Emergence of chiral zero modes in the Lorentzian type IIB matrix model

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Based on collaboration with
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Introduction

Type IIB matrix model

Ishibashi-Kawai-Kitazawa-A.T. ('96)

A proposal for nonperturbative formulation of superstring theory

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^M, A^N][A_M, A_N] + \frac{1}{2}\bar{\psi}\Gamma^M[A_M, \Psi]\right)$$

Kawai's talk
Nishimura's talk

$N \times N$ Hermitian matrices

A_M : 10D Lorentz vector $(M = 0, 1, \ldots, 9)$

Ψ : 10D Majorana-Weyl spinor

Large-N limit is taken

Space-time does not exist a priori, but is generated dynamically from degrees of freedom of matrices

Evidences for nonperturbative formulation

(1) Manifest SO(9,1) symmetry and manifest 10D N=2 SUSY

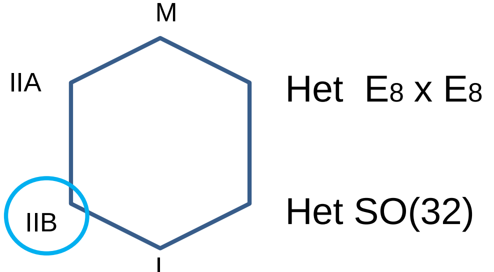
(2) Correspondence with Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

(3) Long distance behavior of interaction between D-branes is reproduced

(4) Light-cone string field theory for type IIB superstring from SD equations for Wilson loops under some assumptions

Fukuma-Kawai-Kitazawa-A.T. ('97)

(5) Believing string duality, one can start from anywhere with nonperturbative formulation to tract strong coupling regime

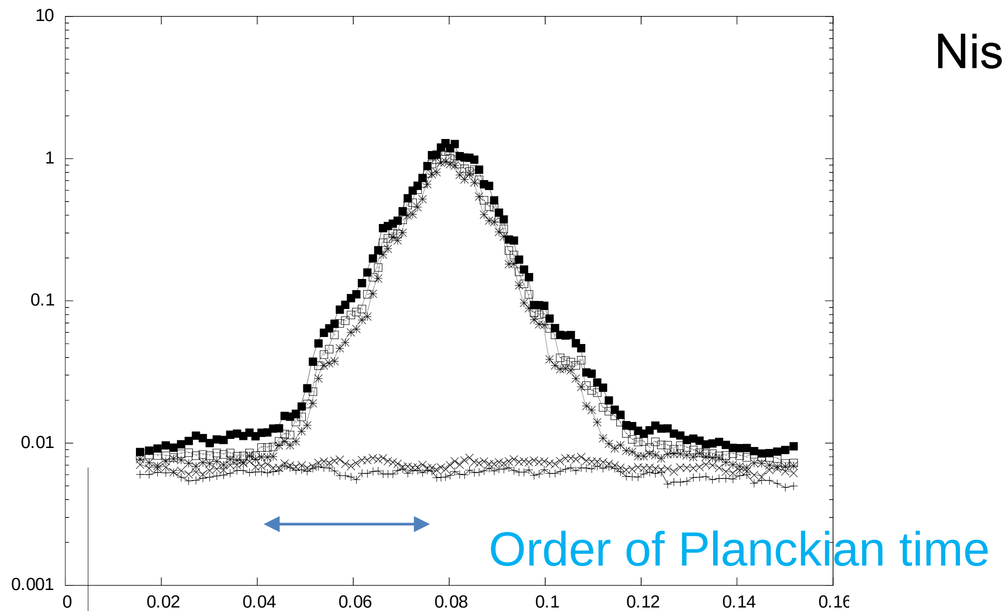


Emergence of expanding (3+1)d universe

Kim-Nishimura-A.T. ('11)

Nishimura-A.T. ('18)

Our numerical simulation suggests that expanding (3+1)-dimensional Universe emerges in the Lorentzian version of the model



Nishimura' talk

Questions

At late times

- (3+1)d expanding space-time emerges?
- How it expands?
- (3+1)d space-time structure is smooth?
- SM or BSM appears?

Structure of extra dimensions ⇨ Chiral fermions

Plan of the present talk

1. Introduction
2. Analysis of classical EOM
3. Space-time and chiral zero modes from Classical solutions
4. Conclusion and discussion

Analysis of classical EOM

Classical dynamics dominates at late times

CF.) Stern's talk, Steinacker's talk

- The late-time behaviors are difficult to study by direct Monte Carlo methods, since larger matrix sizes are required.

- But the classical equations of motion are expected to become more and more valid at later times, since the value of the action increases with the cosmic expansion.

- We develop a numerical algorithm for searching for classical solutions satisfying the most general ansatz with "quasi direct product structure"

  ～nontrivial because of no time a priori in the model

Defining the Lorentzian model

Nishimura's talk

- Lorentzian model

$$S_b \propto \mathrm{Tr}(F^{MN}F_{MN}) = \boxed{-2\mathrm{Tr}F_{0i}^2} + \boxed{\mathrm{Tr}F_{ij}^2}$$

opposite sign
not bounded below

$$F_{MN} = -i[A_M, A_N]$$

Introduce IR cutoffs

Kim-Nishimura-A.T. ('11)

$$\frac{1}{N}\mathrm{Tr}(A_0)^2 \leq \kappa$$
$$\frac{1}{N}\mathrm{Tr}(A_i)^2 \leq L^2$$

removed in $N \to \infty$

Equation of motion

$$S = -\frac{1}{4}\mathrm{Tr}([A^M, A^N][A_M, A_N]$$

$$[A^M, [A_M, A_0]] + \alpha A_0 = 0$$

$$[A^M, [A_M, A_i]] - \beta A_i = 0 \quad (i = 1, \ldots, 9)$$

$\alpha, \ \beta$: Lagrange multiplier

constraints

$$\frac{1}{N}\mathrm{Tr}(A_0^2) = \kappa$$

$$\frac{1}{N}\mathrm{Tr}(A_i^2) = 1$$

corresponding to IR cutoffs

Configuration with "quasi direct product structure"

Nishimura-A.T.('13)

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3)$$
$$A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9)$$

$M = 1$: direct product space-time

$N_X \times N_X$ $\quad N_Y \times N_Y$ $\qquad N = N_X \times N_Y$

Each point on (3+1)d space-time has the same structure in the extra dimensions

This ansatz is compatible with Lorentz symmetry to be expected at late time

$$O_{\mu\nu} X_\nu = g[O]\, X_\mu\, g[O]^\dagger$$

$$O \in \mathrm{SO}(3,1) \qquad g[O] \in SU(N_X)$$

Chiral fermions in type IIB matrix model

It is reasonable that one can analyze massless modes of fermions from Dirac equation in 10d

(1) $\Gamma^M [A_M, \Psi] = 0$

Ψ is Majorana-Weyl in 10d ⟹ $\Gamma_\chi \Psi = \Psi$

we demand Ψ to be chiral in 4d $\Gamma_\chi = \gamma_\chi^{(4d)} \gamma_\chi^{(6d)}$

(2) $\gamma_\chi^{(4d)} \Psi = \pm\Psi$ ⟹ $\gamma_\chi^{(6d)} \Psi = \pm\Psi$ also chiral in 6d

It is easy to show

(1), (2) ⟺
$$\begin{cases} \Gamma^\mu [A_\mu, \Psi] = 0 \\ \Gamma^a [A_a, \Psi] = 0 \\ \Psi \text{ is chiral in 4d and 6d} \end{cases}$$

