Some real-time aspects of quantum black hole

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QFT

Holography

=

Black
Hole

Holography

QFT = Black Hole

↑

For imaginary time, lattice simulation is powerful
and probably the only practical tool in generic situation.
(e.g. Danjoe's talk)

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Can we study the *real-time* dynamics?

We mainly consider **D0-brane matrix model** and **SYK model** in this talk.

de Wit-Hoppe-Nicolai; Witten; Banks-Fischler-Shenker-Susskind; Itzhaki-Maldacena-Sonnenschein-Yankielowicz

Sachdev-Ye; Kitaev

- Thermalization of BH from classical matrix model
- Evaporation of BH from quantum matrix model
- New universality in classical and quantum chaos

- In AdS/CFT, weak and strong couplings are often very similar.

- D0, D1, D2: weak coupling $\sim$ high temperature;

  $T\lambda^{-1/(3-p)}$ is dimensionless for Dp

  classical simulation can be useful.

- Studies of classical D0-brane matrix model suggested it is useful at least for thermalization and equilibrium physics.

Asplund, Berenstein, Trancanelli,…, 2011—

D0-brane quantum mechanics

$$S = \frac{N}{\lambda}\int_0^{\beta=1/T} dt\, Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi]\Big\}$$

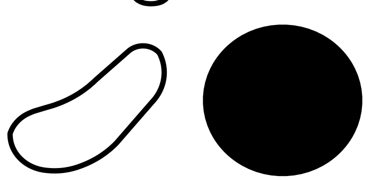
negligible at high-T

(dimensional reduction of 4d N=4 SYM)

effective dimensionless temperature $T_{eff} = \lambda^{-1/3} T$

($\lambda^{-1/2}T$ for D1, $\lambda^{-1}T$ for D2)

high-T = weak coupling = stringy (large α' correction)

string  BH

$$L = \frac{1}{2g_{YM}^2} \mathrm{Tr} \left(\sum_i (D_t X^i)^2 + \frac{1}{2} \sum_{i \neq j} [X_i, X_j]^2 \right)$$

$$\longrightarrow \begin{cases} \dfrac{d^2 X^i}{dt^2} - \sum_j [X^j, [X^i, X^j]] = 0 \\ \sum_i \left[X^i, \dfrac{dX^i}{dt} \right] = 0 \end{cases}$$

(A=0 gauge)

discretize & solve it numerically.

black p-brane solution
(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N} U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$$T_{D0} = \frac{7}{4\pi\sqrt{d_0 \lambda}} U_0^{\frac{5}{2}}$$

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$>>1$ at $U=U_0$ for low-T

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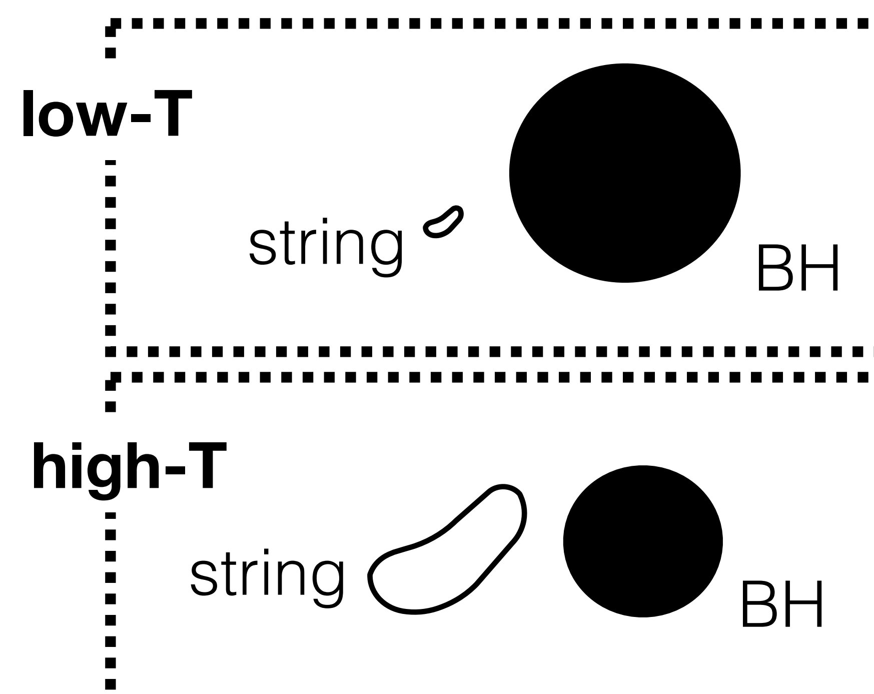

Matrix Model 101

- Flat directions at classical level $[X_M, X_{M'}] = 0$
- Lifted by quantum effect (when fermion is negligible)

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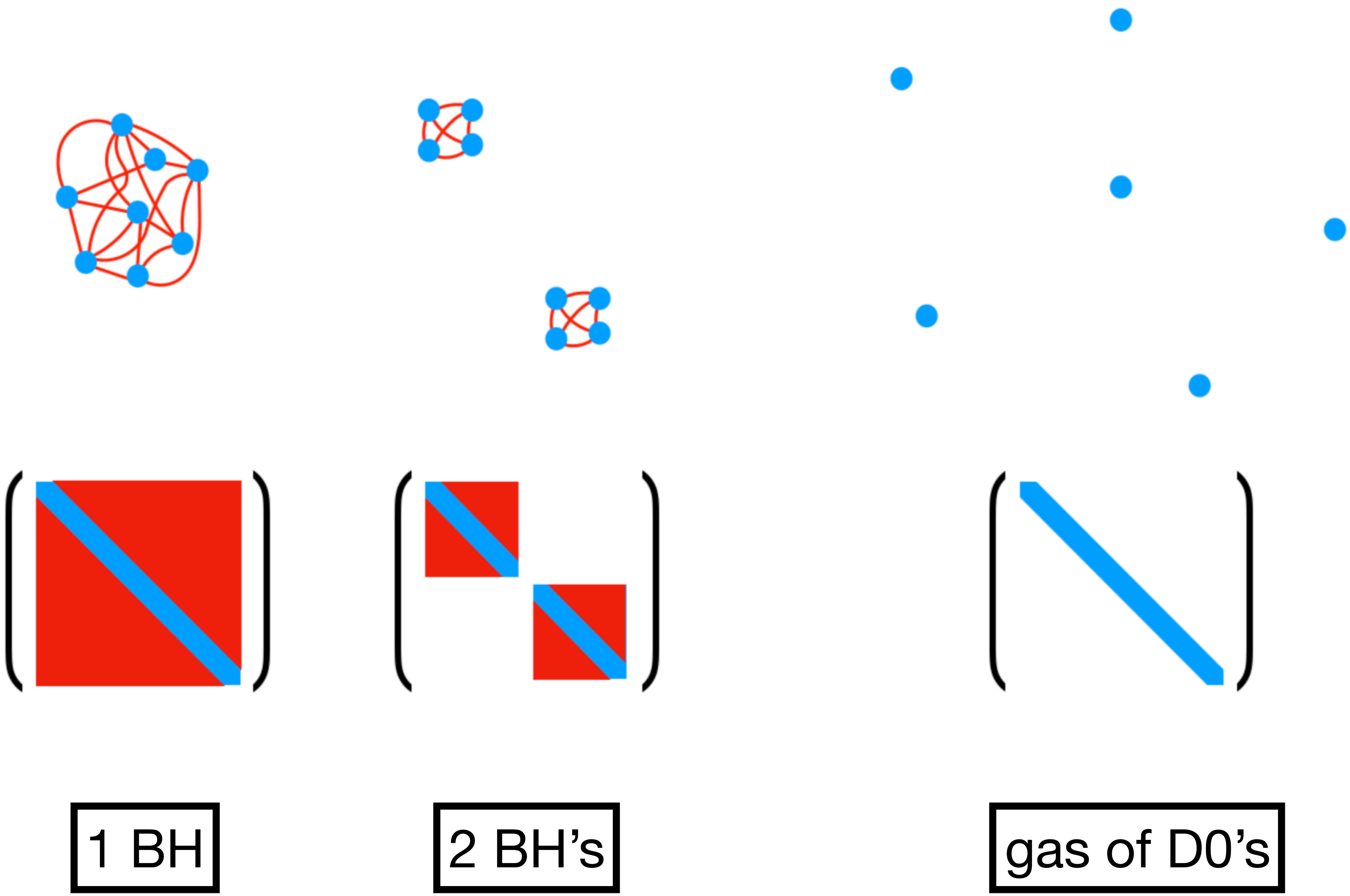
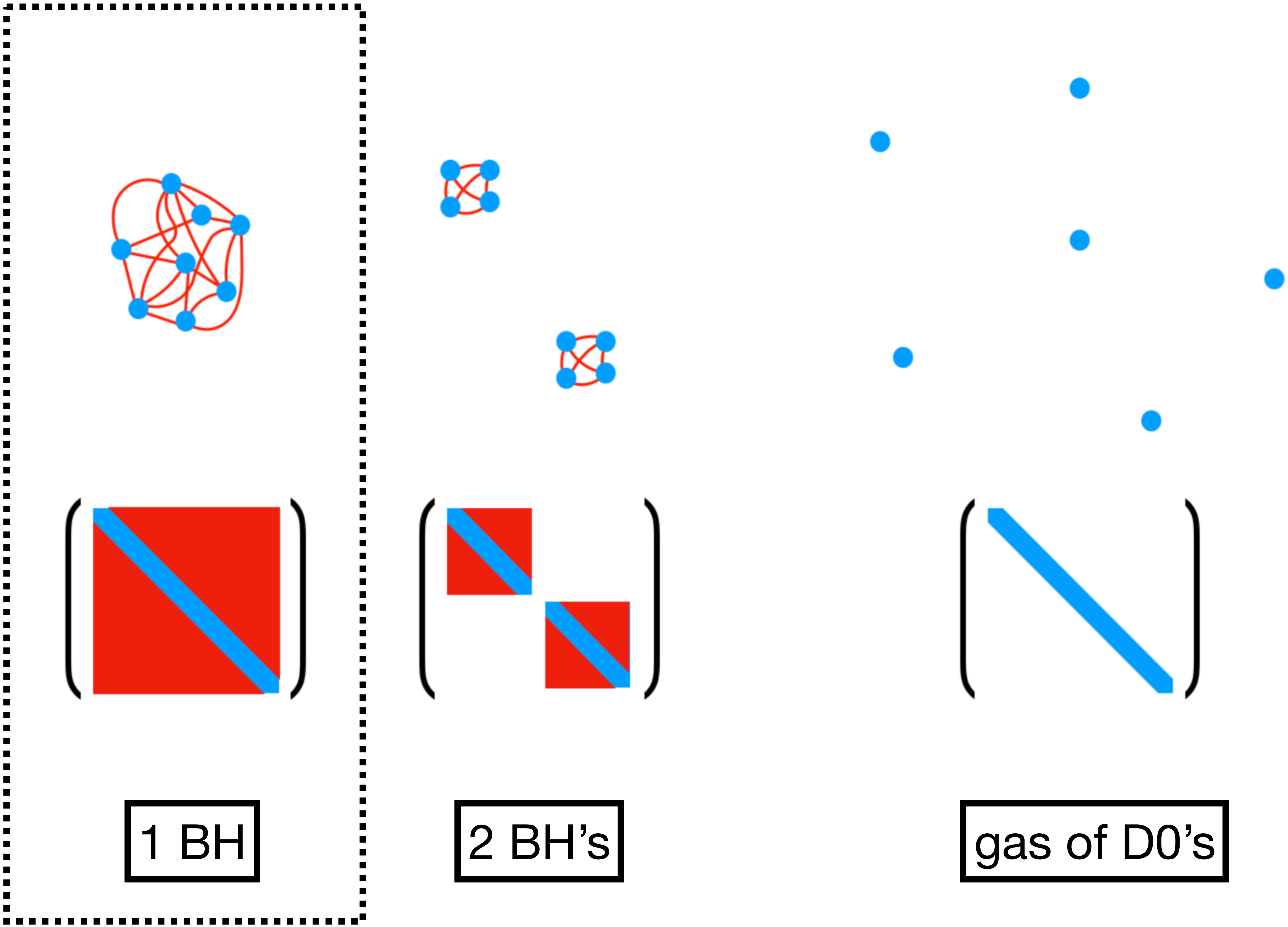
Flat direction is measure zero already in the classical theory

(Gur Ari-MH-Shenker; Berkowitz-MH-Maltz)

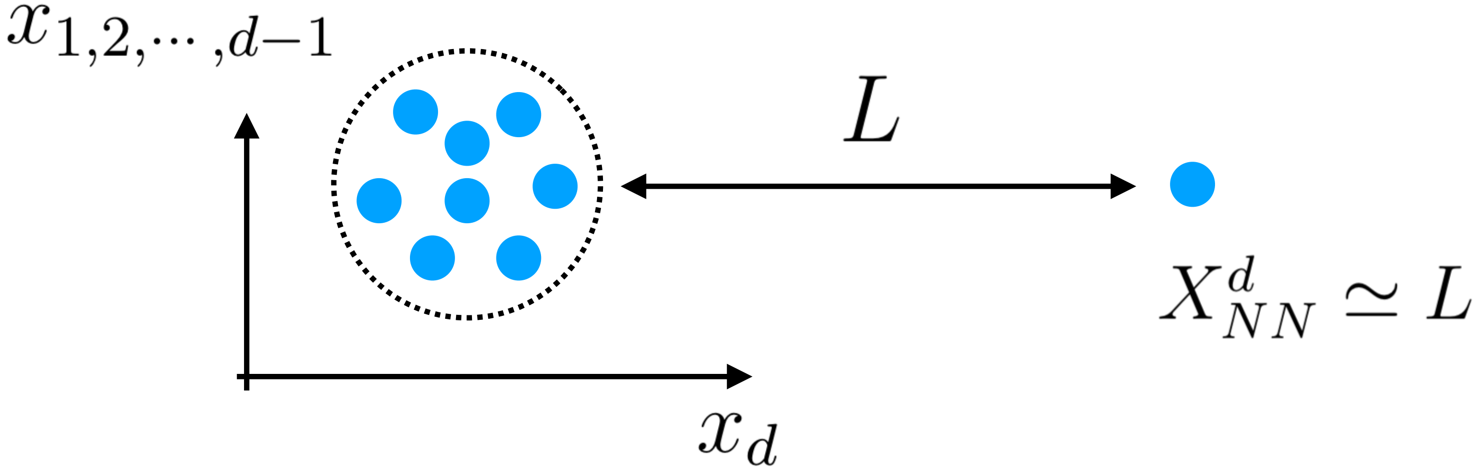
(also, probably Savvidy and Berenstein knew it)


1 BH
2 BH's
gas of D0's

Let’s study this one.



