 0.01$, $\beta = 32$, $(s,k) = (1,1)$, $n = 16$

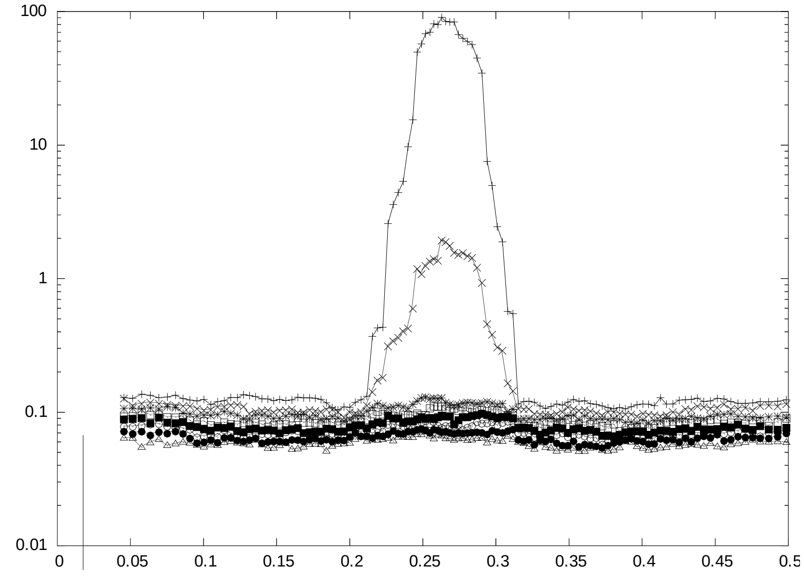
corresponds to stretching the spatial directions

Decreasing *s* from *s=1*

eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$



eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$

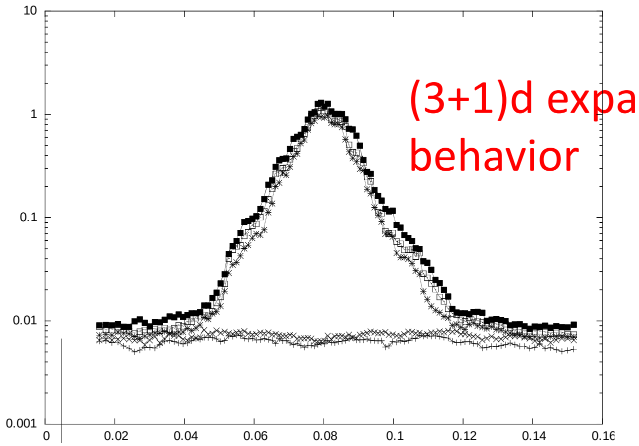


$N = 128$, $\kappa = 0.06$, $\beta = 32$, $(s, k) = (0.195, 0.5975)$, $n = 16$

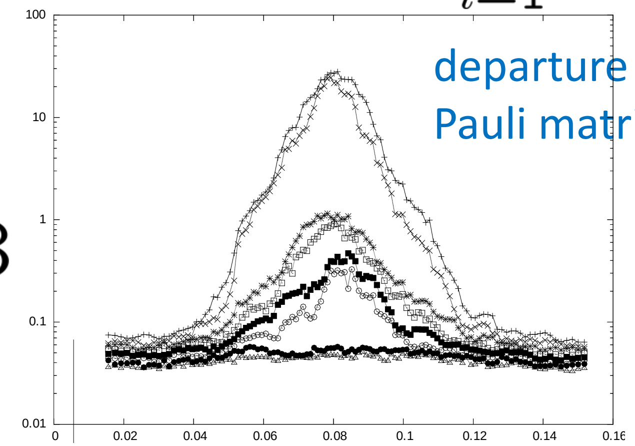
Gradual transition to (3+1)d expanding behavior with Pauli matrices.

N=128 v.s. N=192

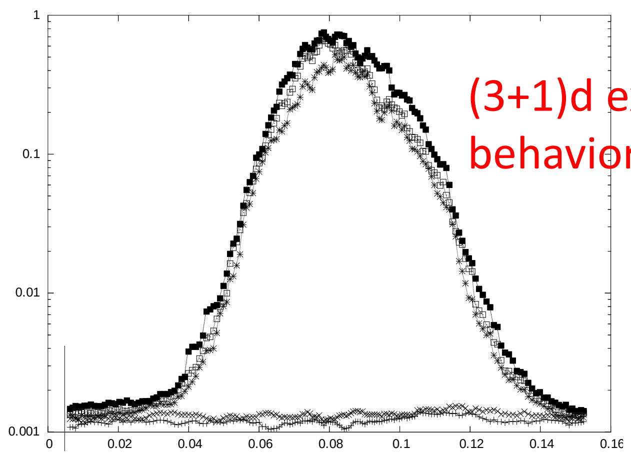
eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$