Massless Dirac equations in 6d

We consider the following (3+1)d background

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3)$$

$$A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9)$$

We decompose Ψ as $\Psi = \varphi^{(4d)} \otimes \varphi^{(6d)}$

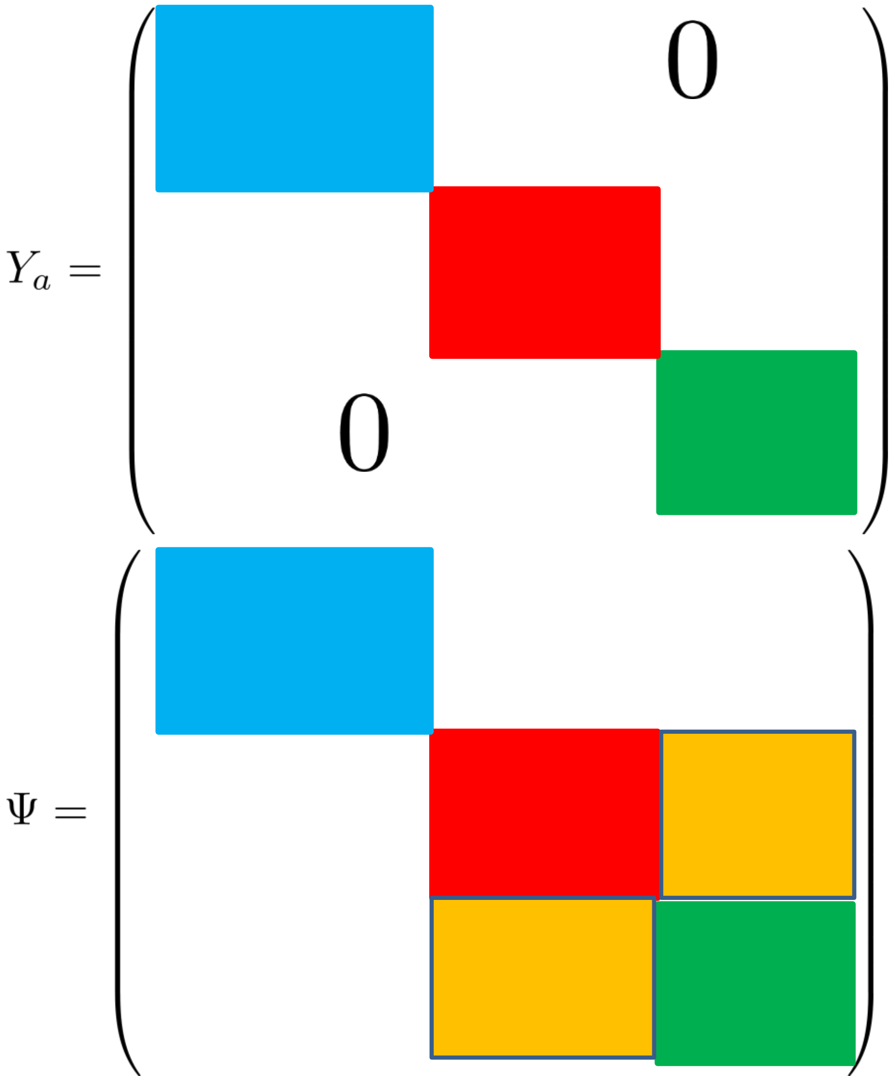
$$\begin{cases} \Gamma^a [A_a, \Psi] = 0 \\ \gamma_\chi^{(6d)} \Psi = \pm \Psi \end{cases} \Rightarrow \begin{cases} \Gamma^a [Y_a, \varphi^{(6d)}] = 0 \\ \gamma_\chi^{(6d)} \varphi^{(6d)} = \pm \varphi^{(6d)} \end{cases}$$

We examine spectrum of 6d Dirac operator $\Gamma^a [Y_a, *]$

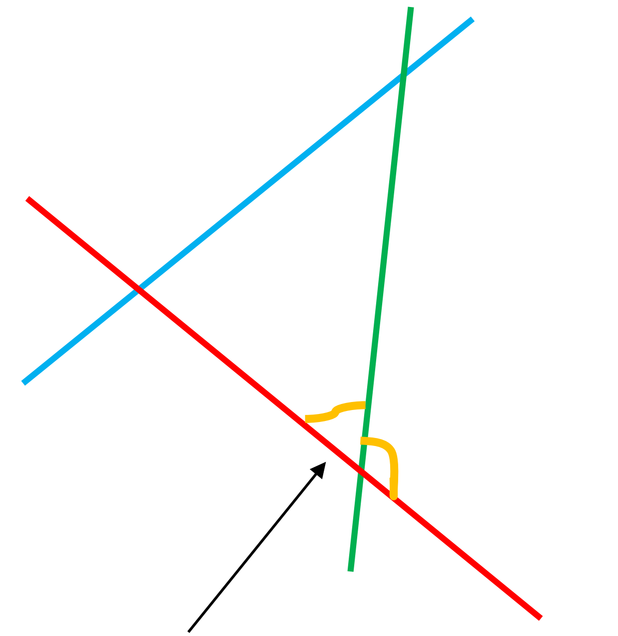
zero eigenvectors ~ chiral zero modes

$$\varphi^{(6d)} \Rightarrow \Psi$$

Structure of Ya and chiral zero modes



Intersecting D-branes



chiral zero modes

Algorithm for finding solutions

$$I = \mathrm{Tr}([A^M, [A_M, A_0]] + \alpha A_0)^2 + \mathrm{Tr}([A^M, [A_M, A_i]] - \beta A_i)^2$$

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3)$$

$$A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9)$$

We search for configurations that gives $I = 0$

gradient descent algorithm

update configurations following

$$\delta X_\mu = -\epsilon \frac{\partial I}{\partial X_\mu^\dagger} \qquad \delta Y_a = -\epsilon \frac{\partial I}{\partial Y_a^\dagger} \qquad \delta M = -\epsilon \frac{\partial I}{\partial M^\dagger}$$

➡ $\delta I \leq 0$

Space-time and chiral zero modes from classical solutions

Our solutions

Our ansatz

$$\begin{cases} A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3) \\ A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9) \end{cases}$$

$$\begin{cases} M^3 = M \\ [M, Y_a] = 0 \\ [X^\nu, [X_\nu, X_0]] + \alpha X_0 = 0 \\ [X^\nu, [X_\nu, X_i]] - \beta X_i = 0 \quad (i = 1, 2, 3) \\ [Y^b, [Y_b, Y_a]] - \beta Y_a = 0 \end{cases}$$

$M^3 = M$ → eigenvalues of M: -1, 0, 1

⇒

$$\begin{cases} [A^M, [A_M, A_0]] + \alpha A_0 = 0 \\ [A^M, [A_M, A_i]] - \beta A_i = 0 \quad (i = 1, \ldots, 9) \end{cases}$$

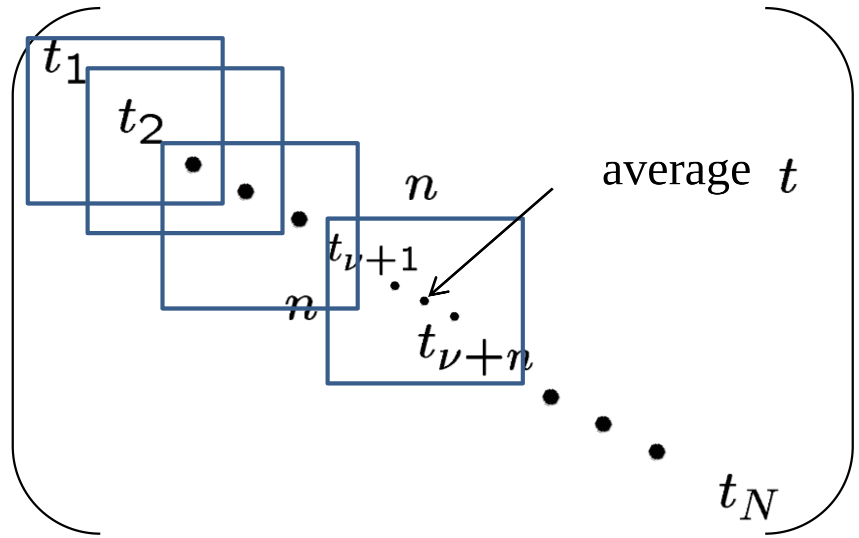
Structure of M and Ya

$$M = \begin{pmatrix} \begin{matrix} -1 & & \\ & \ddots & \\ & & -1 \end{matrix} & & 0 \\ & \begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix} & \\ 0 & & \begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix} \end{pmatrix}$$

$$Y_a = \begin{pmatrix} Y_a^{(-1)} & & 0 \\ & Y_a^{(0)} & \\ 0 & & Y_a^{(1)} \end{pmatrix}$$