Why no flat direction?



energy of *N*-th row & column $\sim \frac{1}{g^2}\sum_{i=1}^{d-1}\sum_{a=1}^{N-1} L^2 |X^i_{aN}|^2$

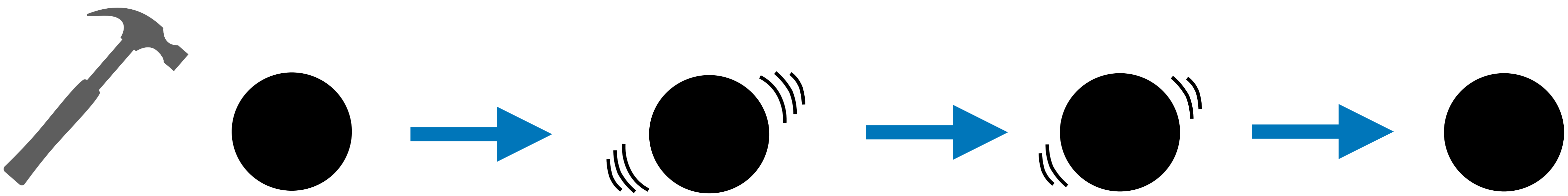
phase space suppression $\sum_{i=1}^{d-1}\sum_{a=1}^{N-1} |X^i_{aN}|^2 \lesssim g^2 E/L^2$

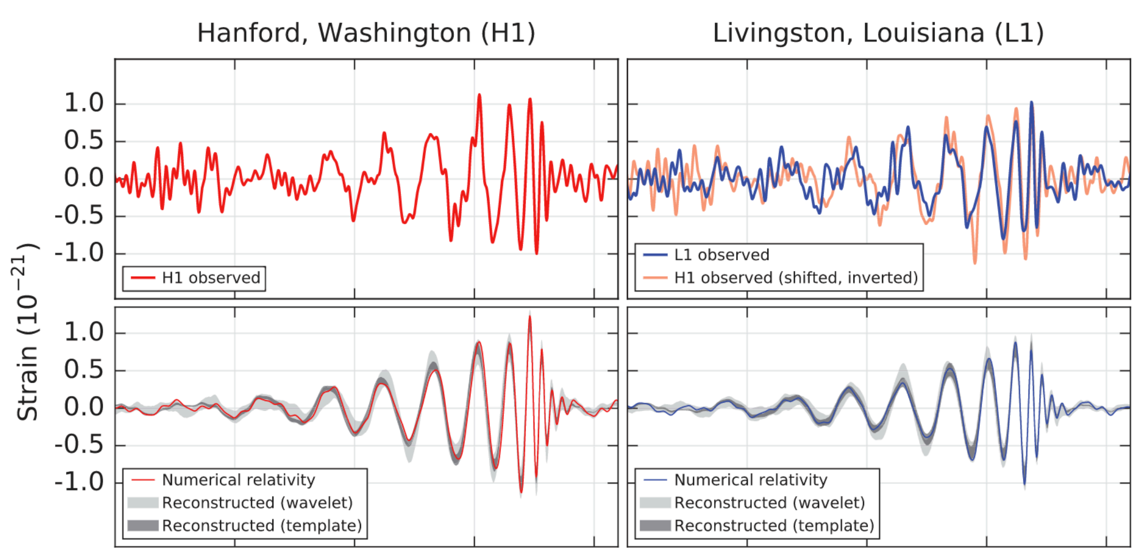
phase space volume at $L > L_0$

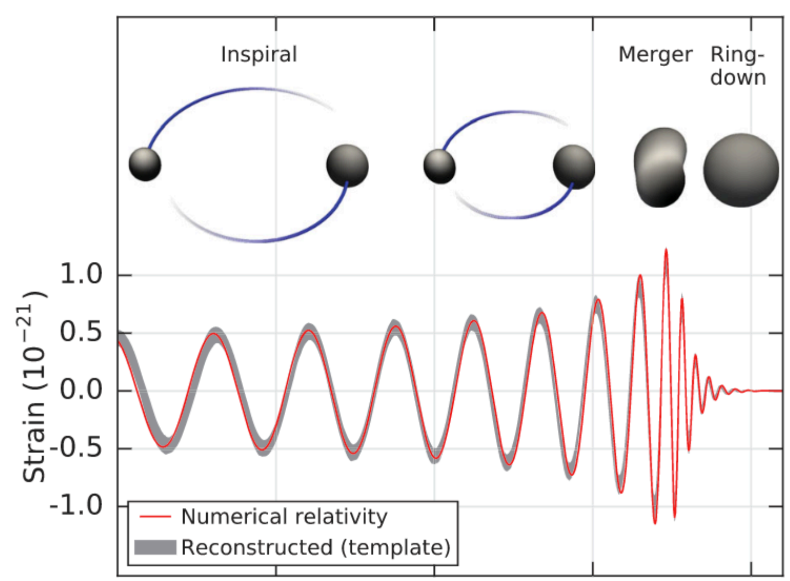
$$\int_{L_0}^{\infty} \frac{L^{d-1}dL}{L^{2(d-1)(N-1)}} \sim \int_{L_0}^{\infty} \frac{dL}{L^{(d-1)(2N-3)}}$$

Finite. (exception: *d*=2, *N*=2)

Quasinormal mode





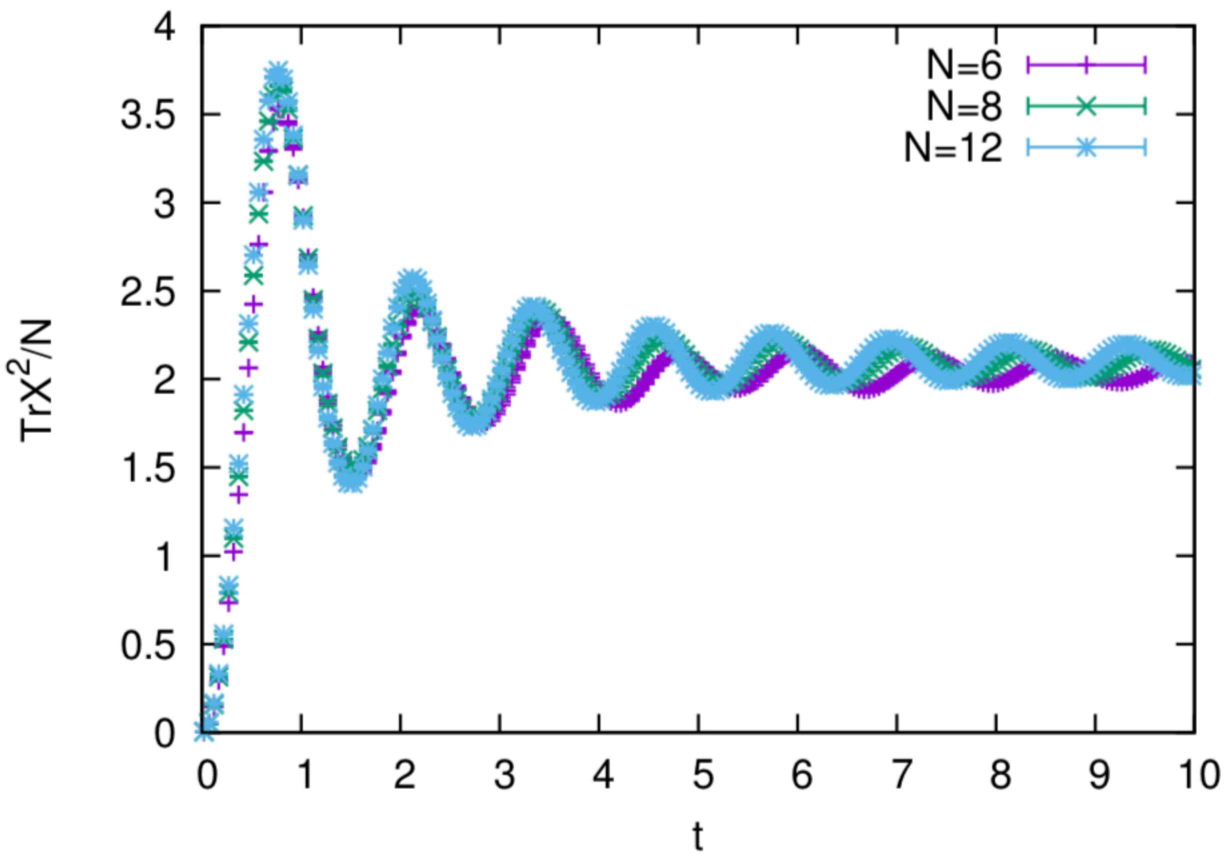


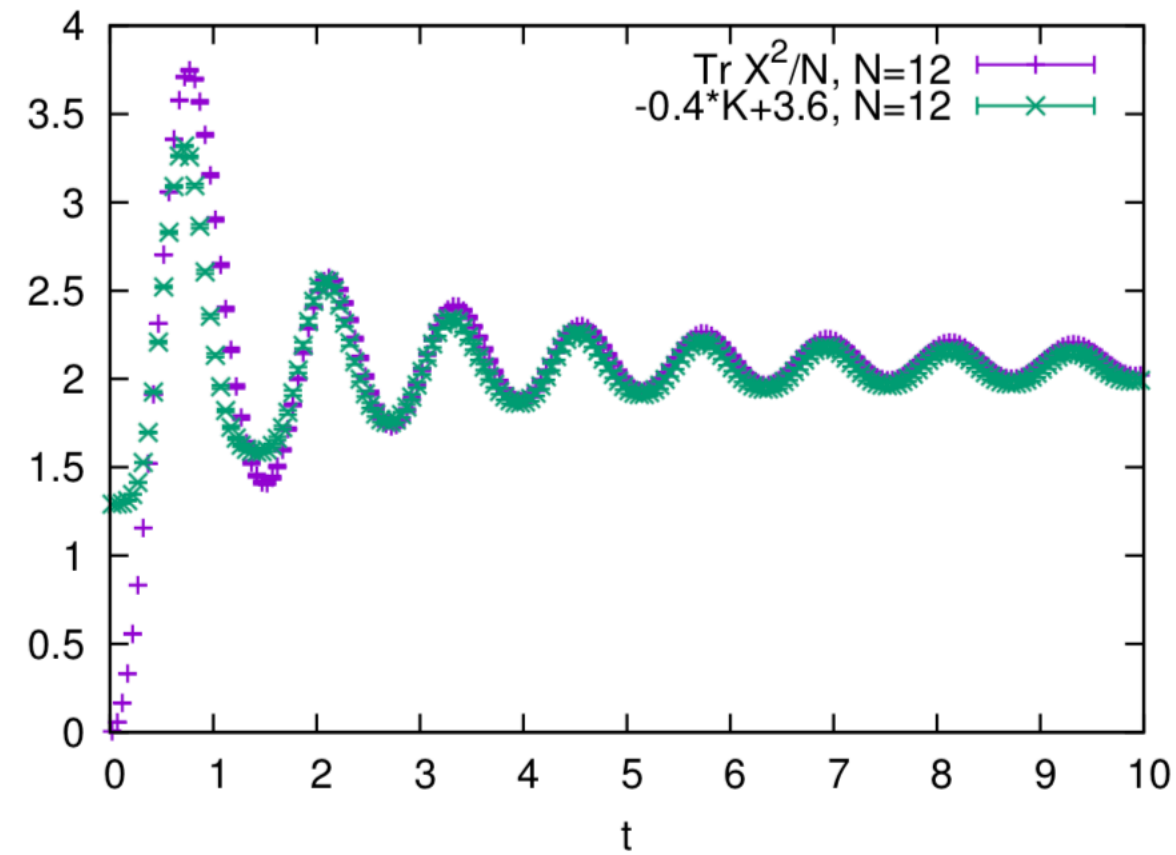
(LIGO Scientific Collaboration and Virgo Collaboration, 2016)

Quasinormal mode

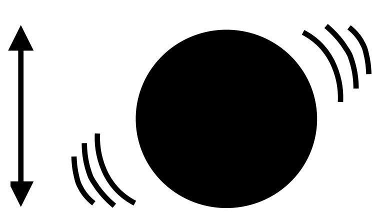
Aoki-MH-Iizuka
MH-Romatschke

$X_M = 0, \dot{X}_M \neq 0$ —thermalize→ generic configuration





$\sqrt{\frac{1}{N}\mathrm{Tr}X^2}$



$$\mathrm{Re}(e^{i\nu t}) \sim \cos(\nu_R t)e^{-\nu_I t}$$

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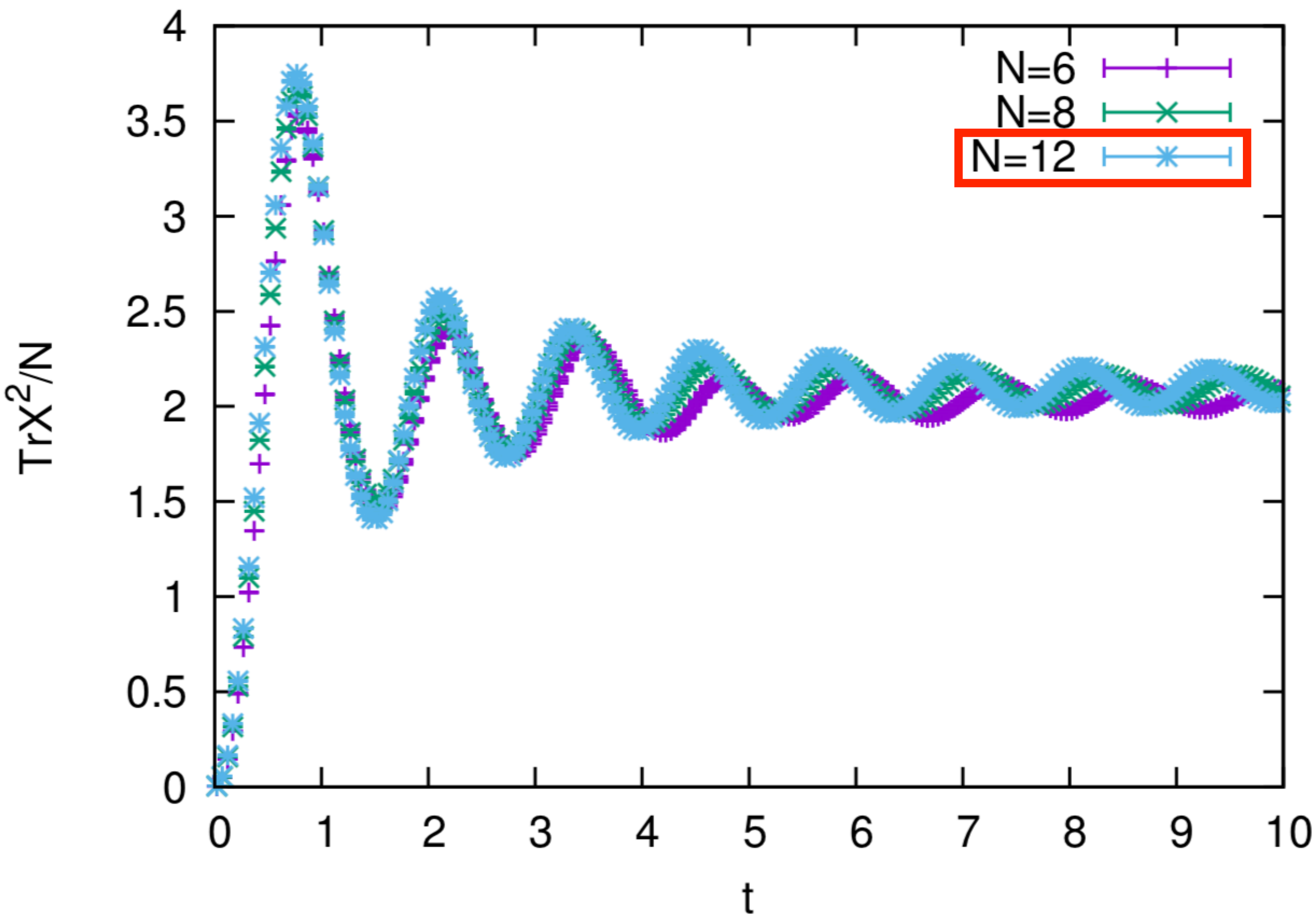

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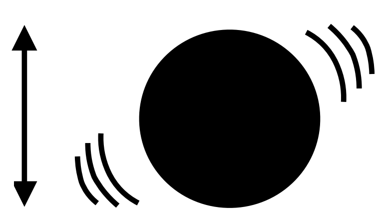
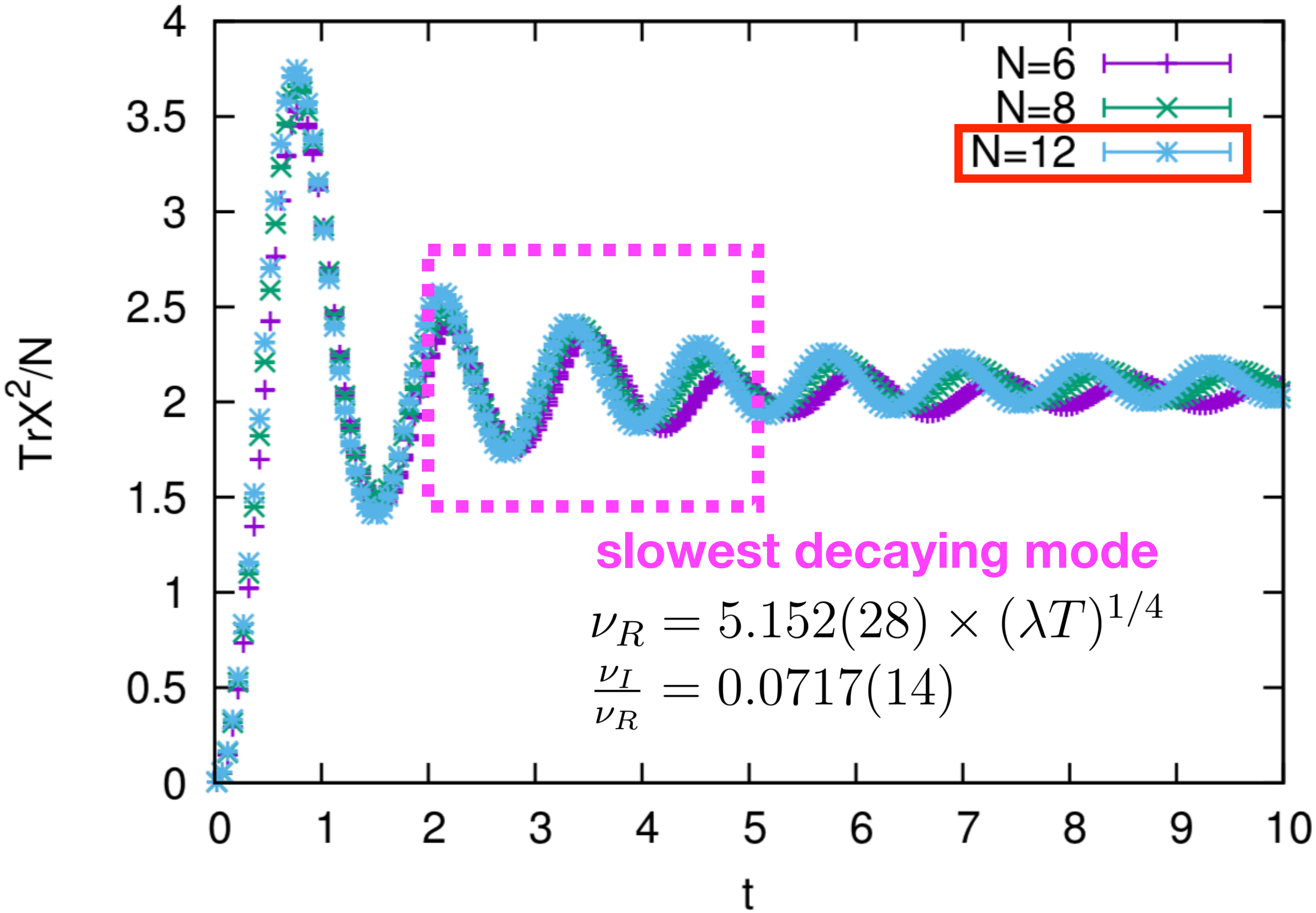