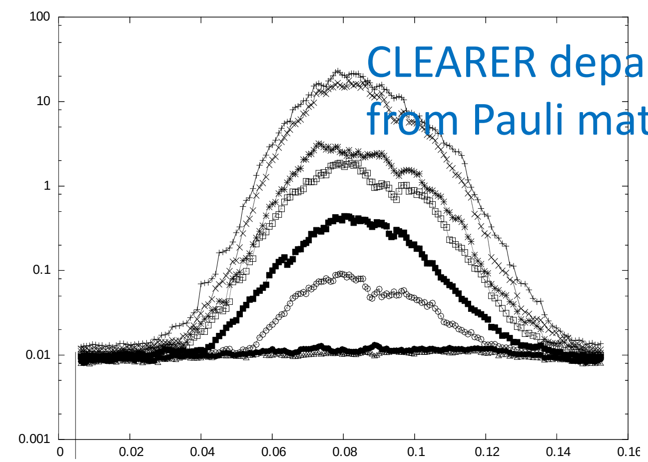
$N = 128$



$N = 128\ ,\quad \kappa = 0.0037\ ,\quad \beta = 32\ ,\quad (s,k) = (0.0076, 0.5038)\ ,\quad n = 16$



$N = 192$



$N = 192\ ,\quad \kappa = 0.0044\ ,\quad \beta = 64\ ,\quad (s,k) = (0.0118, 0.5059)\ ,\quad n = 24$

5. Summary and Discussions

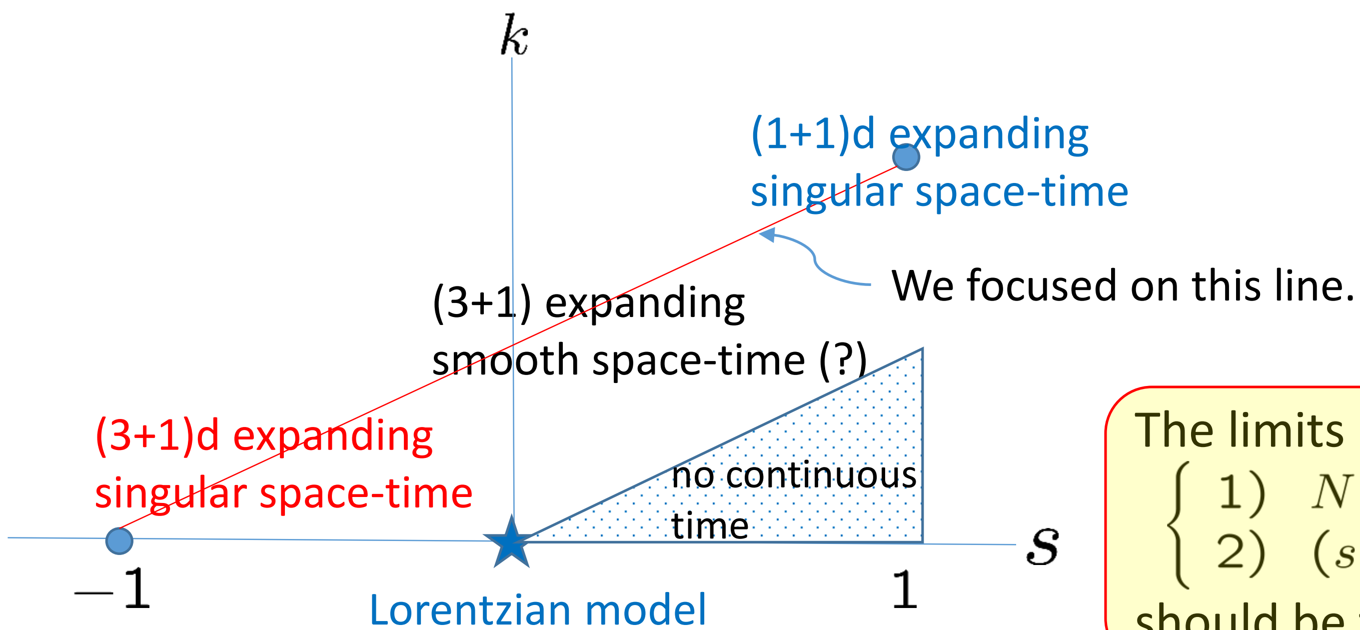
Summary

The Lorentzian version of the type IIB matrix model with certain generalization

$$S(A) = N\beta\, e^{-i\frac{\pi}{2}(1-s)} \left\{ \frac{1}{2}\mathrm{tr}\,[A_0, A_i]^2 - \frac{1}{4}\mathrm{tr}\,[A_i, A_j]^2 \right\}$$