Emergence of concept of ``time evolution”

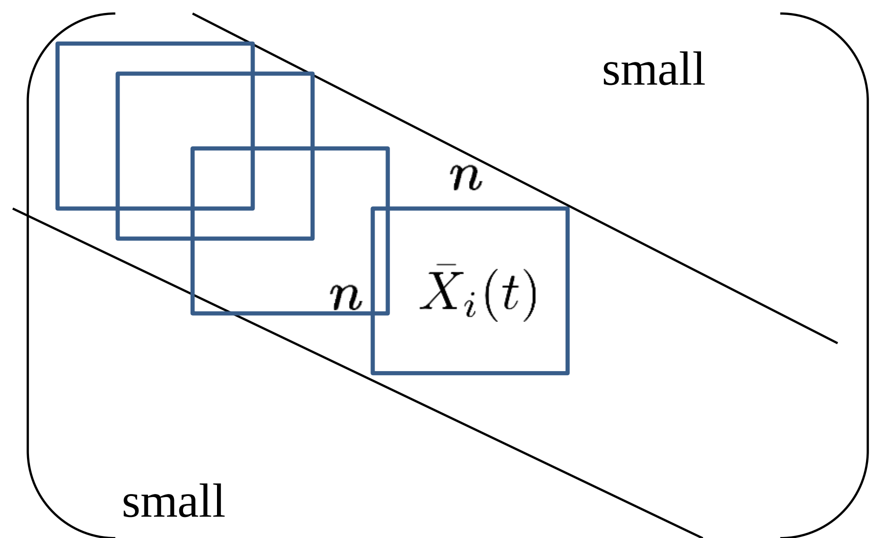
$X_0 =$



$t_1 < t_2 < \cdots < t_N$

These values are dynamically determined

$X_i =$



$(i = 1, 2, 3)$

Band-diagonal structure is observed, which is nontrivial

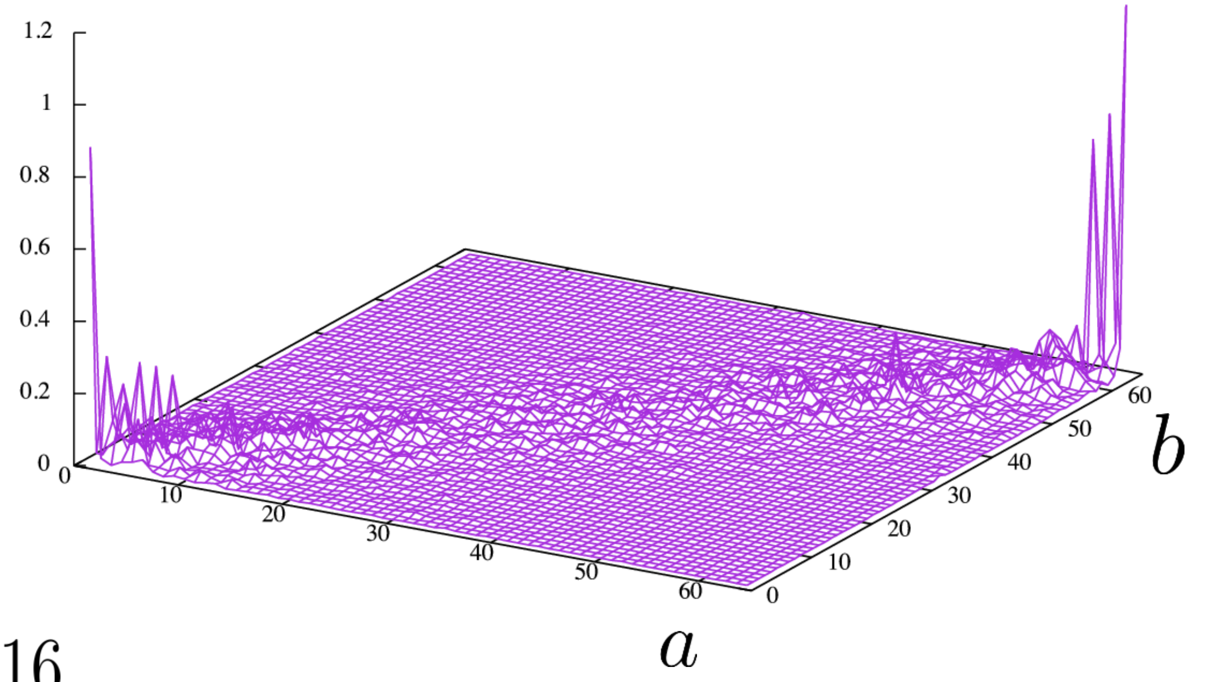
$\bar{X}_i(t)$ represents space structure at fixed time t