MH-Romatschke

$$\mathrm{Re}(e^{i\nu t}) \sim \cos(\nu_R t) e^{-\nu_I t}$$



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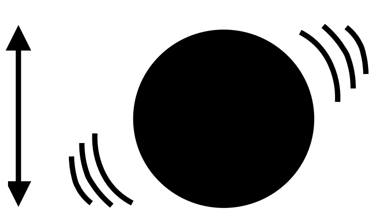

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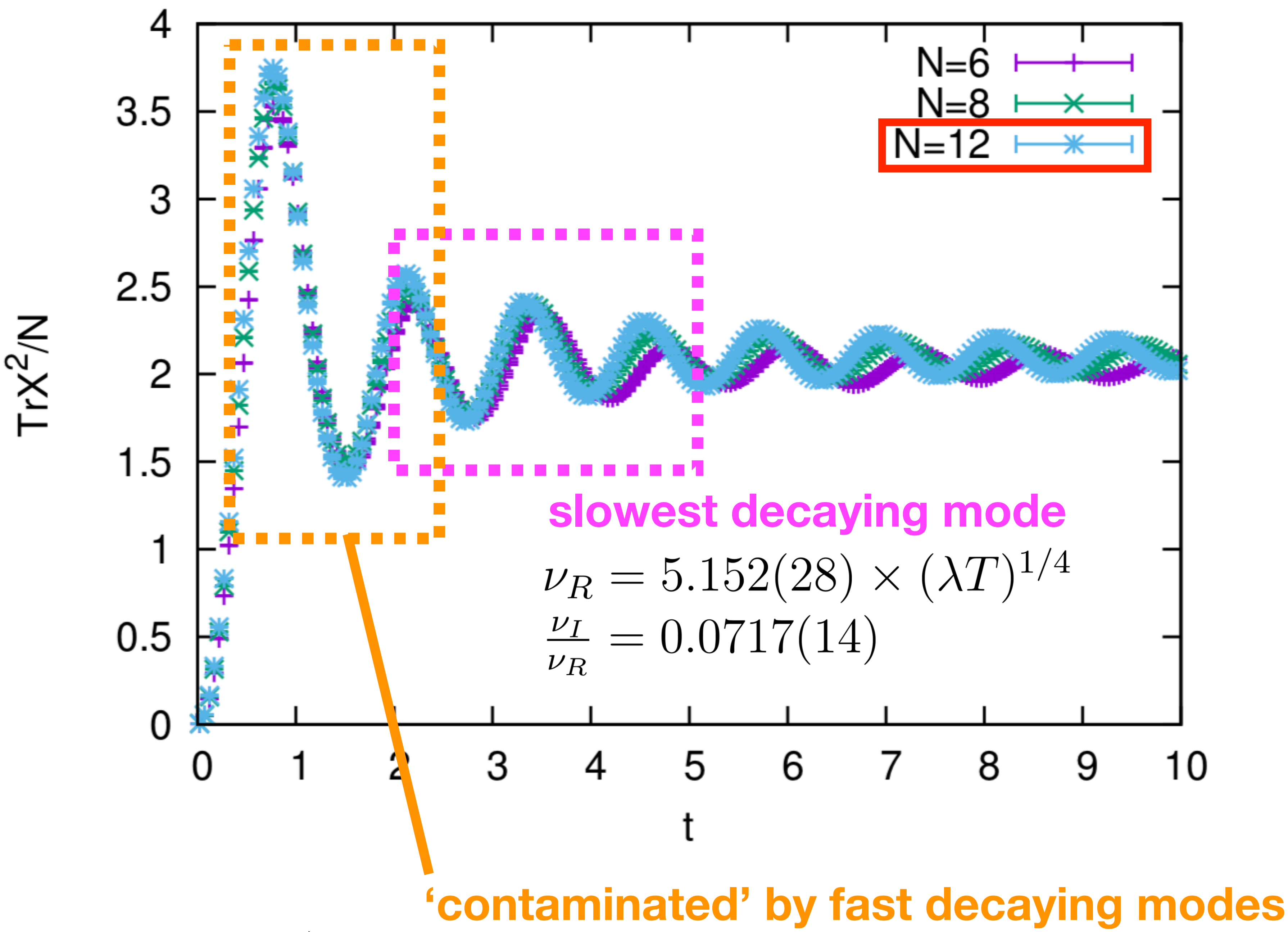
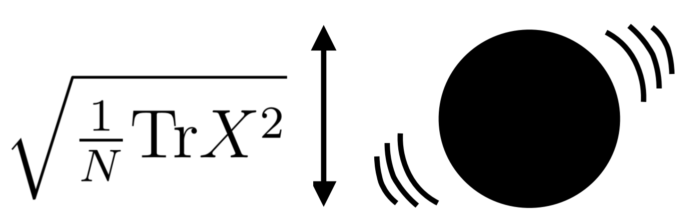


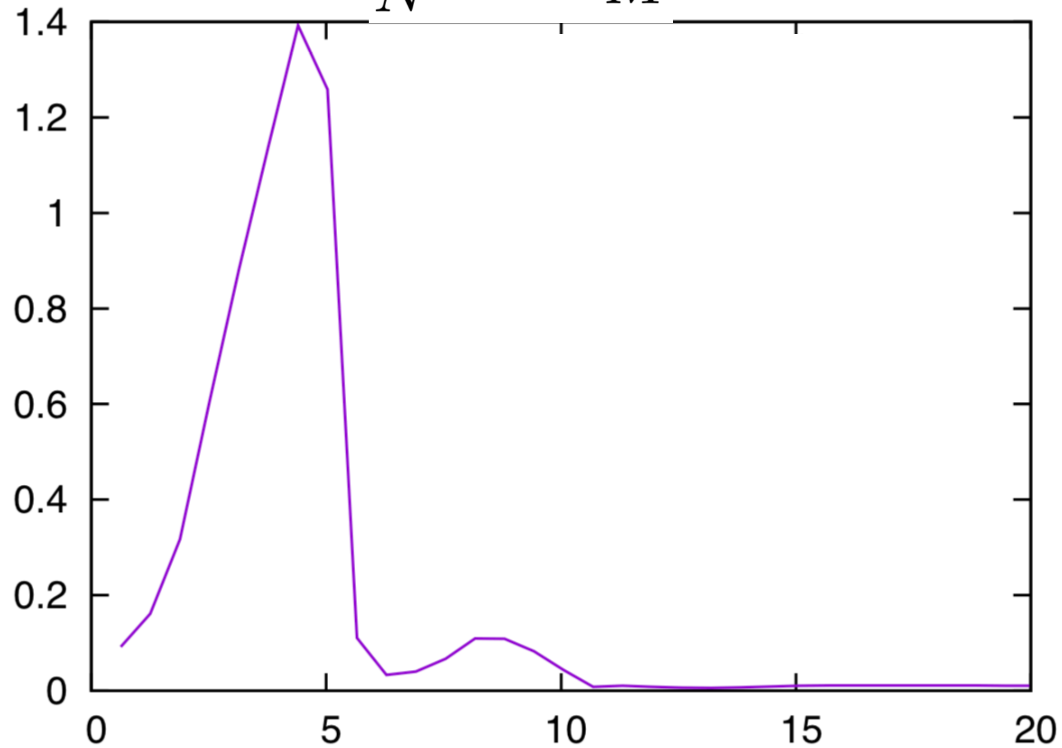


$$\nu_R = 4.63(22) \times (\lambda T)^{1/4}$$
$$\frac{\nu_I}{\nu_R} = 0.183(33)$$

MH-Romatschke

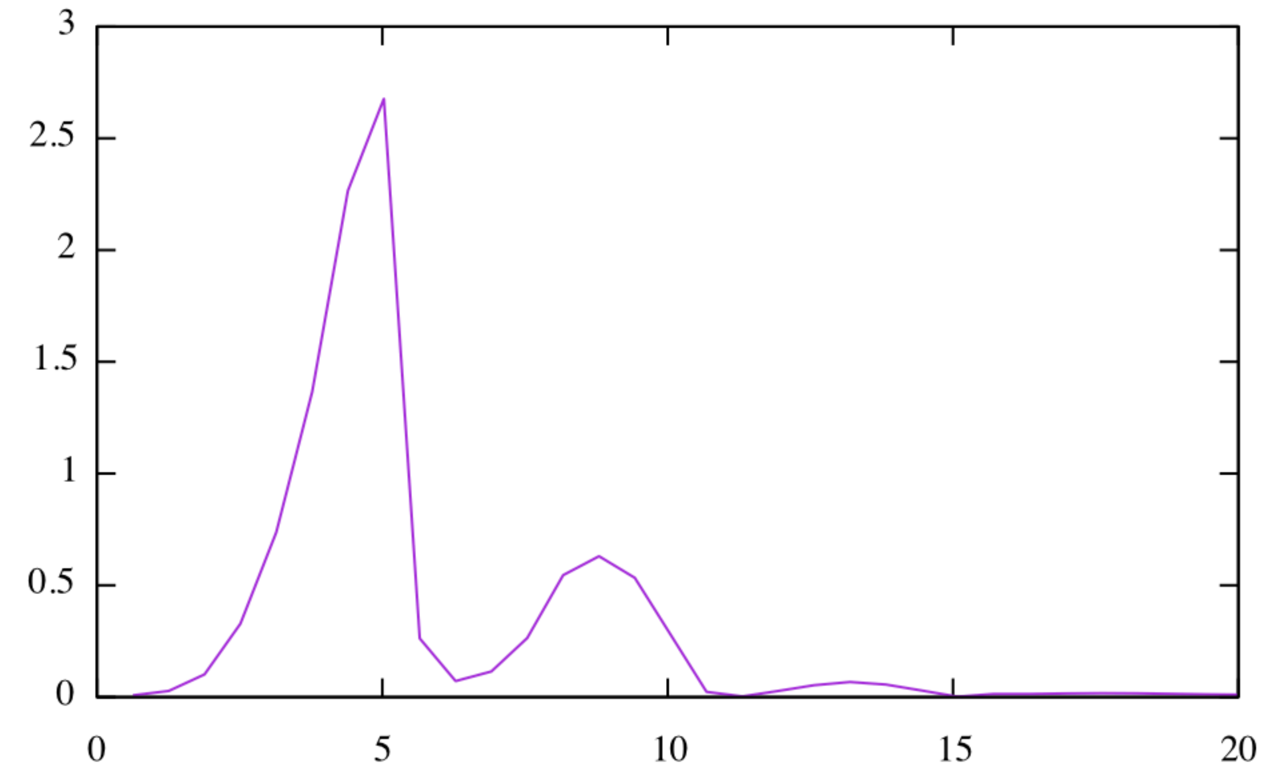
Fourier modes

$\frac{1}{N}\mathrm{Tr}X_M^2$



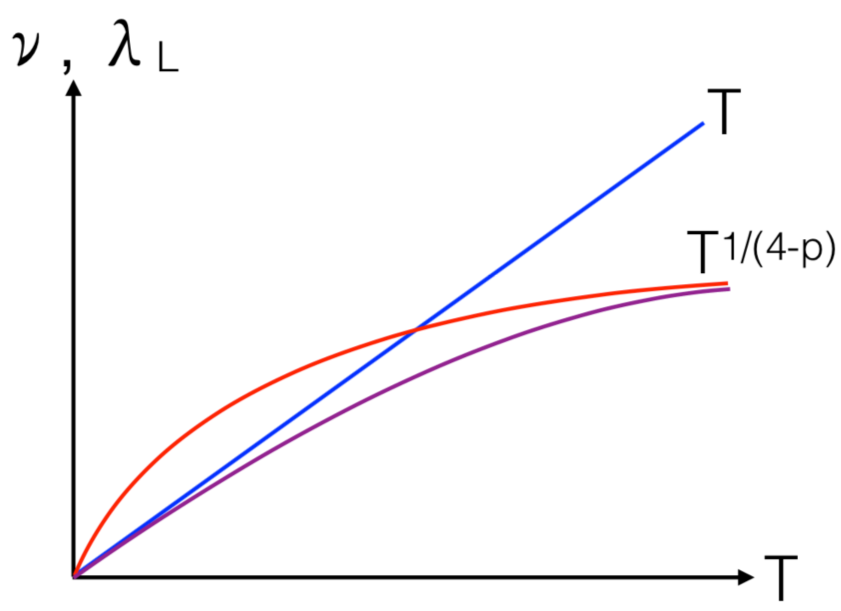
ν_R

kinetic energy

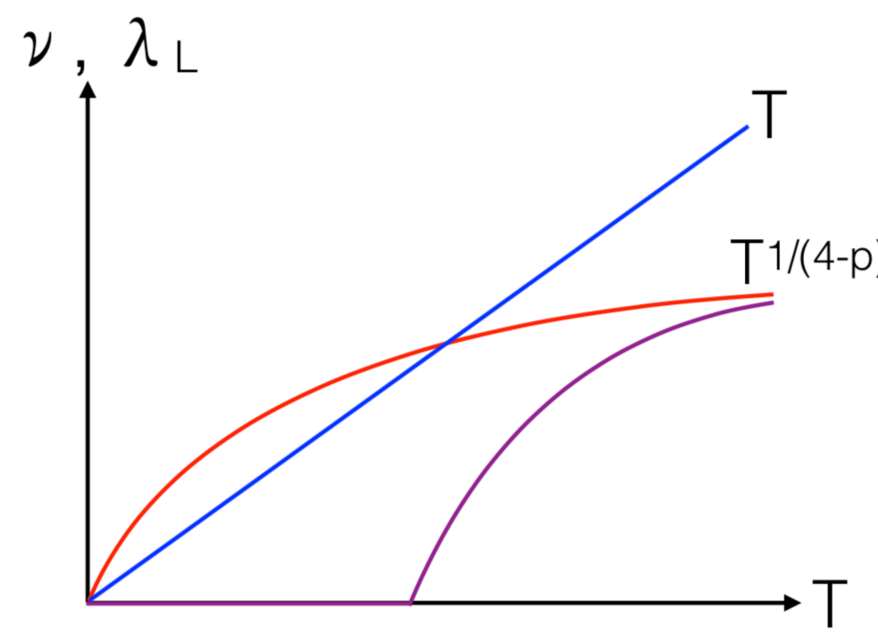


ν_R

MH-Romatschke



SYM



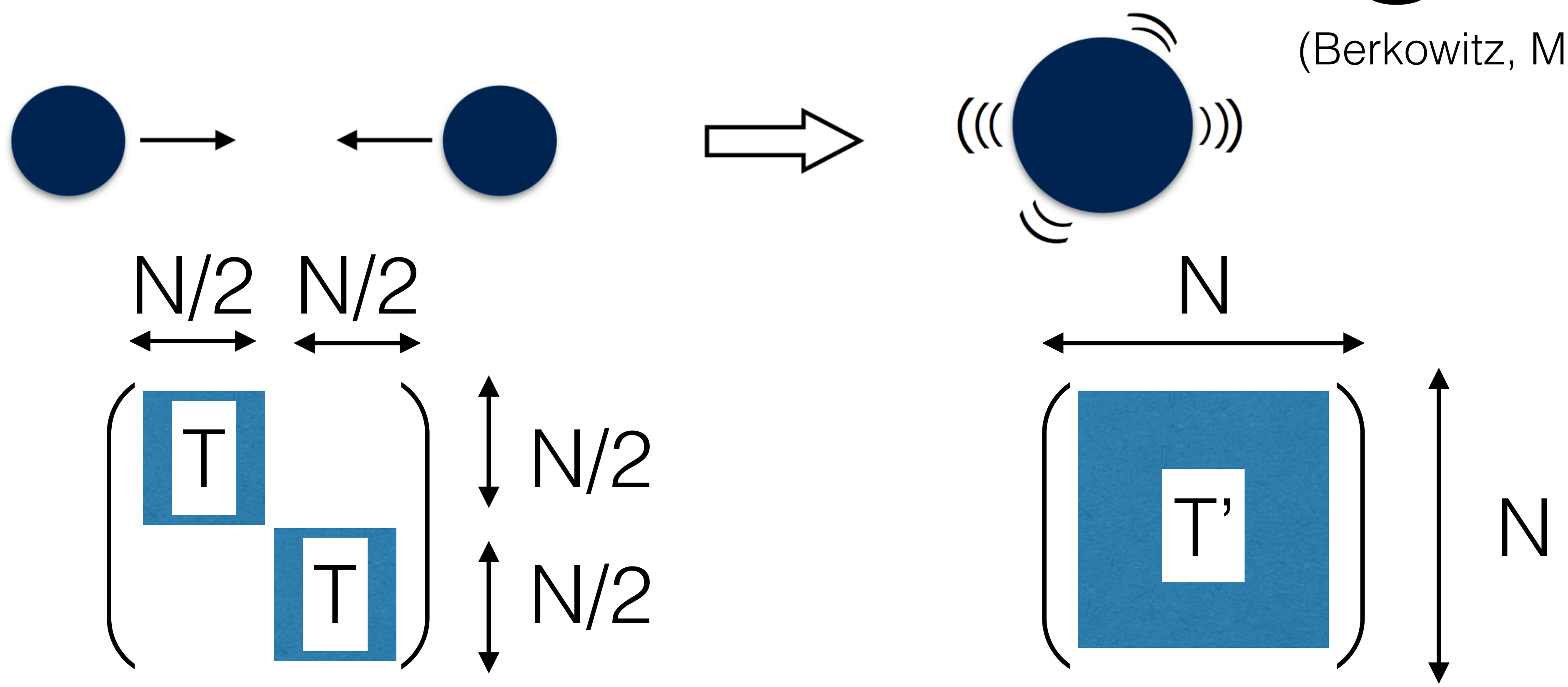
pure YM +scalar

‘Gaussian state approximation’ supports this picture.