$$A_0 = e^{-ik\pi/2}\tilde{A}_0$$

IR cutoffs : $\frac{1}{N}\mathrm{tr}\,(\tilde{A}_0)^2 = \kappa\ , \quad \frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$



$$k = \frac{1+s}{2}$$

The limits

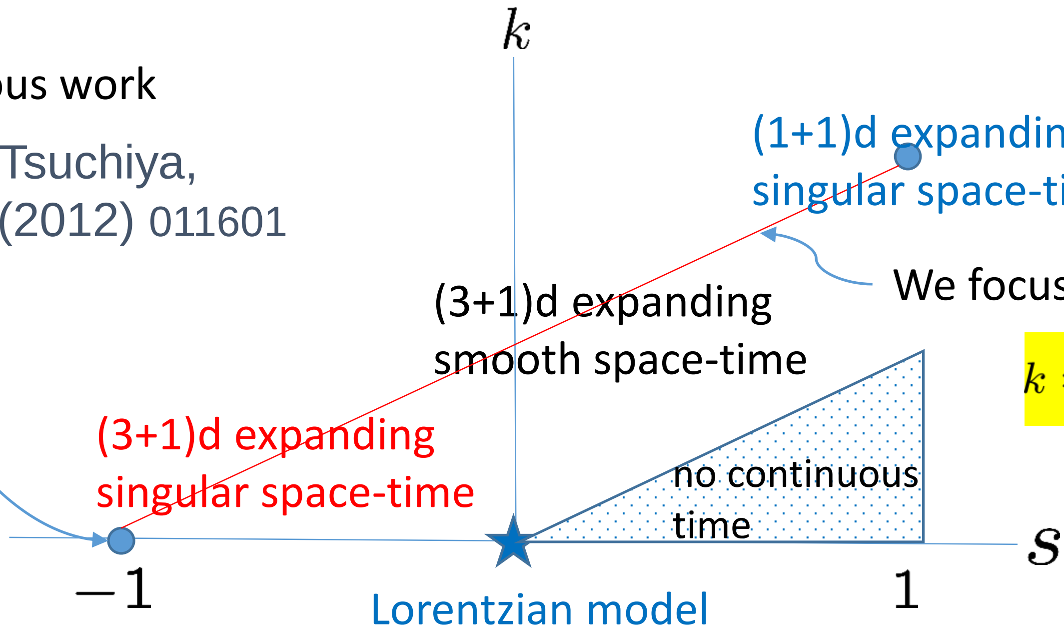
$$\begin{cases} 1) \quad N \to \infty \\ 2) \quad (s,k) \to (0,0) \end{cases}$$

should be taken eventually.

Discussions

Our previous work

Kim-J.N.-Tsuchiya,
PRL 108 (2012) 011601



We focused on this line.

$$k = \frac{1+s}{2}$$

- Transition from the Pauli matrices to a smooth space-time occurs at slightly positive *s* for *N*=128, 192.

Does the transition point approach *s=0* at larger *N* ?

- Complex Langevin simulation becomes unreliable due to growing non-hermiticity when we decrease *k* from *k*=(1+*s*)/2 too much.

Can we approach the target (s,*k*)=(0,0) at larger N ?
Does the (3+1)d expanding smooth space-time survive there ?

Discussions (cont'd)

- Hermiticity of spatial matrices emerges as the space expands.

  This suggests that a classical solution is dominating there.
  If so, solving the classical eq. of motion is a sensible way
  to explore the late time behavior of this model.

  c.f.) Asato Tsuchiya's talk

- Effects of the fermionic matrices ?

  Not straightforward due to the "singular-drift problem" in the CLM
  caused by the near-zero eigenvalues the Dirac operator,
  but maybe possible.

- Generalizing the IR cutoffs to

$$\frac{1}{N}\mathrm{tr}\,\{(\tilde{A}_0)^2\}^p = \kappa^p \ , \qquad \frac{1}{N}\mathrm{tr}\,\{(A_i)^2\}^p = 1$$

  Previous studies suggest that we obtain universal results for $p \sim 1.5$,
  but the model becomes pathological for larger p.