concept of “time evolution” emerges

Band diagonal structure of Xi

$$N_X = 64$$

$$\sum_{i=1}^{3} |(X_i)_{ab}|^2$$

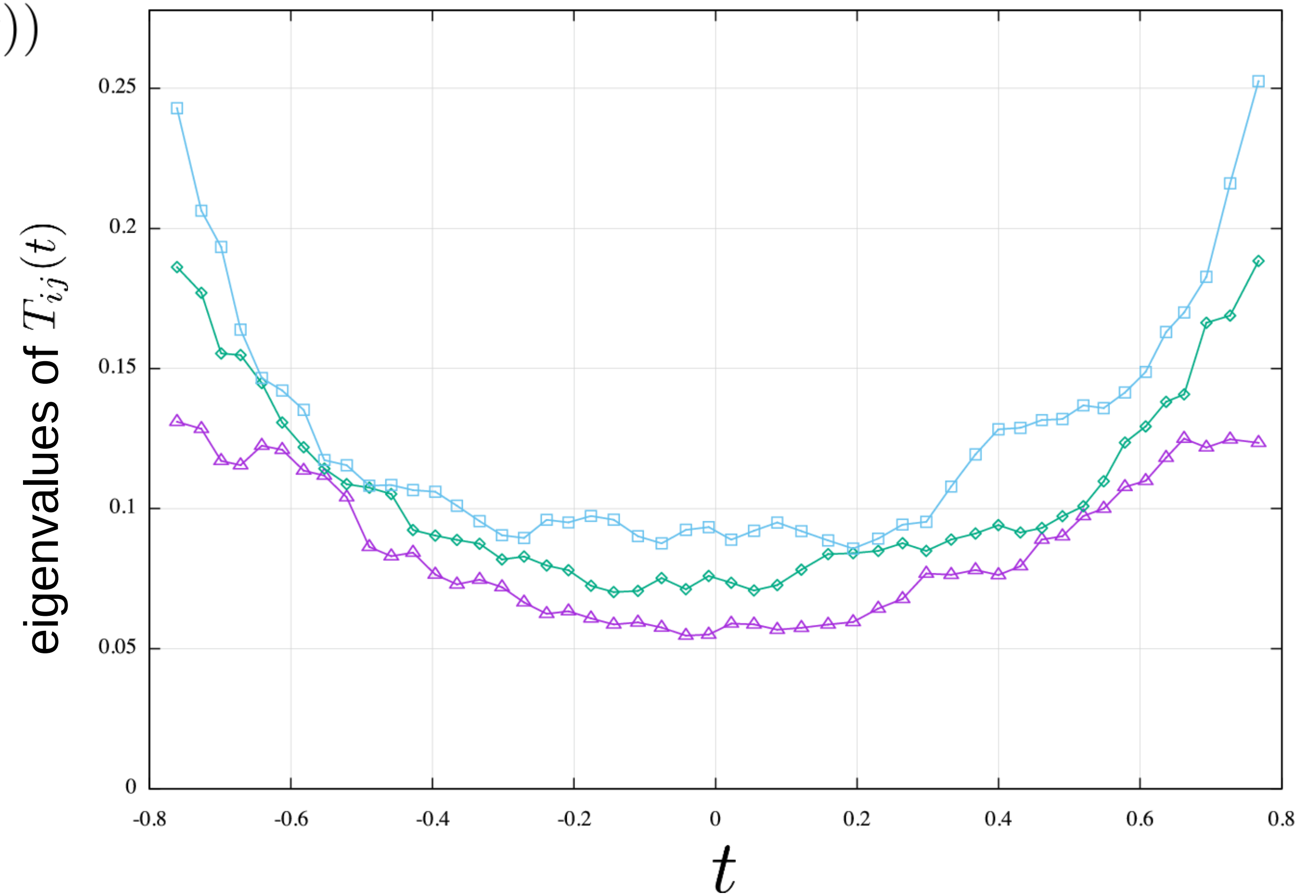


$$n = 16$$

Eigenvalues of Tij

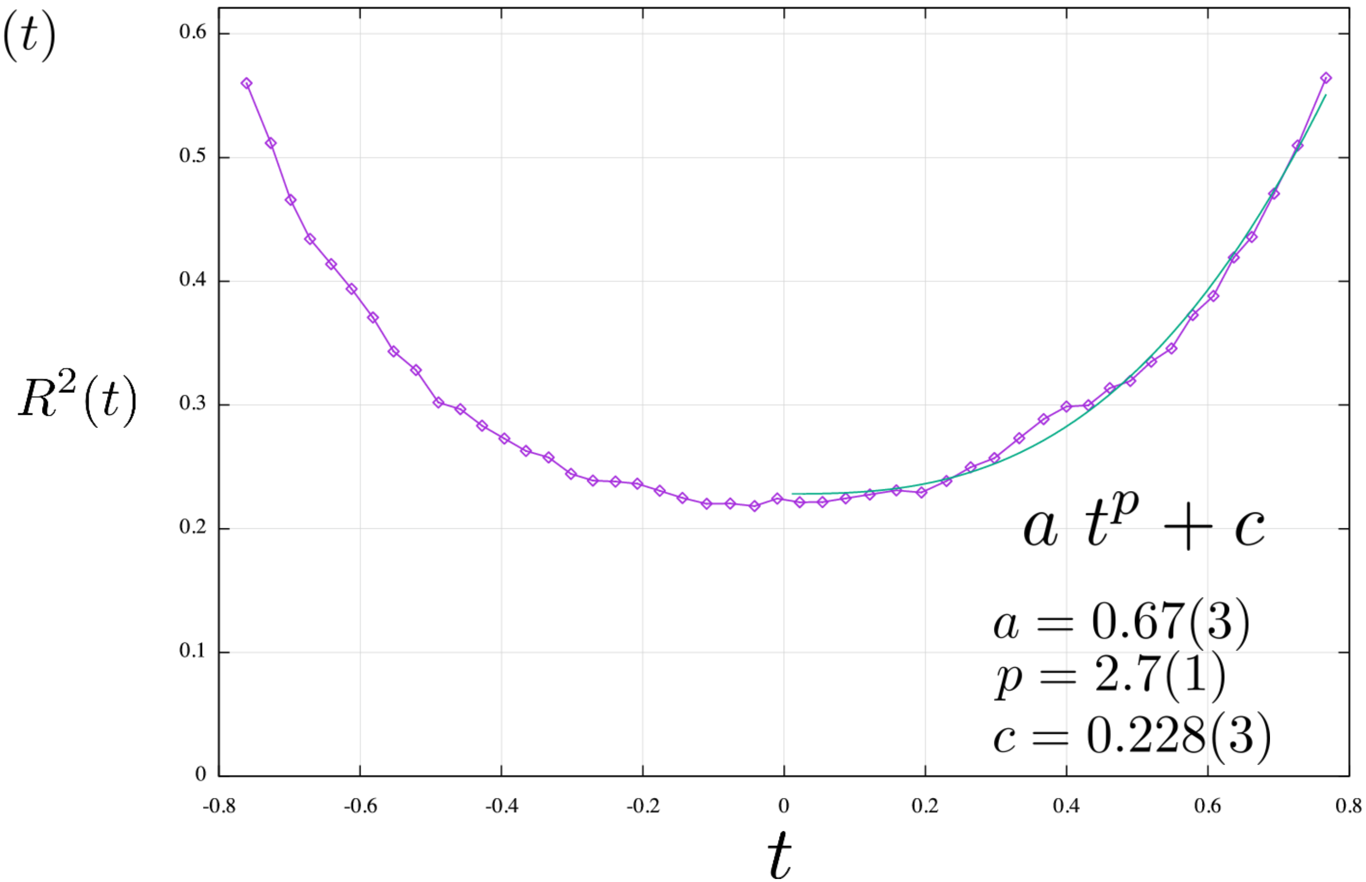
$$T_{ij}(t) = \frac{1}{n}(\bar{X}_i(t)\bar{X}_j(t))$$

SO(3) symmetric



R^2(t)