(Buividovich-MH-Schaefer, in preparation)

BH cools down as it grows

(Berkowitz, M.H., Maltz, 2016)



T ∼ (energy)/(# d.o.f)

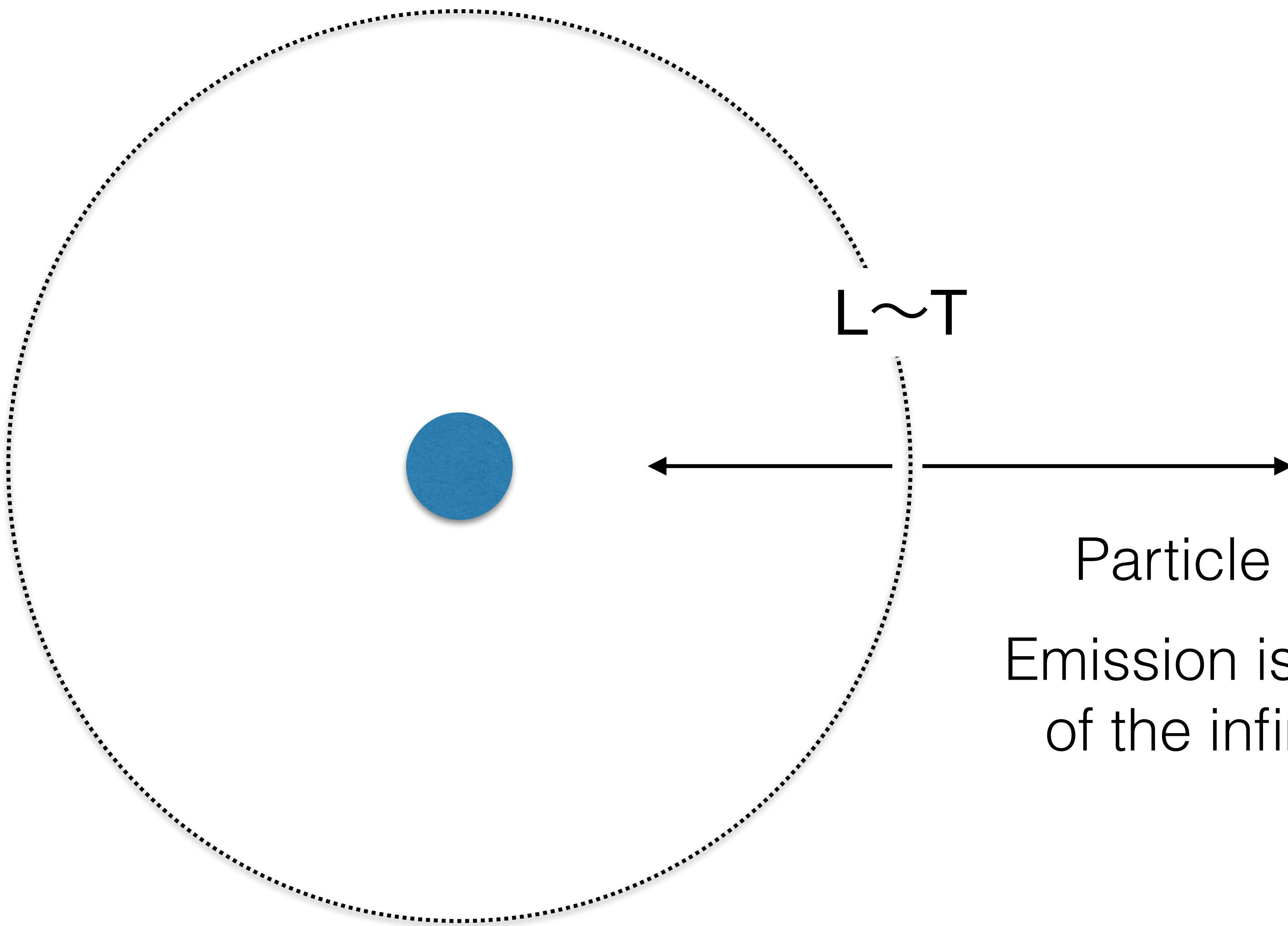
- Energy does not change
- # d.o.f. increases

→ Black hole cools down

<u>high-T</u> $E = 2 \times 6T\,(N/2)^2 = 6T'N^2$ $T' = T/2$

- Thermalization of BH from classical matrix model
- Evaporation of BH from quantum matrix model
- New universality in classical and quantum chaos

(David’s talk should be related to this part)

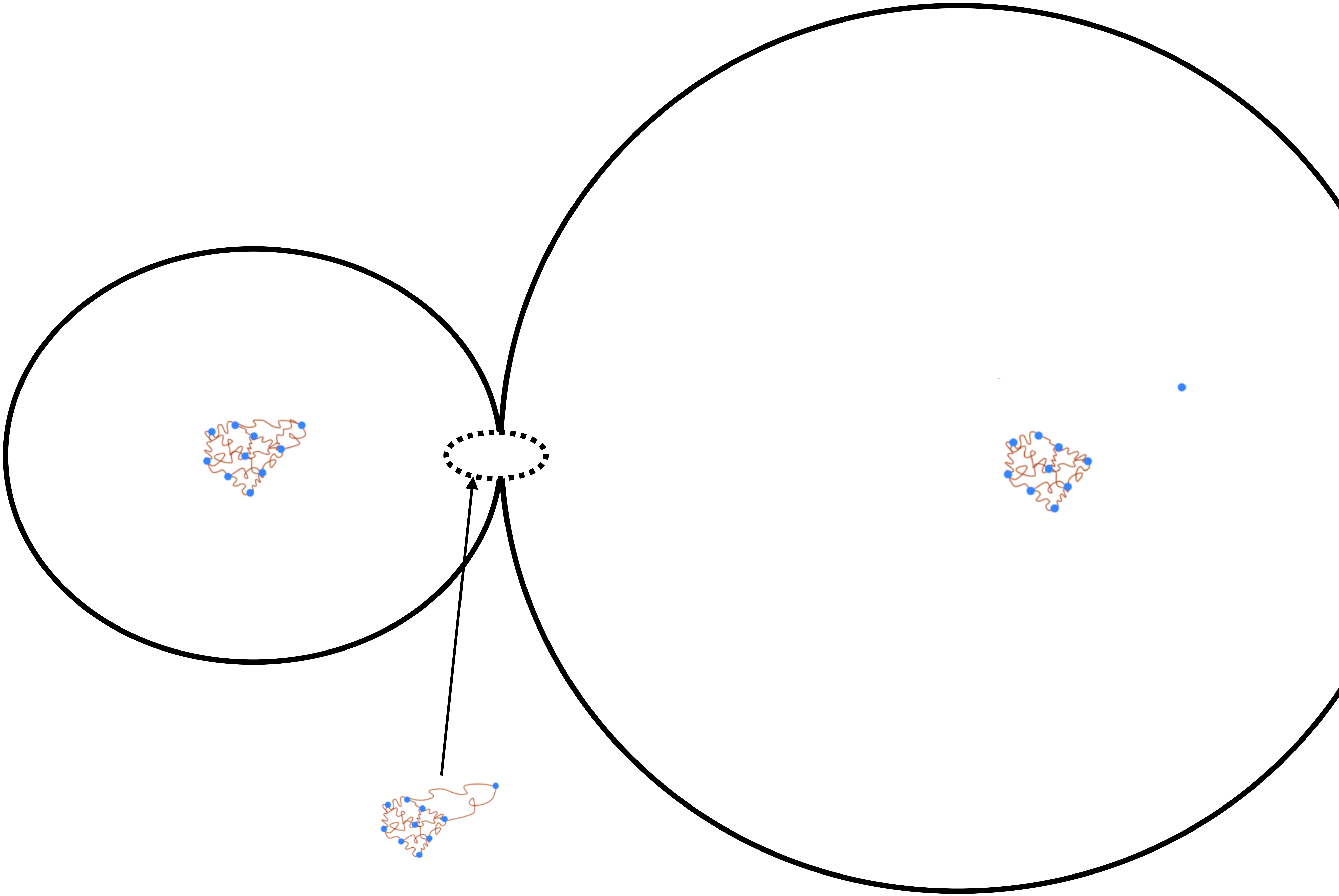


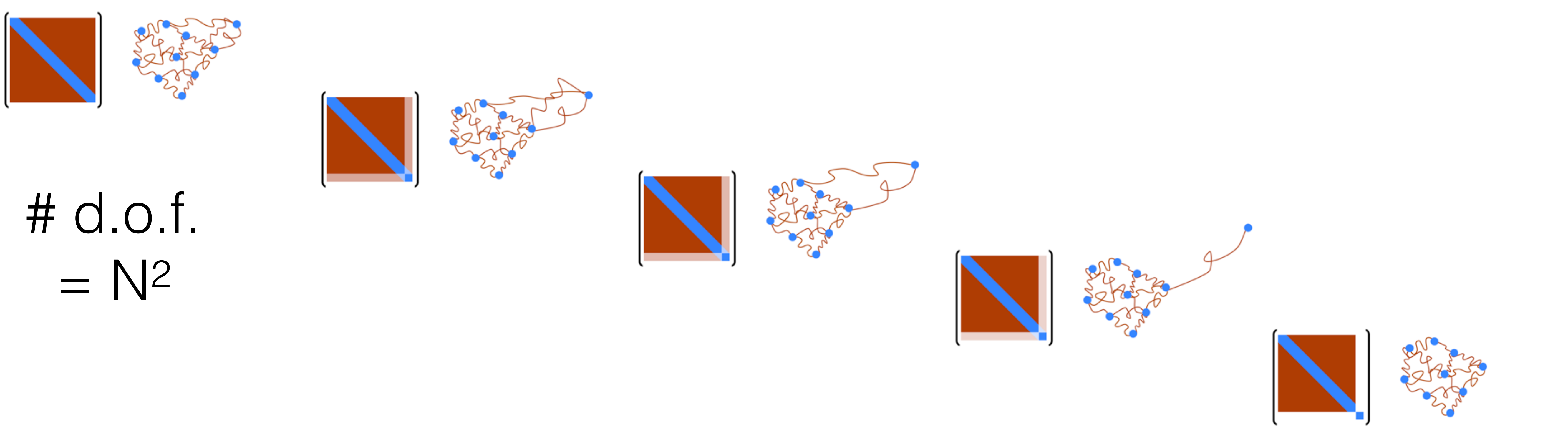
Particle travels almost freely.

Emission is preferred because of the infinite volume factor.

$$\# \text{ d.o.f.} = (N-1)^2 + 1 \times \log(\text{volume})$$

- Emission is entropically disfavored at short distance.
- Beyond some point, it is entropically favored.



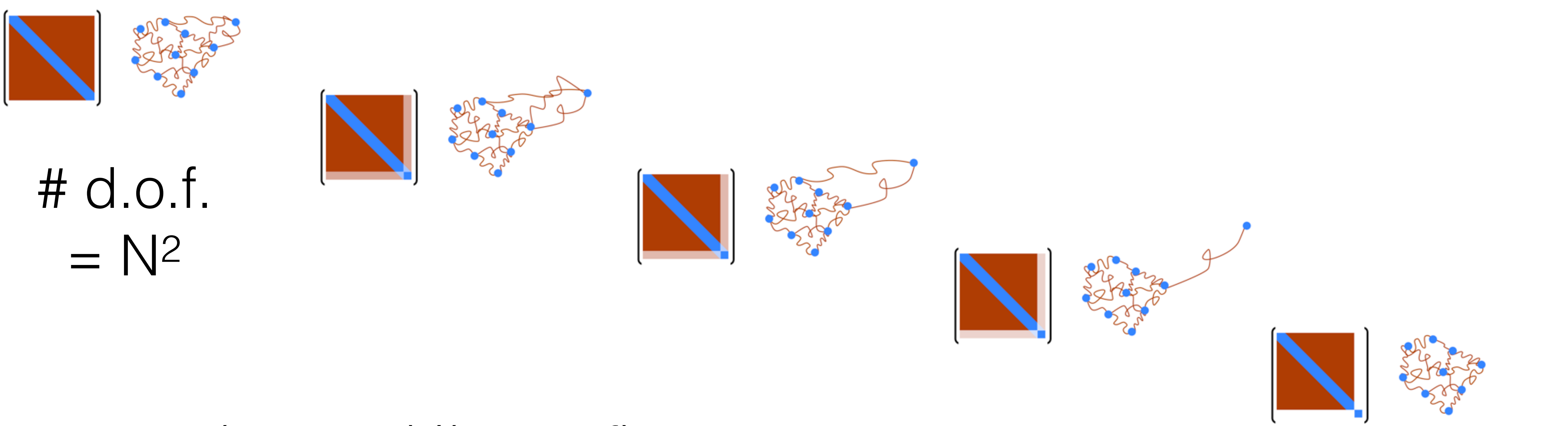


d.o.f. $= N^2$

d.o.f. $= (N-1)^2 + 1$

- Finite probability of particle emission, suppressed at $N=\infty$
- Emission time $\sim \exp(N)$ note: recurrence time $\sim \exp(N^2)$
  scrambling time $\sim \log N$
- k-particle emission is suppressed; $\exp(kN)$
- Temperature goes due to Higgsing.

Black hole becomes hotter as it evaporates



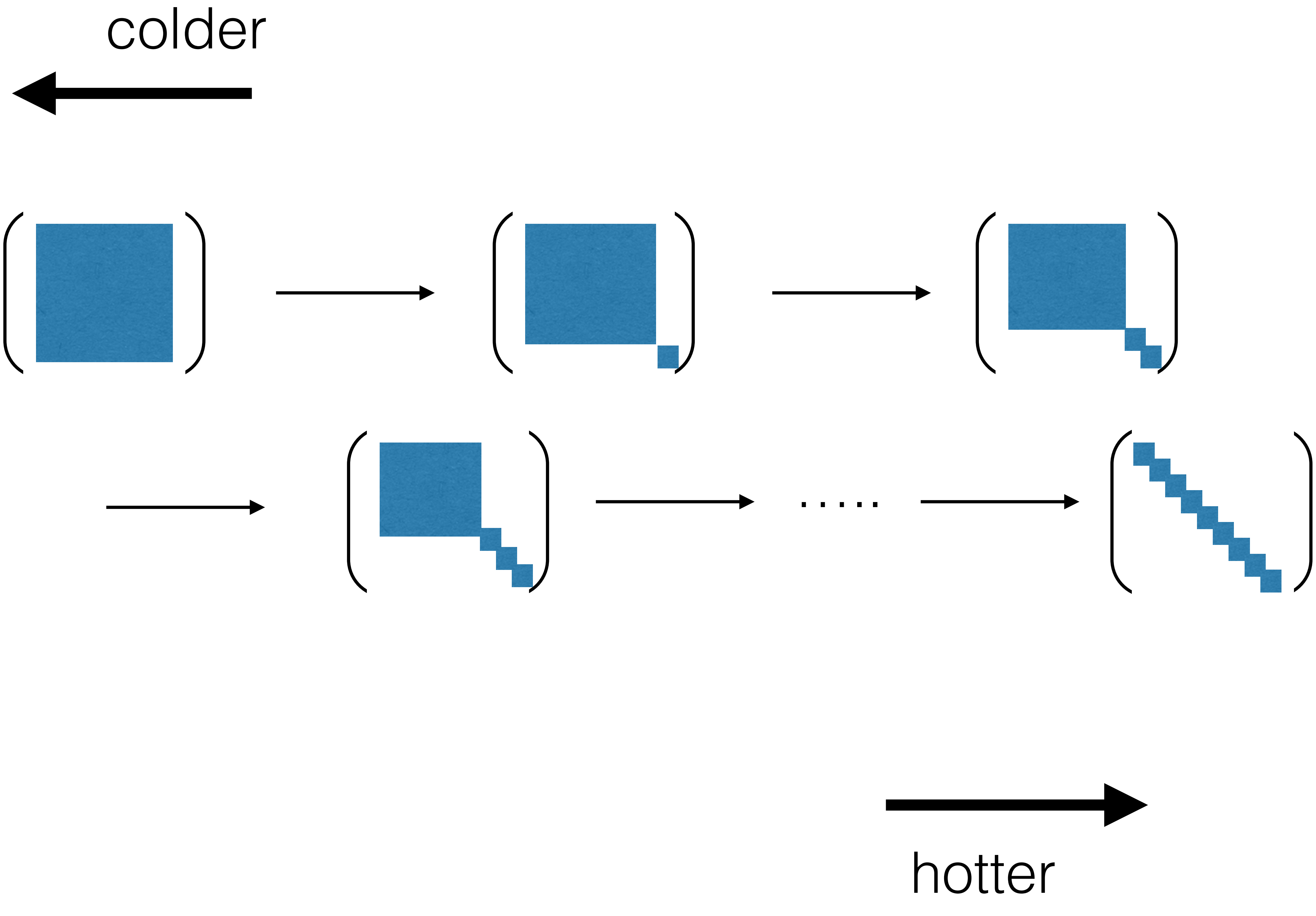
d.o.f. $= N^2$

d.o.f. $= (N-1)^2 + 1$

$T \sim$ (energy)/(# d.o.f)

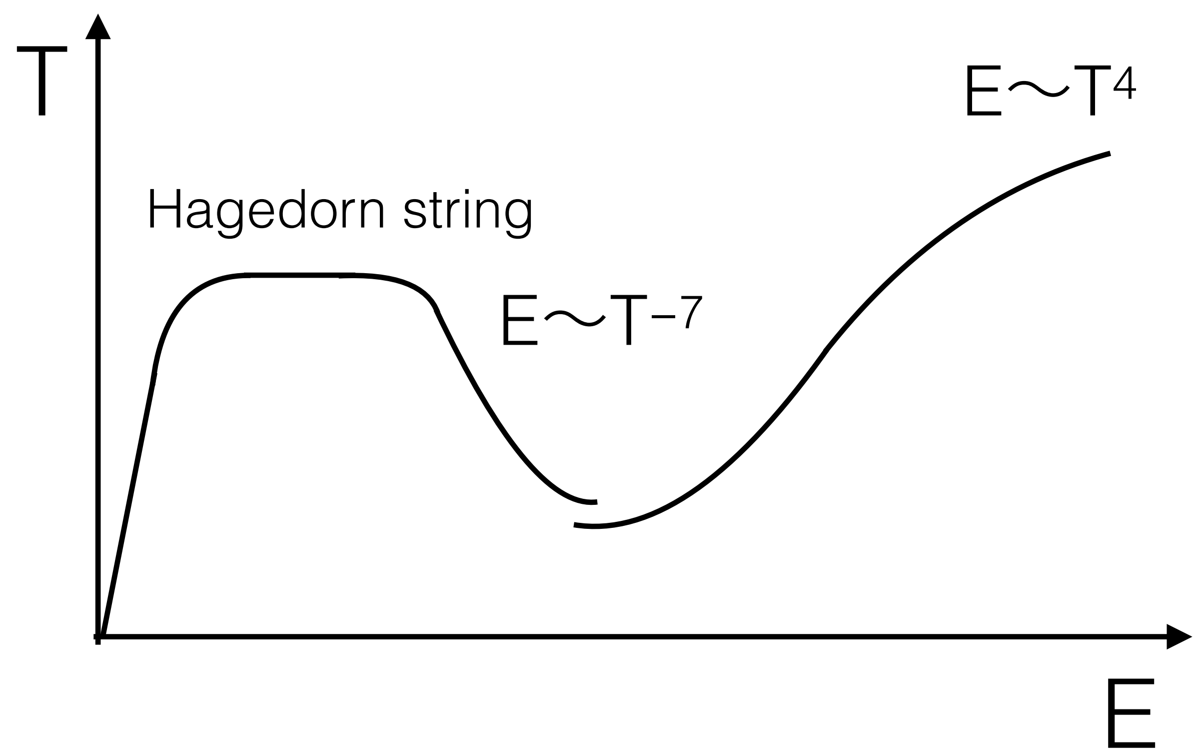
- Energy does not change
- # d.o.f. decreases (Higgsing)

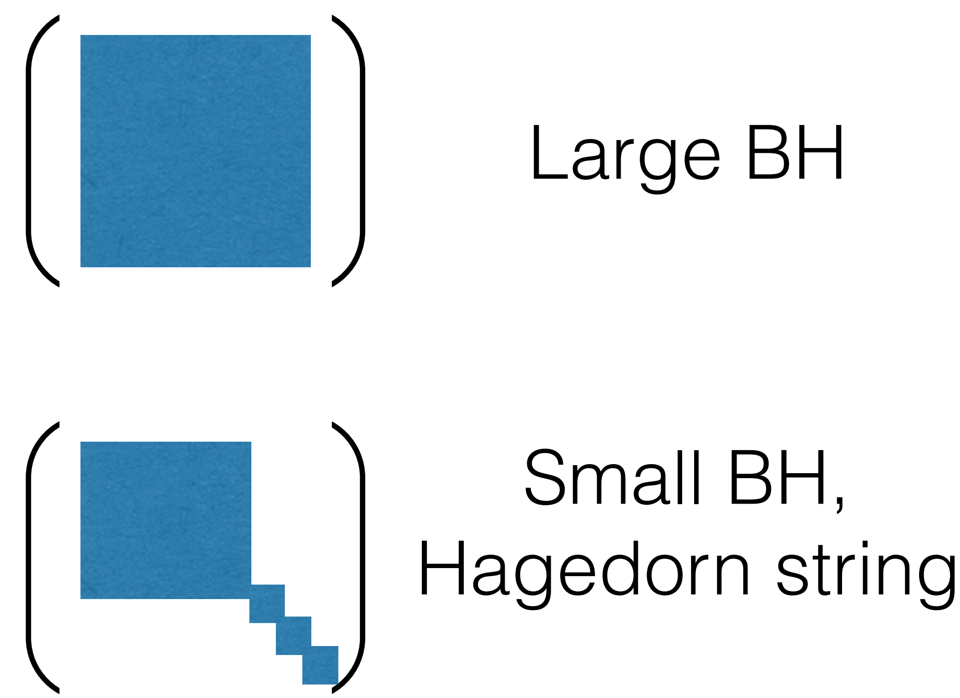
→ Black hole heats up as it evaporates.


colder
hotter

4d N=4 SYM can be understood in a similar manner.

(MH-Maltz, 2016; David's talk)





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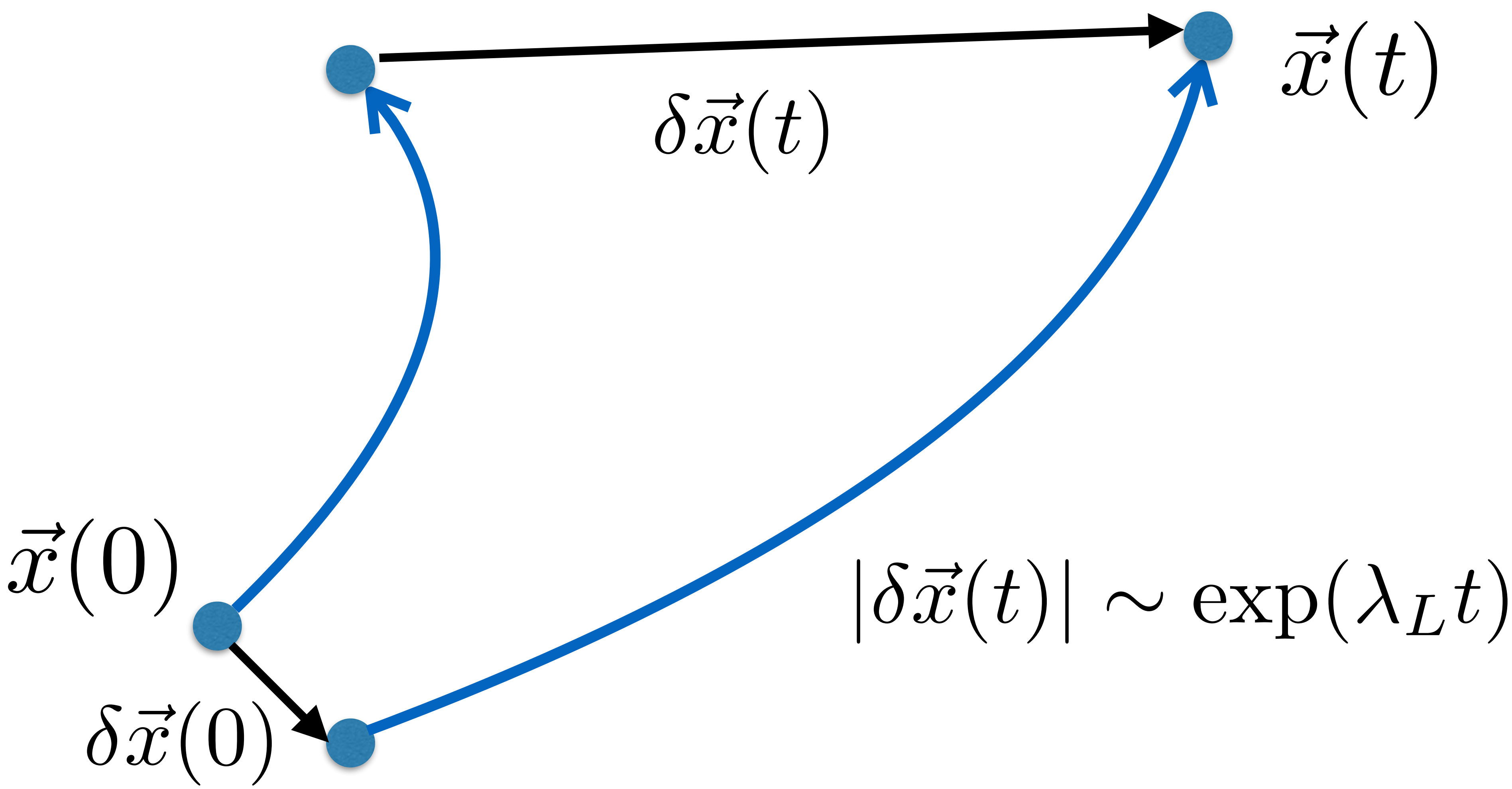
Characterization of *classical* chaos

- Sensitivity to a small perturbation.
  Lyapunov exponent $\lambda_L>0$.



Characterization of *quantum* chaos

Early time

- Sensitivity to a small perturbation.
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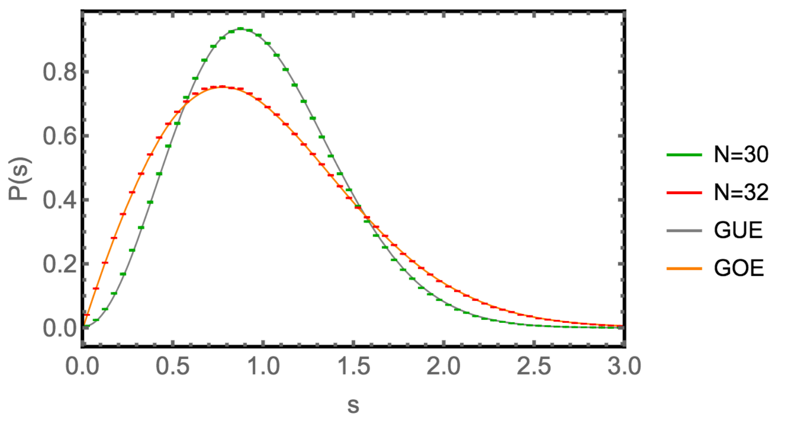
  (Out-of-time-order correlation functions)

Late time

- 'Universal' energy spectrum.
  Fine-grained energy spectrum should agree with Random Matrix Theory (RMT).



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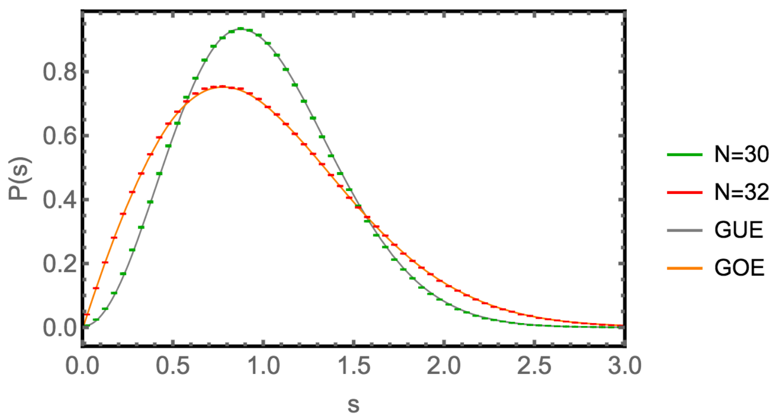

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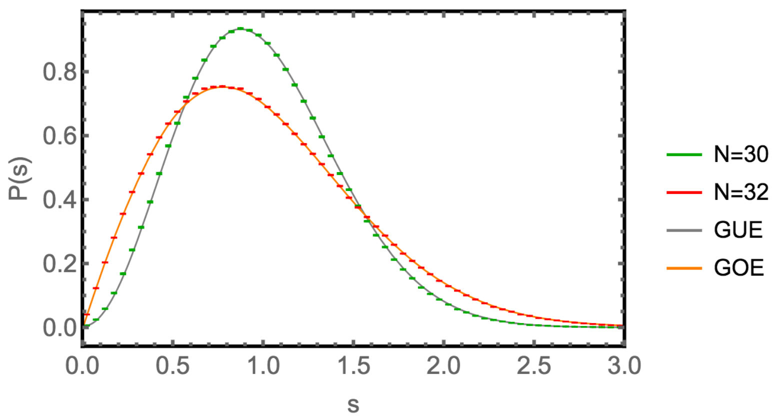

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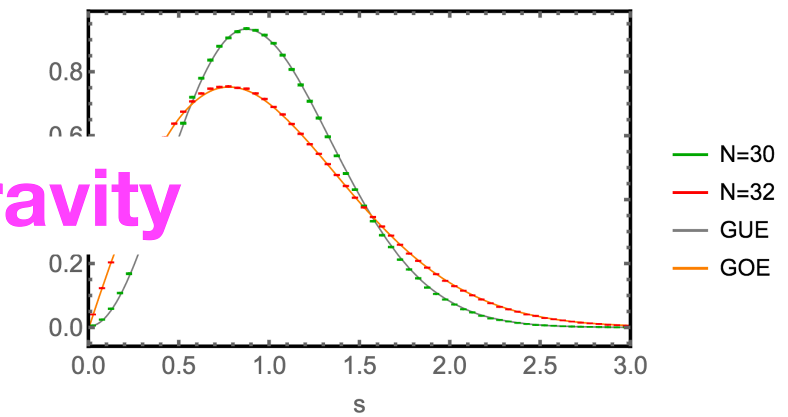
Interesting connection to quantum gravity



Lyapunov exponents

(Lyapunov spectrum)

Lyapunov Spectrum in Classical Chaos

- Classical phase space is multi-dimensional.
- Perturbation can grow or shrink to various directions.