$$R^2(t) = \frac{1}{n}\mathrm{Tr}\bar{X}_i^2(t)$$
$$= T_{ii}(t)$$



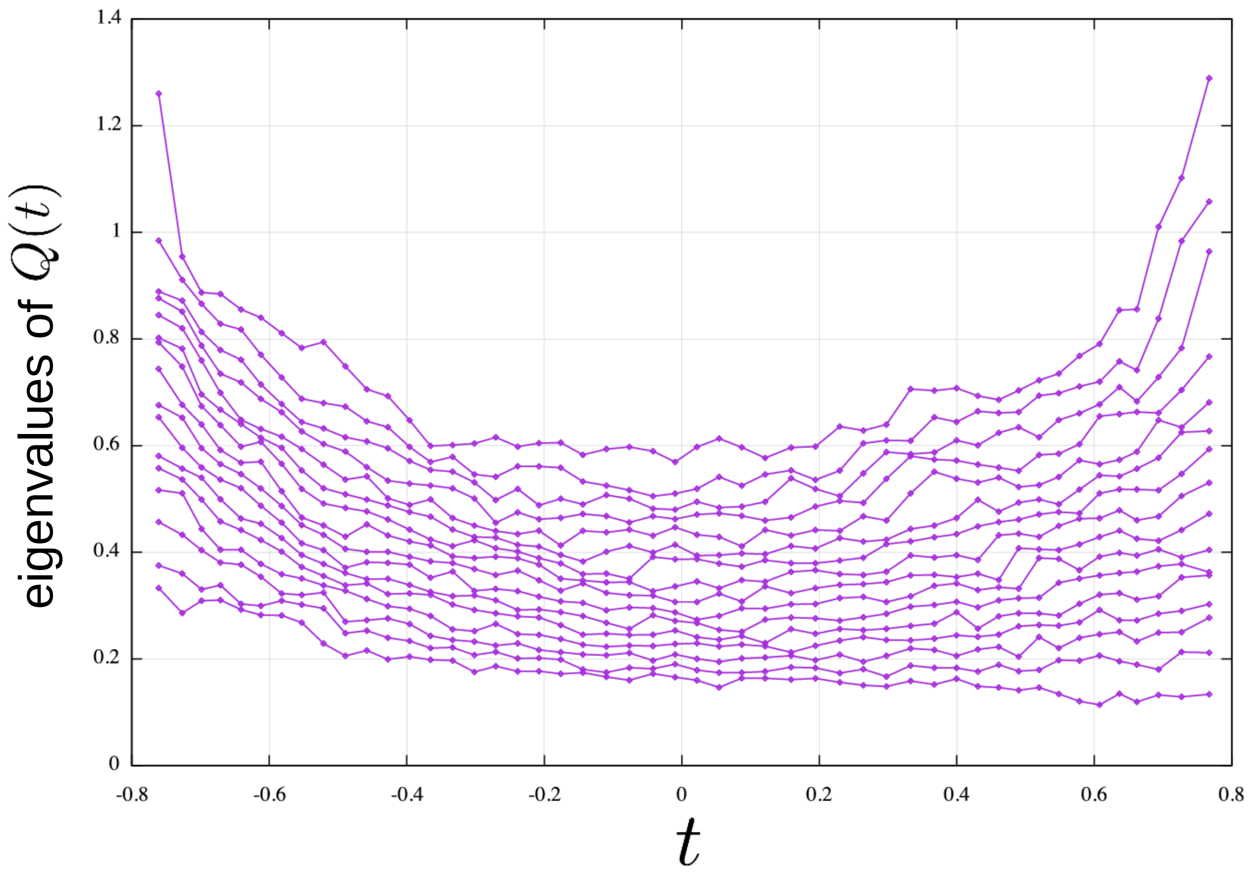
Power-law expansion

Space-time structure

$$Q(t) = \sum_{i=1}^{3} \bar{X}_i(t)^2$$

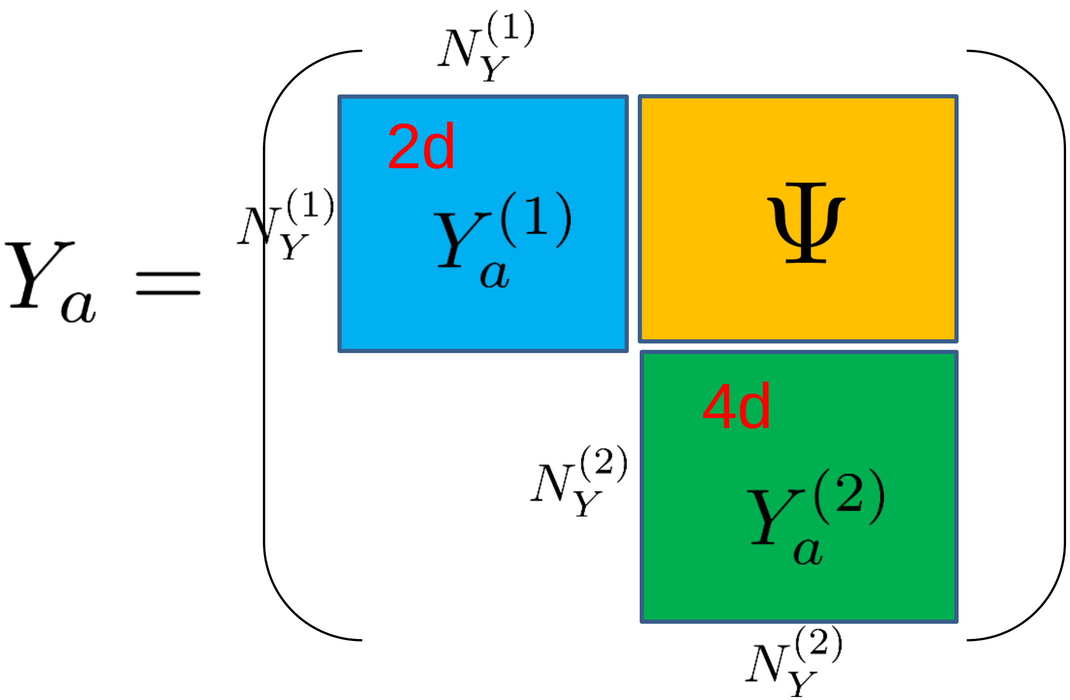
dense distribution

⇨ smooth manifold



2d-4d ansatz

2d manifold and 4d manifold intersects at points



$$Y_1^{(1)} \neq 0, \quad Y_2^{(1)} \neq 0$$

$$Y_3^{(1)} = Y_4^{(1)} = Y_5^{(1)} = Y_6^{(1)} = 0$$

$$Y_1^{(2)} = Y_2^{(2)} = 0$$

$$Y_3^{(2)} \neq 0, \quad Y_4^{(2)} \neq 0$$

$$Y_5^{(2)} \neq 0, \quad Y_6^{(2)} \neq 0$$

2d-4d ansatz

$$[Y_b^{(1)}, [Y_b^{(1)}, Y_a^{(1)}]] - Y_a^{(1)} = 0$$

$$\Rightarrow \quad Y_1^{(1)} = L_1, \quad Y_2^{(1)} = L_2$$

$$[L_i, L_j] = i\epsilon_{ijk} L_l$$

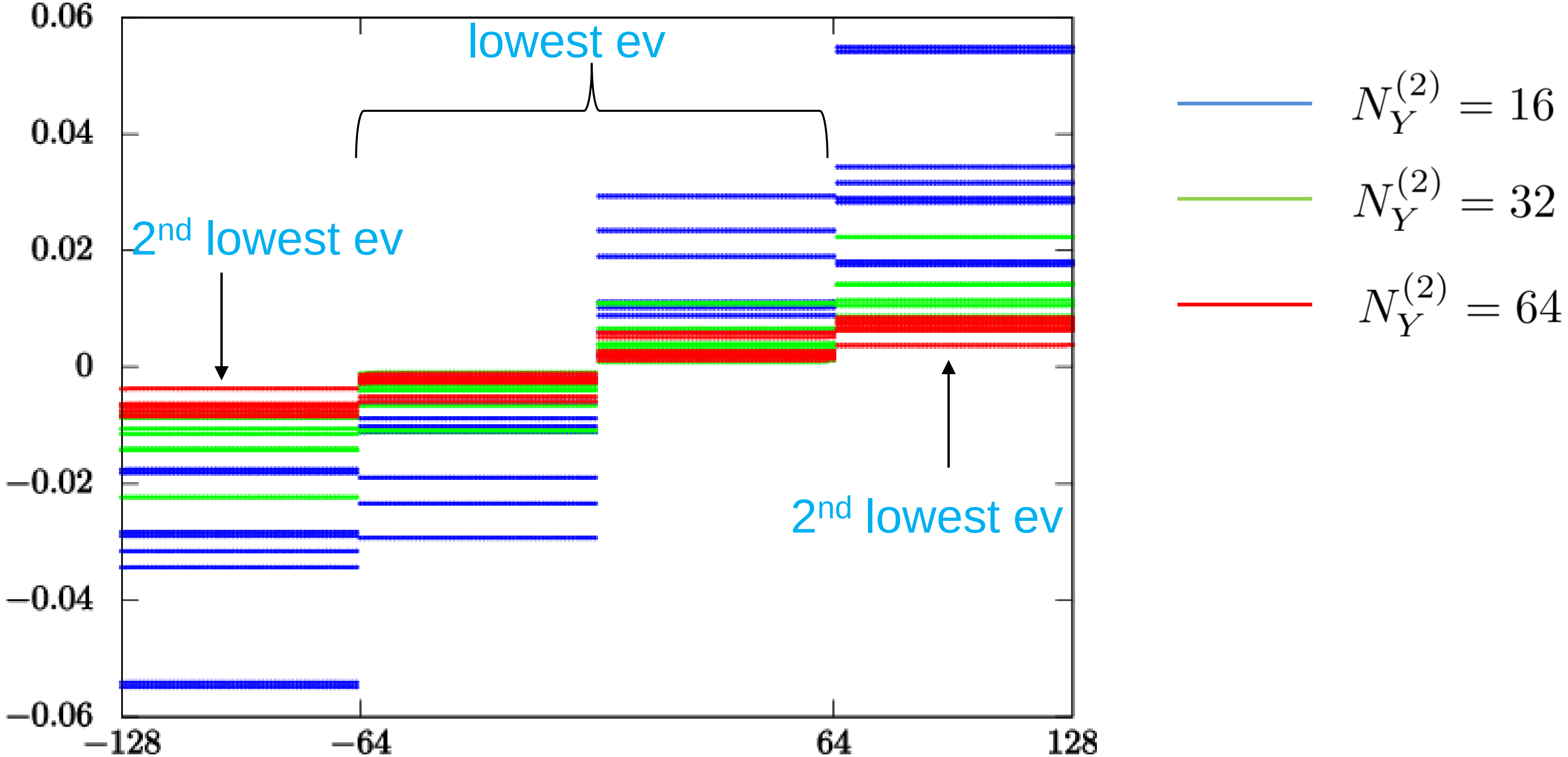
Generators of SU(2)