$$z = (x, p)$$

$$M_{ij}(t) = \frac{\delta z_i(t)}{\delta z_j(0)}$$ singular value $s_i(t)$

$$L_{ij}(t) = \left[M^\dagger(t) M(t)\right]_{ij} = M^*_{ki}(t) M_{kj}(t)$$ eigenvalue $s_i(t)^2$

finite-time Lyapunov exponents $$\lambda_i(t) = \frac{1}{t} \log s_i(t)$$

Largest Exponent is not enough

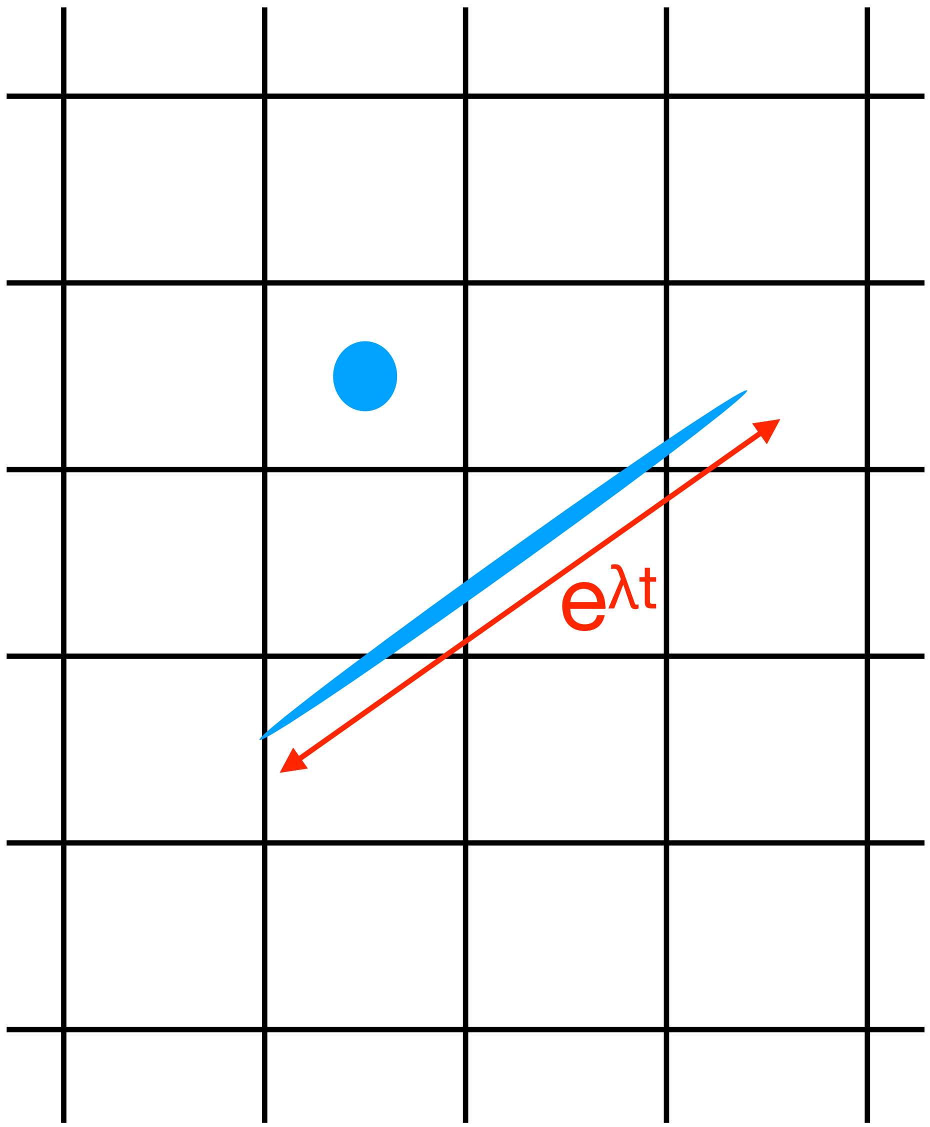
$\lambda_1=100$

$\lambda_2=\lambda_3=\ldots\lambda_{1000}=0$

$\lambda_1=\lambda_2=\ldots\lambda_{1000}=1$

Which is more chaotic?

Coarse-grained entropy and Kolmogorov-Sinai Entropy



# of cells to cover the region $\sim \prod_{\lambda>0} \exp(\lambda t)$

Coarse-grained entropy

$= \log[\text{\# of cells to cover the region}]$

$\sim (\text{sum of positive } \lambda) \times t$

KS entropy $=$ (sum of positive λ)

$=$ entropy production rate

Largest Exponent is not enough

$\lambda_1=100$

$\lambda_2=\lambda_3=\ldots\lambda_{1000}=0$

$\lambda_1=\lambda_2=\ldots\lambda_{1000}=1$

Which is more chaotic?

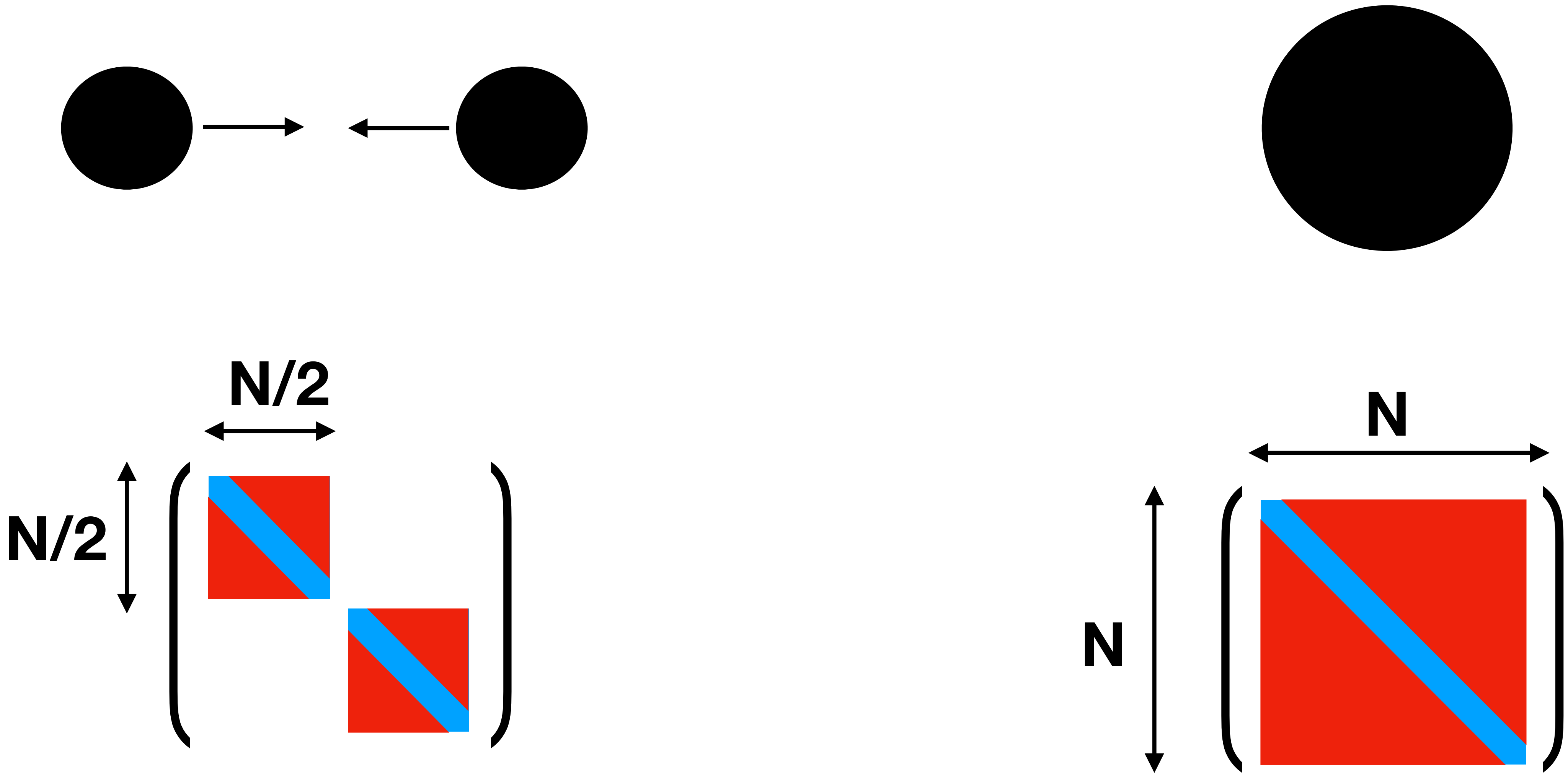
$\lambda_{1+}+\lambda_2+\ldots+\lambda_{1000}=100$

$\lambda_{1+}+\lambda_2+\ldots+\lambda_{1000}=1000$

More chaotic

Bigger black hole is colder.

Bigger black hole is less chaotic?

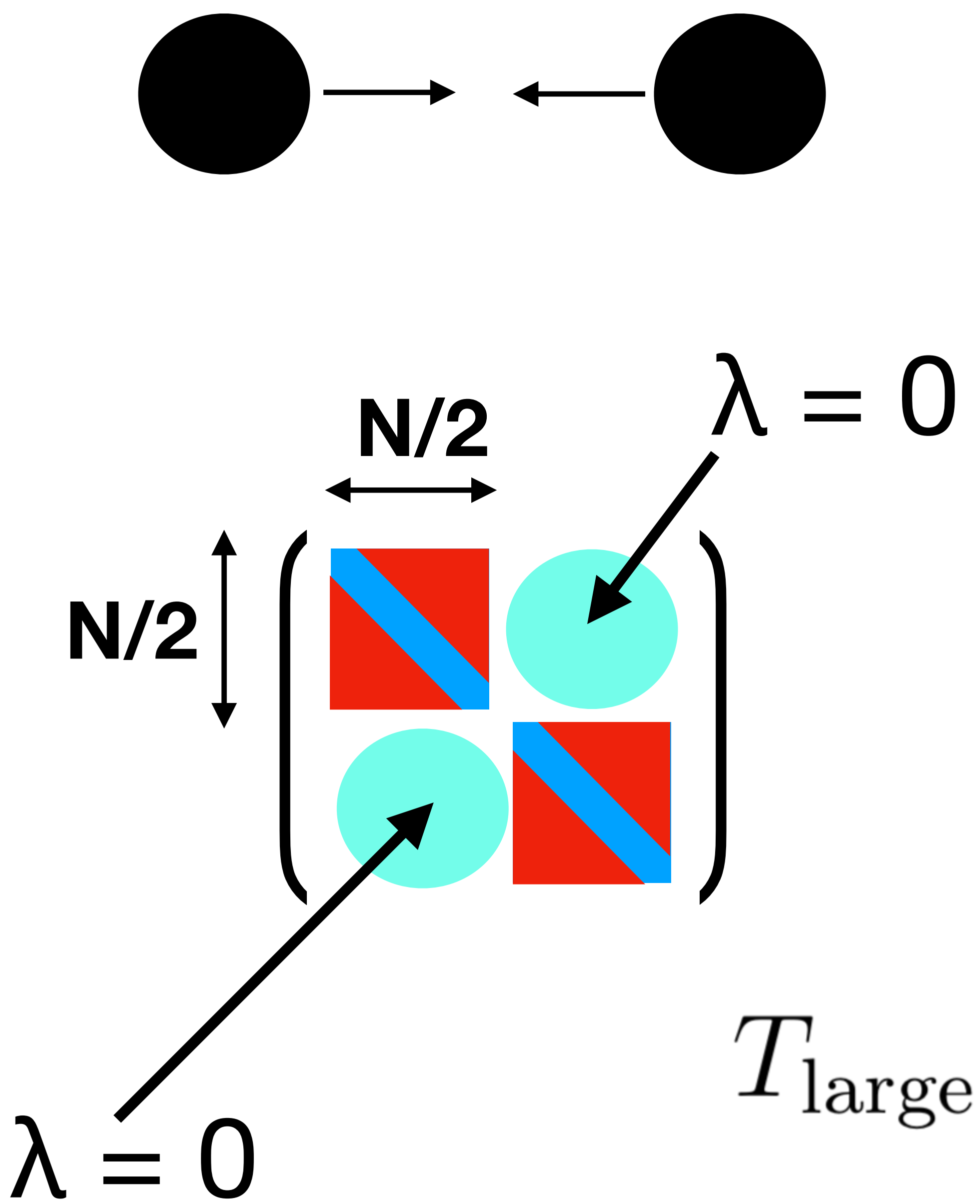


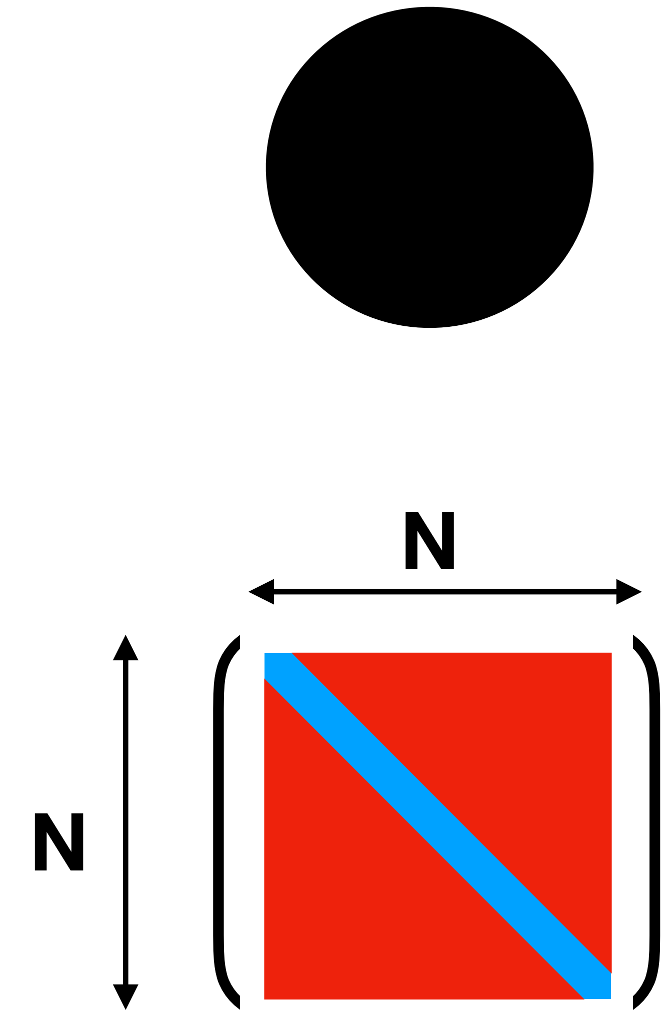
$$E \sim N^2 T_{\text{large}} = \left(\tfrac{N}{2}\right)^2 T_{\text{small}} \times 2$$

$$T_{\text{large}} = \tfrac{1}{2} T_{\text{small}}$$

$$\lambda \sim T^{1/4}$$

(@high-T region)





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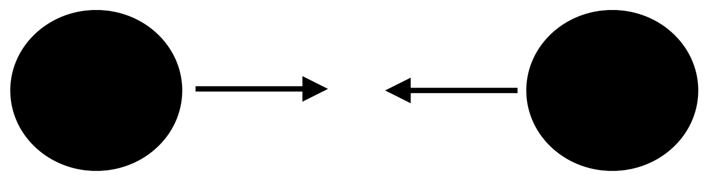