We solve $\Gamma^a (Y_a^{(1)} \Psi - \Psi Y_a^{(2)}) = \lambda \Psi$

$Y_a^{(1)}$	$Y_a^{(2)}$
1) $1^7 \oplus 2^{18} \oplus 3^7$	i) 8 solutions at $N_Y^{(2)} = 16$
2) 64^1	ii) 8 solutions at $N_Y^{(2)} = 32$
$(N_Y^{(1)} = 64)$	iii) 8 solutions at $N_Y^{(2)} = 64$

Spectrum of 6d Dirac operator

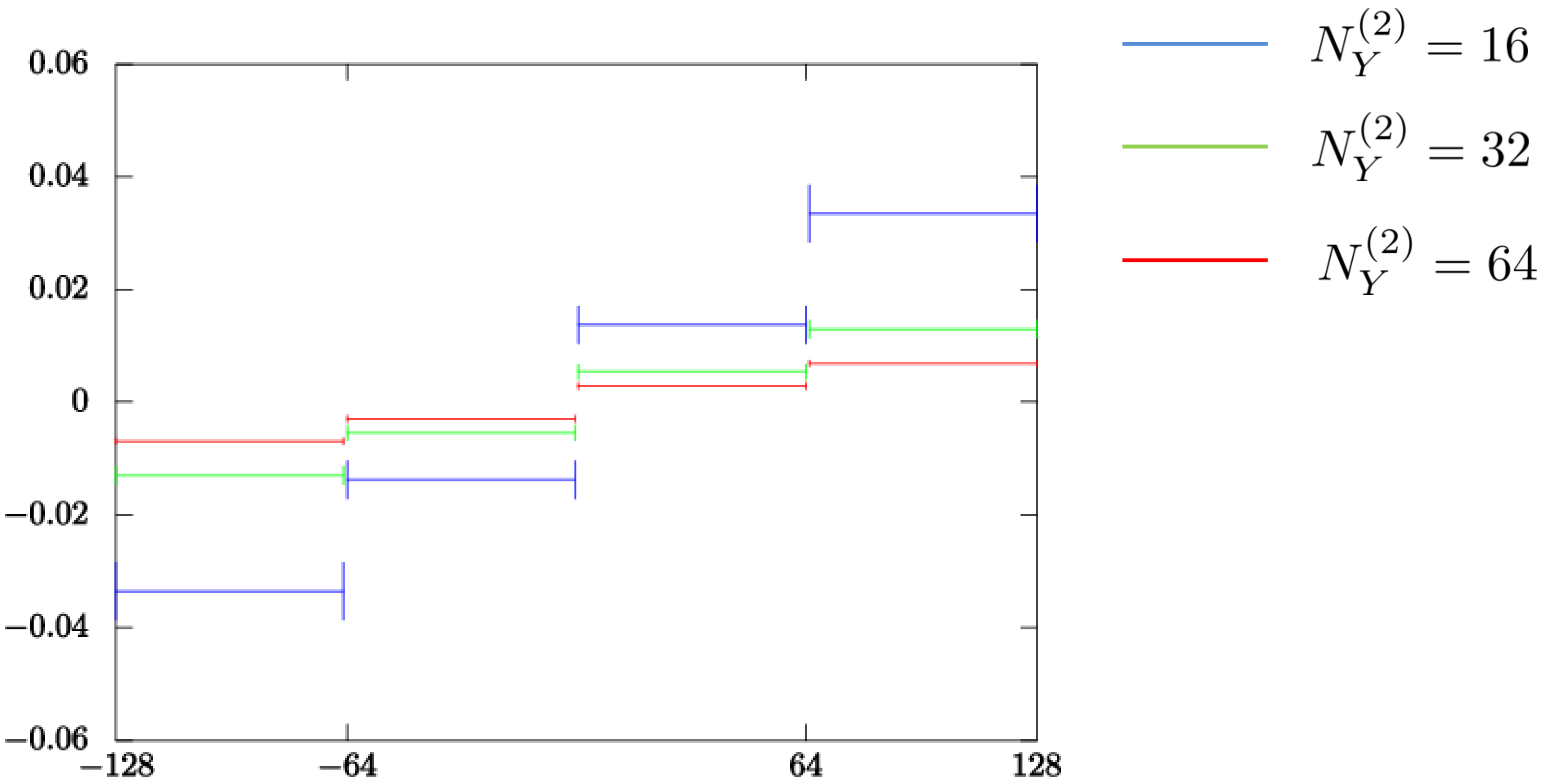
1) $1^7 \oplus 2^{18} \oplus 3^7$



We plot only 256 eigenvalues out of 32768 ones

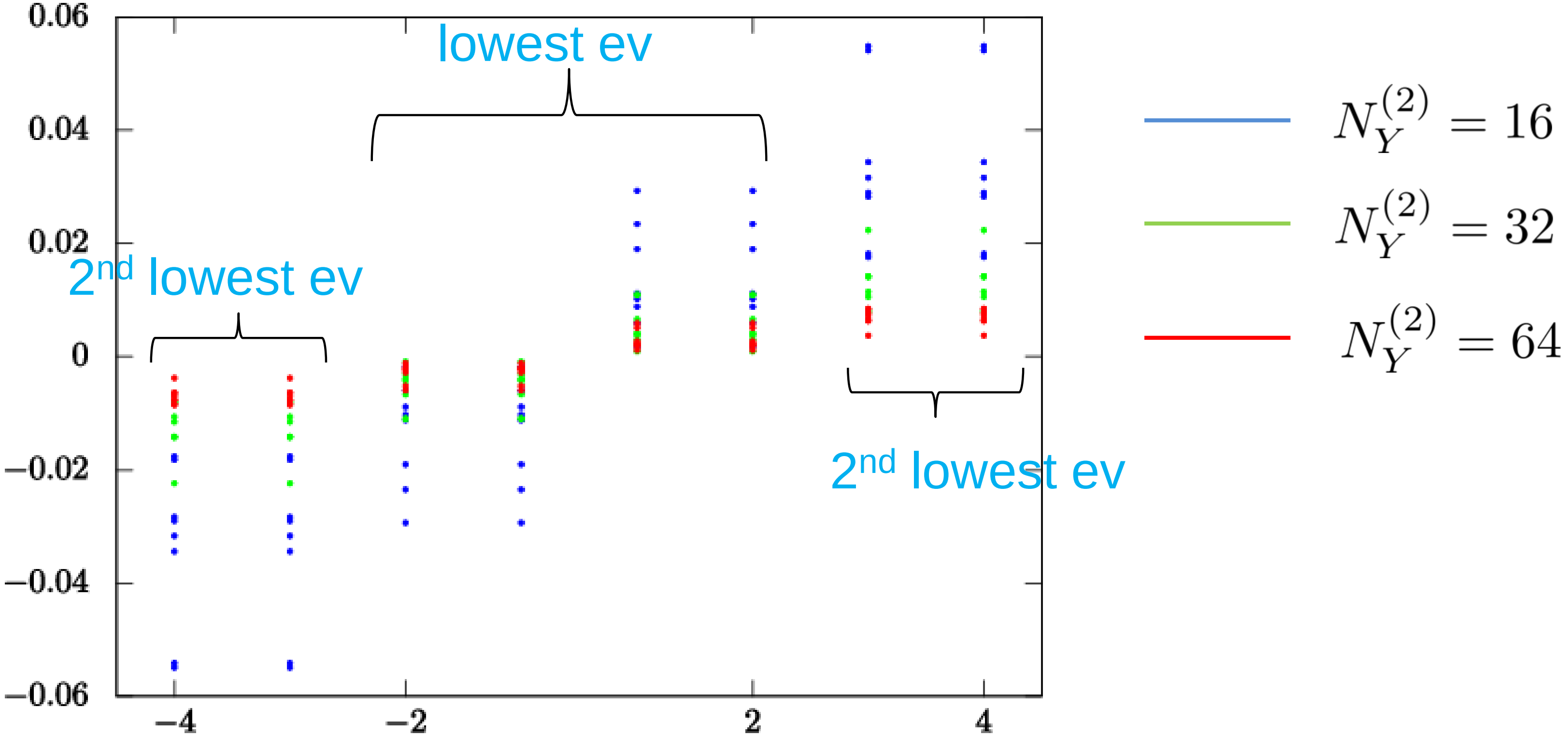
Spectrum of 6d Dirac operator

1) $1^7 \oplus 2^{18} \oplus 3^7$ Average of 8 solutions



Spectrum of 6d Dirac operator

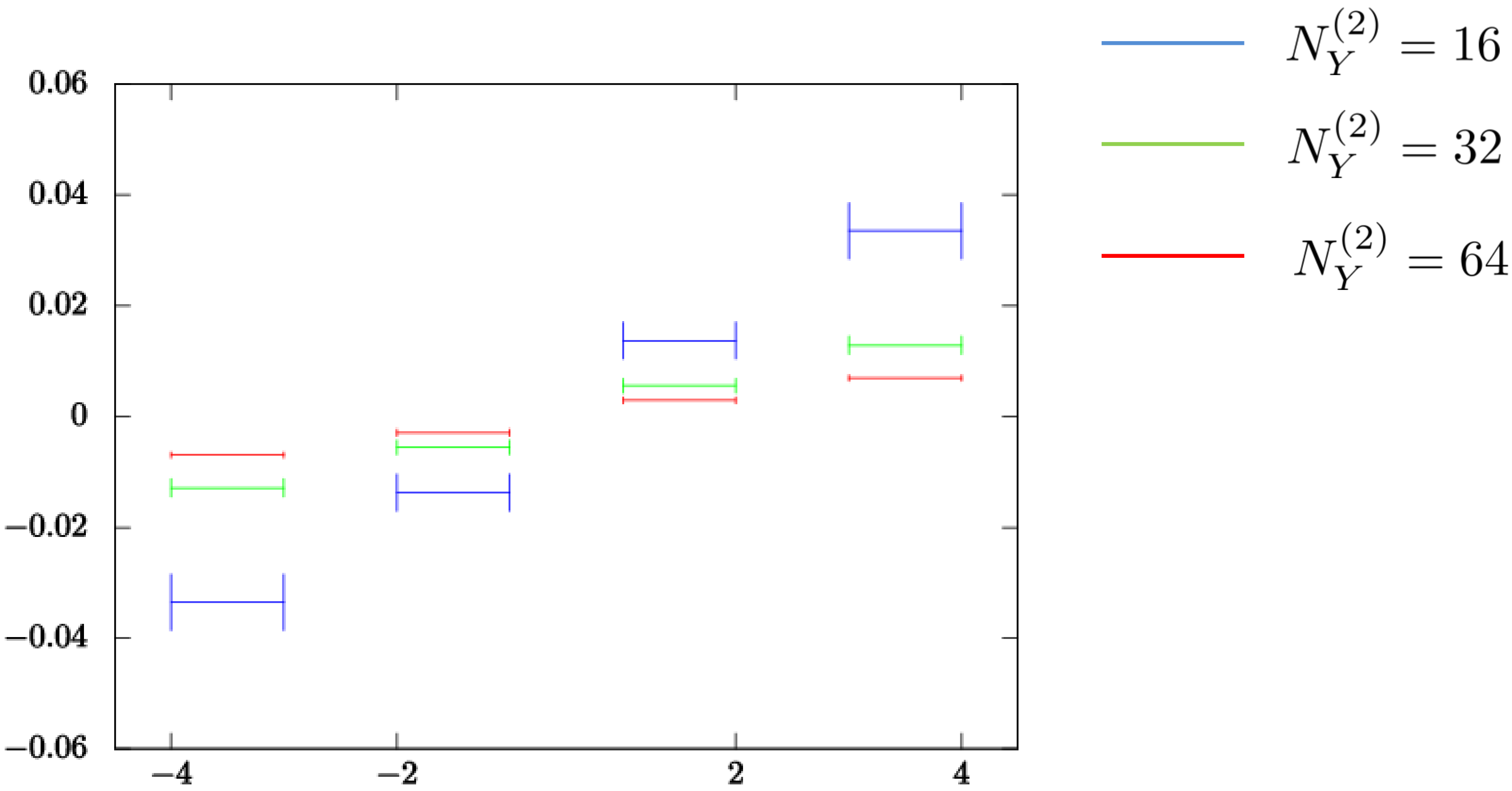
2) 64^1



We plot only 8 eigenvalues out of 32768 ones

Spectrum of 6d Dirac operator

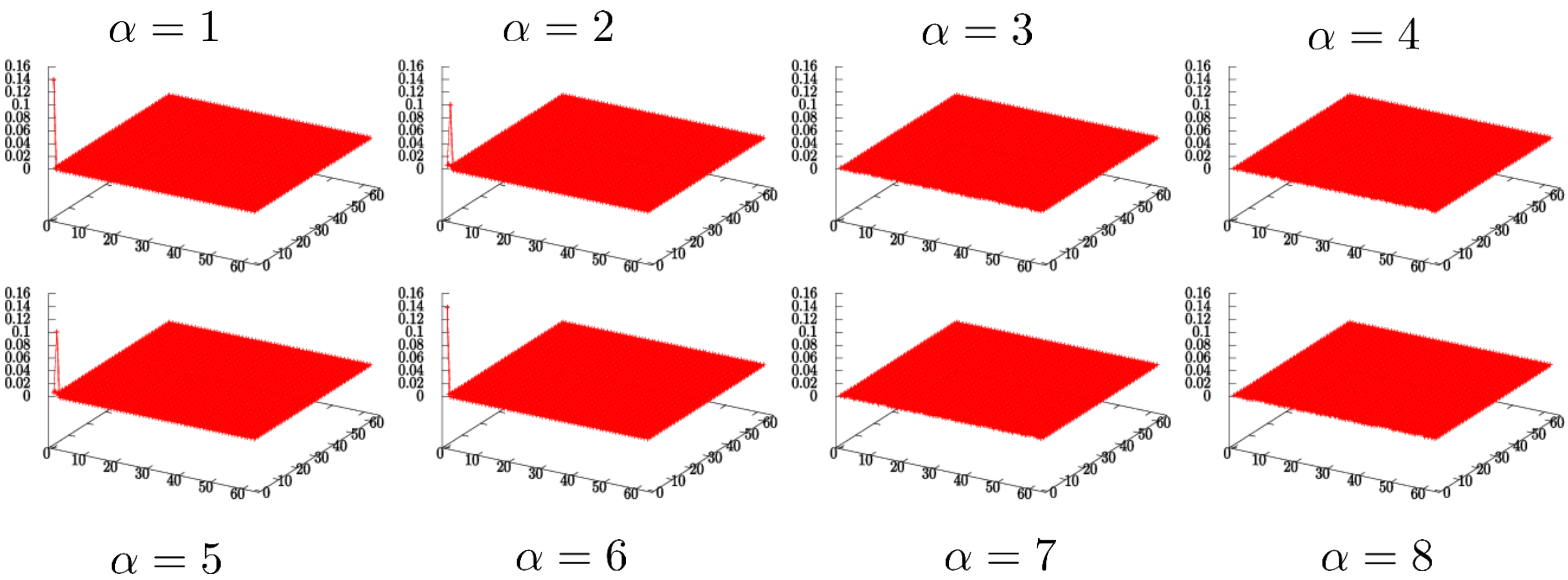
2) 64^1 Average of 8 solutions



Profile of wave function for lowest ev

1) $1^7 \oplus 2^{18} \oplus 3^7 \qquad N_Y^{(2)} = 64$

$\Psi'_\alpha = U \Psi_\alpha V^\dagger$ SVD for $\alpha = 1$



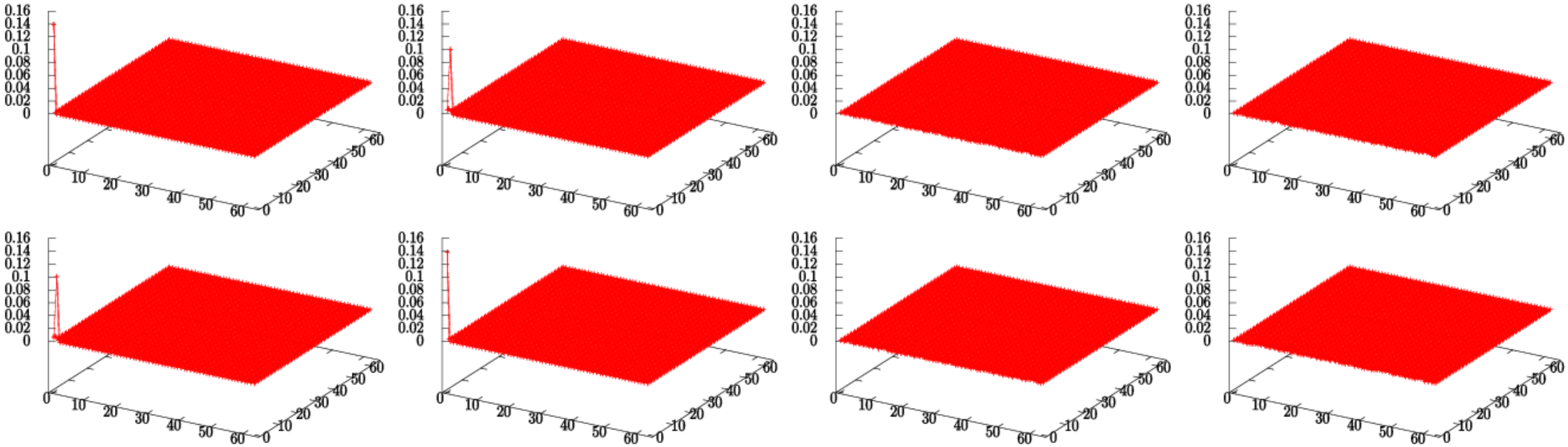
Localized ! → Intersecting at a point

Profile of wave function for lowest ev

2) 64^1 $N_Y^{(2)} = 64$

$\Psi'_\alpha = U \Psi_\alpha V^\dagger$ SVD for $\alpha = 1$

$\alpha = 1$ $\alpha = 2$ $\alpha = 3$ $\alpha = 4$



$\alpha = 5$ $\alpha = 6$ $\alpha = 7$ $\alpha = 8$

Localized ! Intersecting at a point

Conclusion and discussion

Conclusion

- We developed a numerical method to search for classical solutions satisfying the most general ansatz with “quasi direct product structure”. It works well.
- Solutions in general give expanding (and shrinking) (3+1)d space-times, which have smooth structure. Expansion seems to obey power-law.
- Quasi direct product structure favors block-diagonal structure which can yield intersecting branes in extra dimensions. One can obtain chiral zero modes in 6d at intersecting points, which can lead to the chiral fermions in (3+1) dimensions.
- What is important is that chiral zero modes are obtained as solutions of EOM.

Cf.) Aoki(’11) A. Chatzistavrakidis, H. Steinacker and G. Zoupanos (‘11)
Nishimura-A.T.(’13) Aoki-Nishimura-A.T.(’14)

Discussion

- We obtained 128(=4x(7+18+7)) zero modes for $1^7 \oplus 2^{18} \oplus 3^7$ and 4 zero modes for 64^1
  4 zero modes for each brane in 2d?

- We need to further examine dependence of lowest and 2nd lowest eigenvalues on $N_Y^{(1)}, N_Y^{(2)}$ and SU(2) representations.

- Profile of D-branes and geometry of extra dimensions

  Berenstein-Dzienkowski ('12), Ishiki ('15), Schneiderbauer-Steinaker ('16)

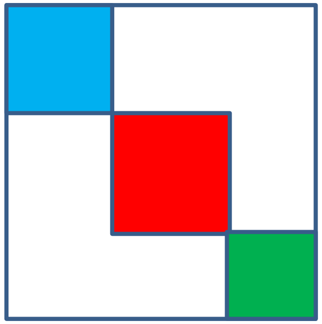
  Gutleb's talk

Discussion

➢ Only 3 blocks?

Indeed, to realize the Standard Model, more blocks seems to be needed.

(1) structure of blocks within a block is allowed for a classical solution, but seems non-generic.
Quantum effect might favor such a structure.



(2) We can generalize IR cutoffs as follows:

$$\frac{1}{N}\mathrm{Tr}((A_0^2)^p) = \kappa \qquad \frac{1}{N}\mathrm{Tr}((A_i^2)^p) = 1$$

We took p=1 in this talk for simplicity.