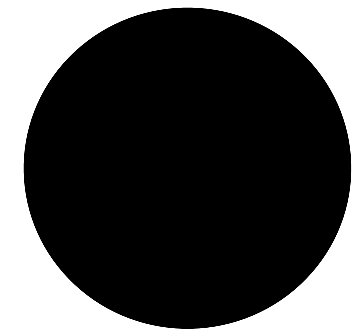

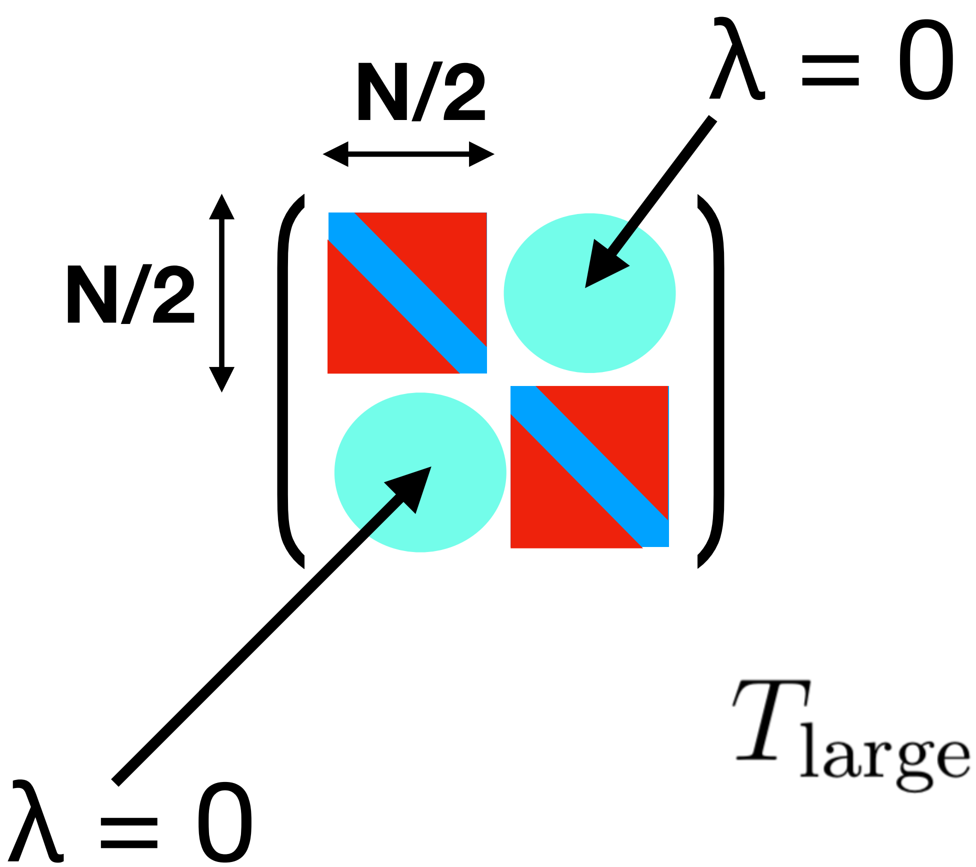

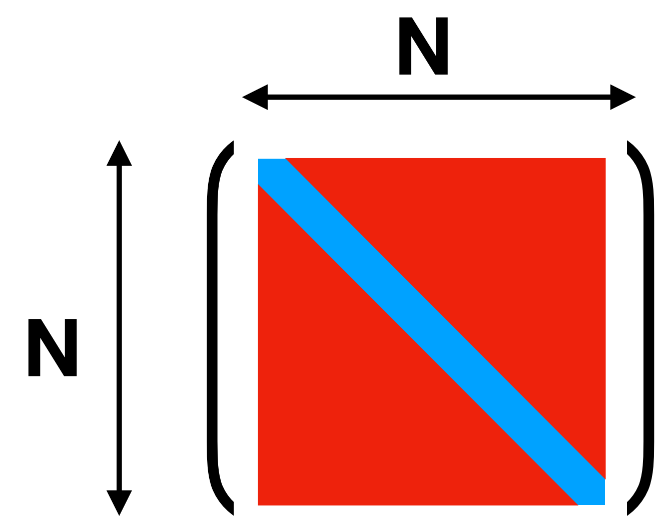

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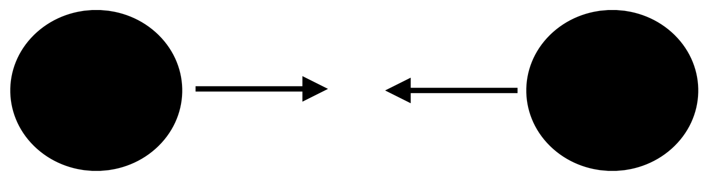
$$\lambda \sim T^{1/4}$$

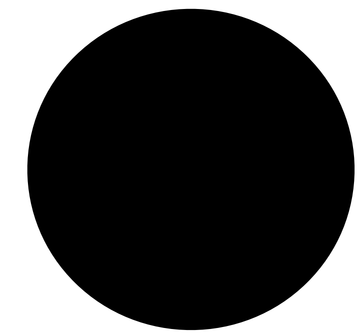
(@high-T region)

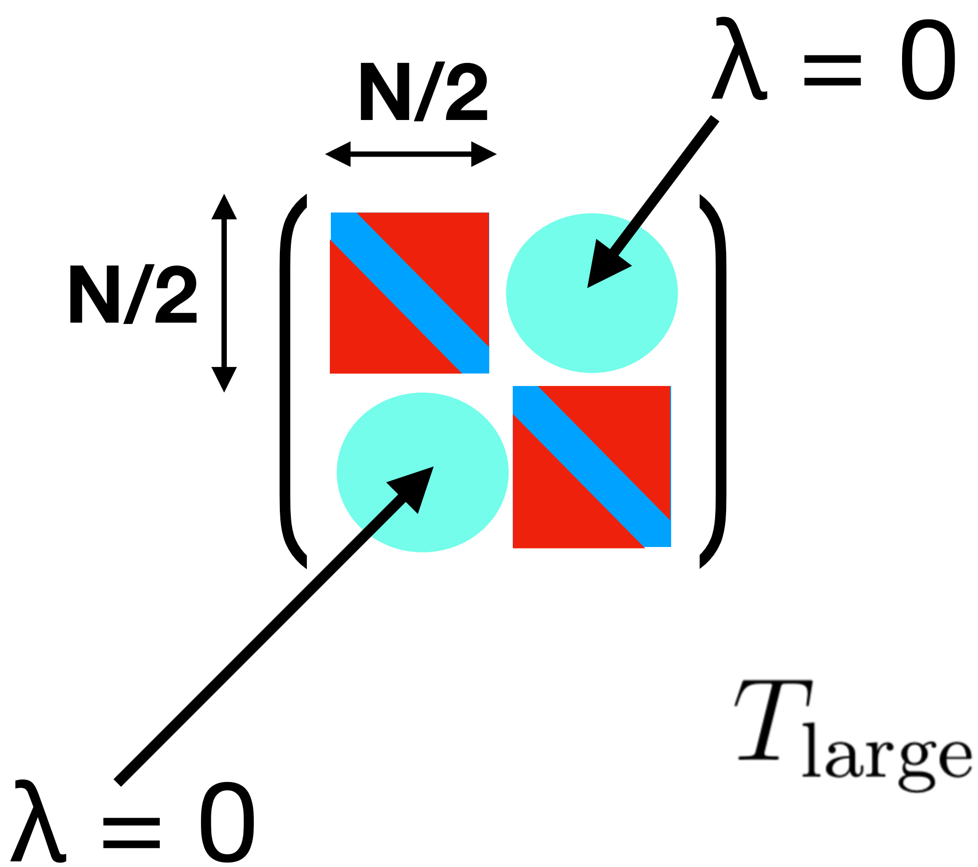
$$\boxed{2 \times \left(\frac{N}{2}\right)^2 \times T_{\text{small}}^{1/4} < N^2 \times T_{\text{large}}^{1/4}}$$

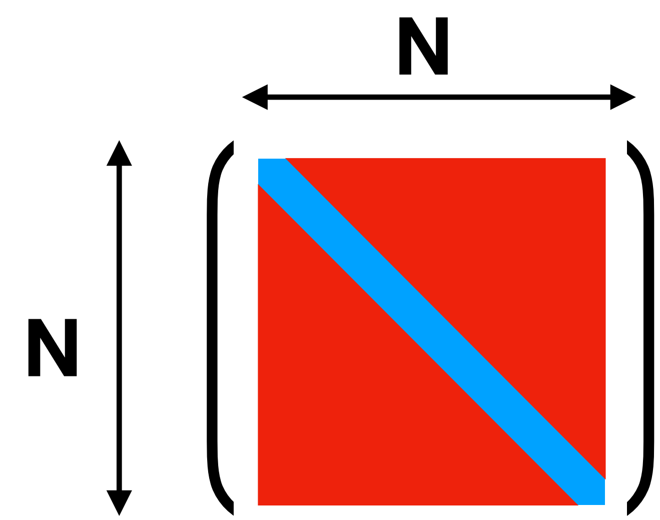
Similar calculation is doable at low-T and also for other theories (Berkowitz-MH-Maltz 2016)

More chaotic









$$T_{\rm large} = \frac{1}{2} T_{\rm small}$$

$$\lambda \sim T^{1/4}$$

(@high-T region)

$$\boxed{2 \times \left(\frac{N}{2}\right)^2 \times T_{\rm small}^{1/4} < N^2 \times T_{\rm large}^{1/4}}$$

Similar calculation is doable at low-T and also for other theories (Berkowitz-MH-Maltz 2016)

Plan

- Universality of *classical* Lyapunov spectrum

MH, Shimada, Tezuka, PRE 2018

- Universality of *quantum* Lyapunov spectrum

Gharibyan, MH, Swingle, Tezuka, in progress

Lyapunov Spectrum

$$z = (x, p)$$

$$M_{ij}(t) = \frac{\delta z_i(t)}{\delta z_j(0)}$$ → singular value $s_i(t)$

$$L_{ij}(t) = \left[M^\dagger(t) M(t)\right]_{ij} = M^*_{ki}(t) M_{kj}(t)$$

→ eigenvalue $s_i(t)^2$

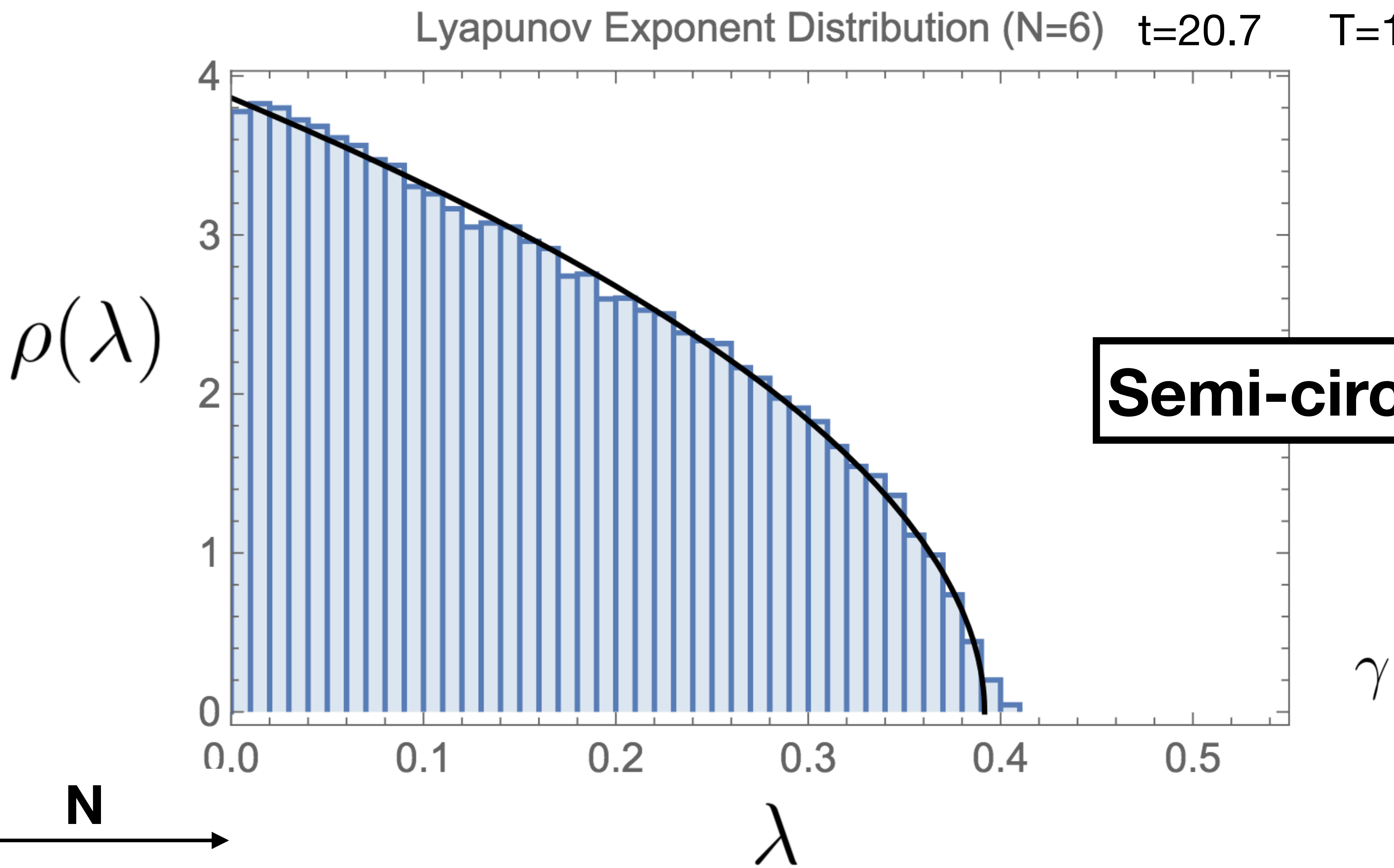
finite-time Lyapunov exponents $$\lambda_i(t) = \frac{1}{t}\log s_i(t)$$

Lyapunov Spectrum

$$\begin{aligned} M_{ij}(t) &= \frac{\delta z_i(t)}{\delta z_j(0)} \\ &= \frac{\delta z_i(t)}{\delta z_k(t-\Delta t)} \cdot \frac{\delta z_k(t-\Delta t)}{\delta z_l(t-2\Delta t)} \cdots \frac{\delta z_m(\Delta t)}{\delta z_j(0)} \end{aligned}$$

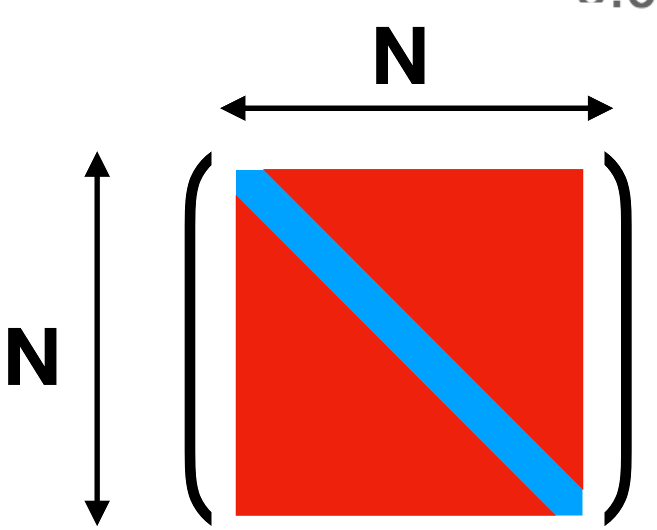
Easily to calculate with good precision

Gur Ari-MH-Shenker, JHEP2016



Semi-circle

$\gamma = 0.513$



Fitting ansatz $\rho(\lambda) = \dfrac{(\gamma+1)(\lambda_{max}-\lambda)^{\gamma}}{\lambda_{max}^{\gamma+1}}$

RMT vs Classical Chaos

- The correlation of the finite-time Lyapunov exponents may have a universal behavior?

(Some hints found in the previous study by Gur-Ari, MH, Shenker)

$$\lambda_1 < \lambda_2 < \cdots < \lambda_N$$

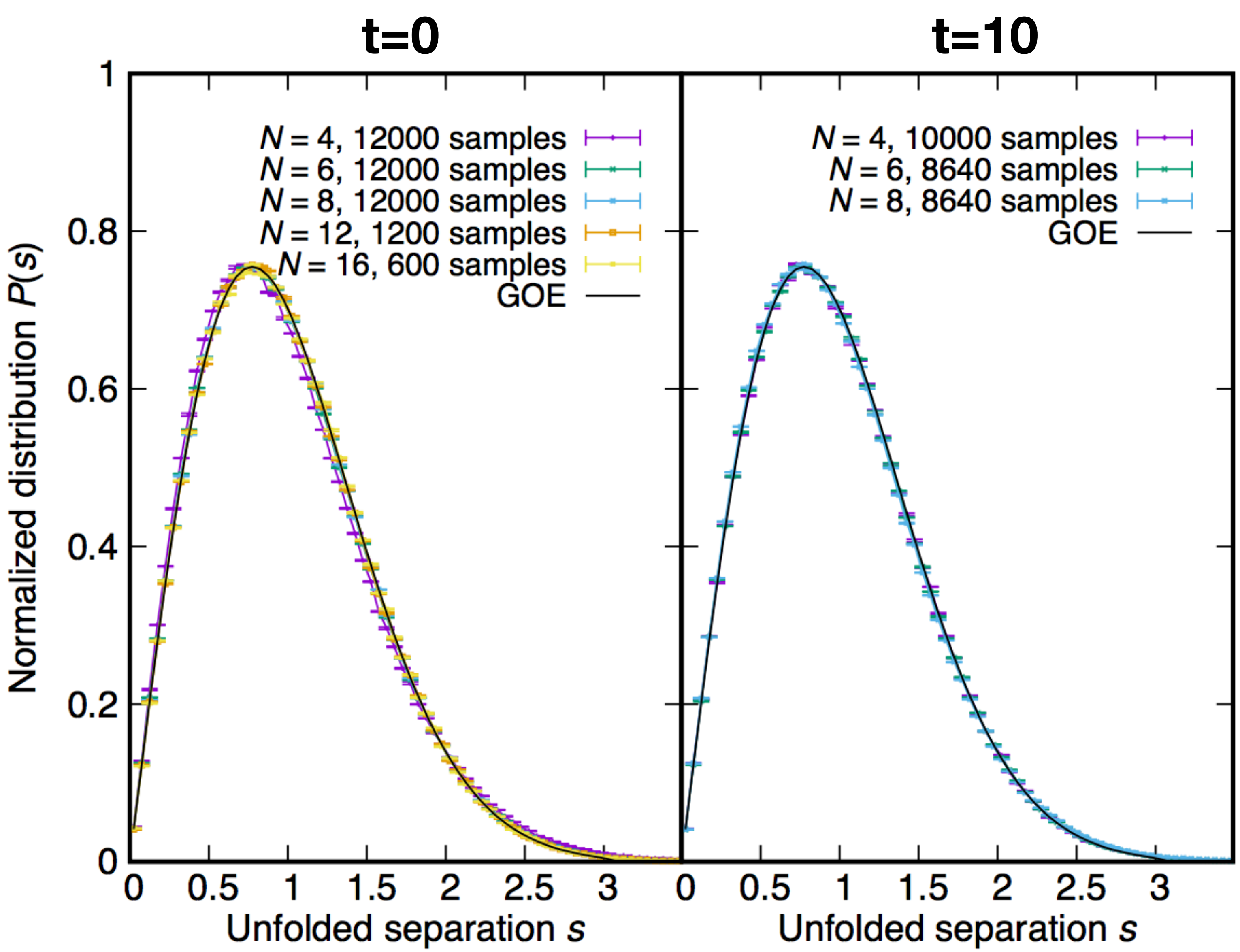
$$s_i = \lambda_{i+1} - \lambda_i$$

(different from $s_i = \exp(\lambda_i t)$, sorry for using the same letter!)