For p=2, arbitrary number of blocks are naturally obtained, because no constraints are obtained from $M^3 = M^3$
Indeed, p >1 seems to be required from universality
Azuma-Ito-Nishimura-A.T. ('17)

Discussion

- Where left-right asymmetry comes from?

  Indeed, wave functions for the left and right modes are different:

  (1) from Yukawa coupling.

  we need to calculate coupling of zero modes to Higgs, which comes from fluctuation of Y_a

  (2) realized in more nontrivial solution having structure as

$$[M, Y_a] \neq 0$$

  action of M on left and right modes are different

  Nishimura-A.T.('13) Aoki-Nishimura-A.T.('14)

- Gauge groups?

  seem to come from a stack of multiple D-branes

  ~ identical blocks within a block

  ~ favored by quantum effect?

Outlook

- We search for solutions by starting with various initial configurations to understand the variety of solutions.
- We expect that there exists a solution that realizes the Standard model or beyond the Standard model and that it is indeed selected in the sense that our Monte Carlo result is connected to such a solution.
- Or we can calculate 1-loop effective actions around classical solutions we have found. We expect the effective action for the solution giving SM or BSM to be minimum.

Outlook

- We perform numerical calculation at Nx ~ Ny ~1000 (N ~ 10^6) by using Kei or post-Kei supercomputers with large-scale parallel computation.
  It is doable since the computation is not more than simulating a bosonic matrix model, which has been done already with matrix size ~1000.