- $N \to \infty$ before $t \to \infty$

(In chaos community, often $t \to \infty$ is taken first.)

GOE-distribution at any time

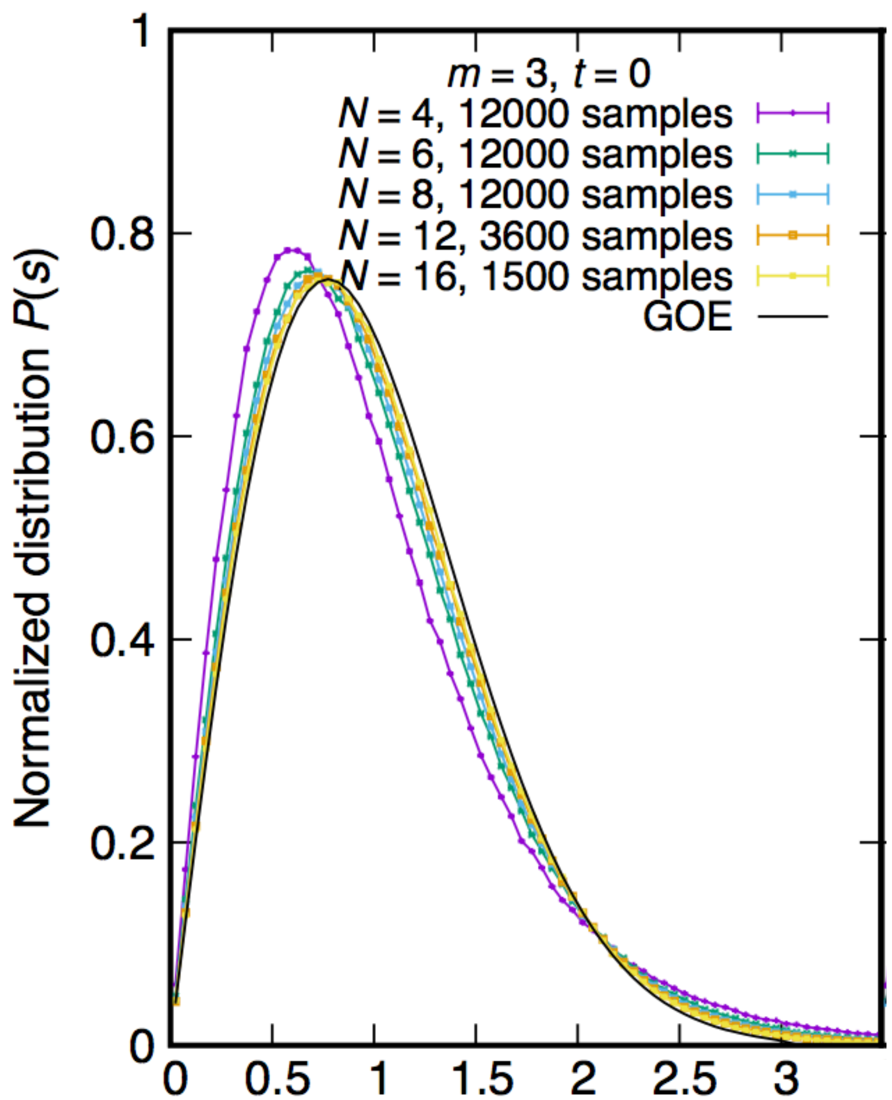


$$s_i = \lambda_{i+1} - \lambda_i$$

Lyapunov exponents are described by RMT

M.H.-Shimada-Tezuka, PRE 2018

with a mass term (→no gravity interpretation), GOE is gone, at t=0.



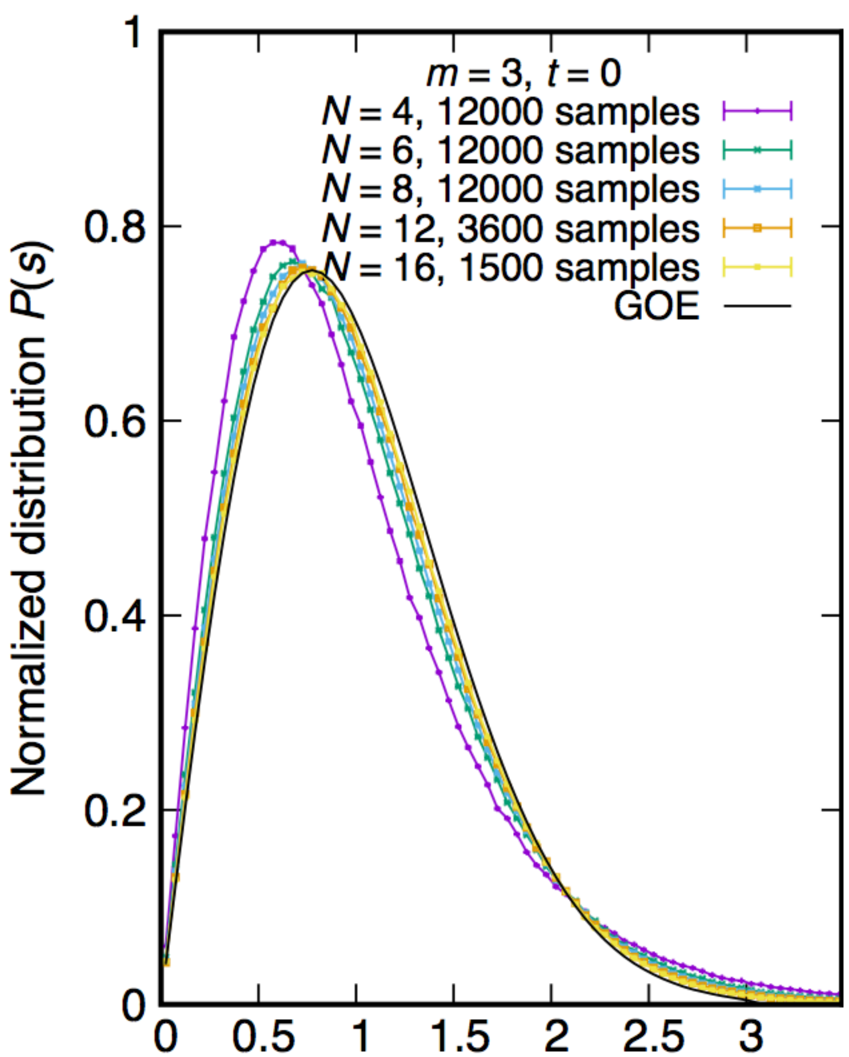
$$s_i = \lambda_{i+1} - \lambda_i$$

m=3, t=0

$$\Delta L = -\frac{Nm^2}{2}\mathrm{Tr}X_M^2$$

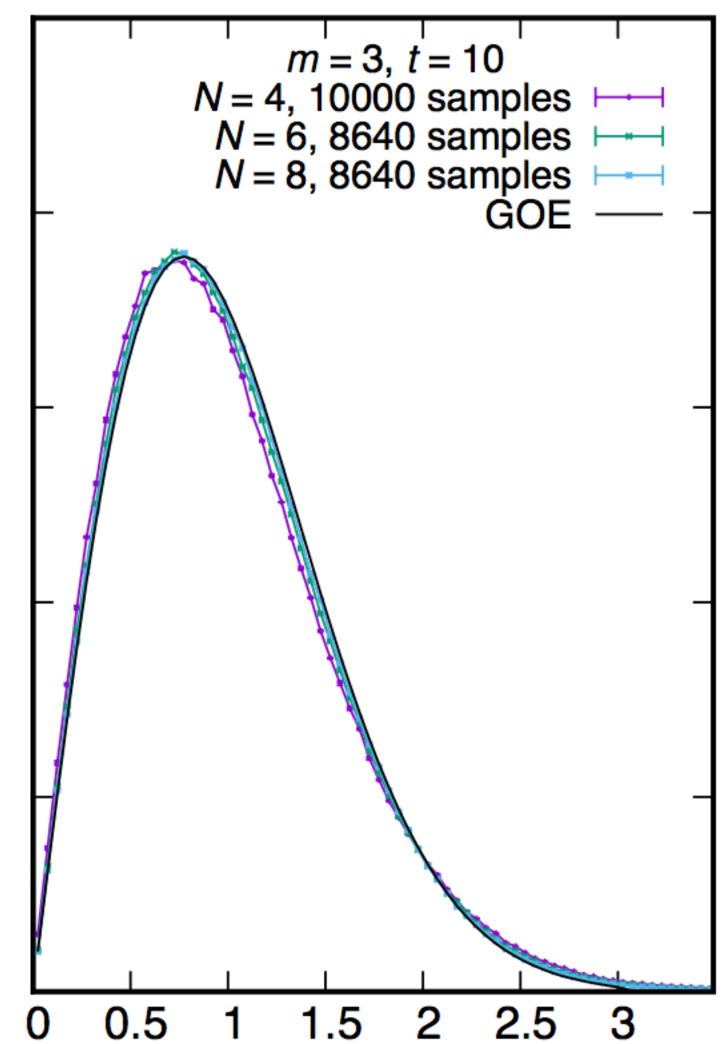
But GOE is back at later time

t=0



$$s_i = \lambda_{i+1} - \lambda_i$$

t=3



$$s_i = \lambda_{i+1} - \lambda_i$$

$$\Delta L = -\frac{Nm^2}{2}\mathrm{Tr}X_M^2$$

Summary of numerical observations

- Universality beyond nearest-neighbor can be checked. (Spectral Form Factor)

- D0-brane matrix model — RMT already t=0

  Maybe a special property of quantum gravitational systems?

- Other systems — not RMT at t=0, but gradually converges to RMT.

  Likely to be a universal property in classical chaos.
  Generalization to quantum theory?

- So far we have looked at only the bulk of the spectrum; not the edge.

Early-time universality in quantum chaos

Gharibyan, MH, Swingle, Tezuka, in progress

- There is no consensus for the definition of ‘quantum Lyapunov spectrum’

- Let’s try the simplest choice:

$$M_{ij}(t) = \frac{\delta z_i(t)}{\delta z_j(0)}$$

$$\hat{M}_{ij} = \sqrt{-1}\left[\hat{z}_i(t), \hat{\Pi}_j(0)\right]$$

$$L_{ij}(t) = M^*_{ki}(t)M_{kj}(t)$$

$$L^{(\phi)}_{ij}(t) = \langle\phi|\hat{M}^*_{ki}(t)\hat{M}_{kj}(t)|\phi\rangle$$

$$\lambda_i(t) = \frac{1}{t}\log s_i(t)$$

$\hat{M}_{ij}(t)|\phi\rangle$ grows exponentially

$\langle\phi|\hat{M}_{ij}(t)|\phi\rangle$ cannot capture the growth

SYK model

$$\hat{H} = \sqrt{\frac{6}{N^3}} \sum_{i<j<k<l} J_{ijkl} \hat{\psi}_i \hat{\psi}_j \hat{\psi}_k \hat{\psi}_l + \frac{\sqrt{-1}}{\sqrt{N}} \sum_{i<j} K_{ij} \hat{\psi}_i \hat{\psi}_j$$

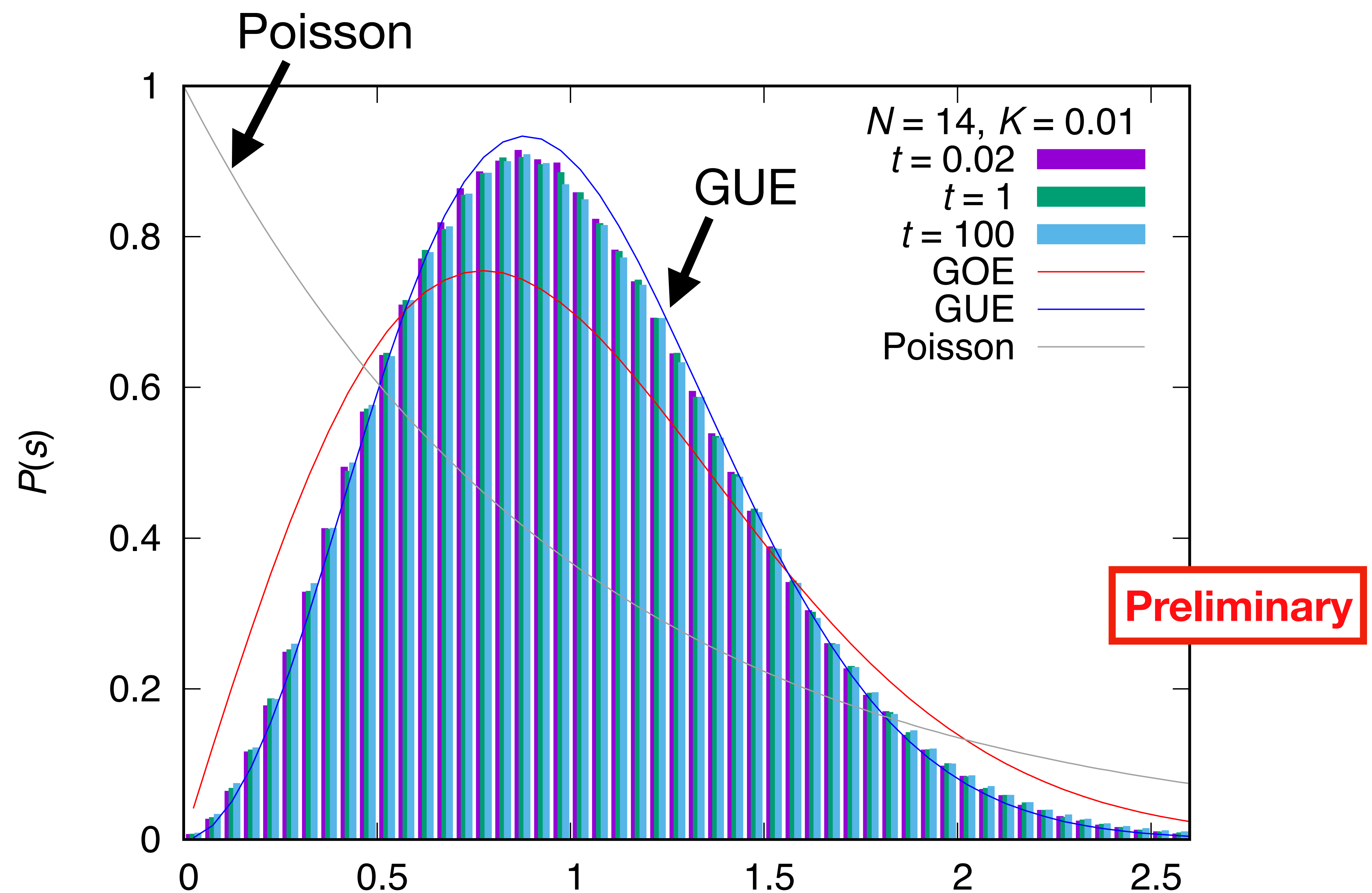
maximally chaotic

integrable

$$\hat{M}_{ij}(t) = \{\hat{\psi}_i(t), \hat{\psi}_j(0)\}$$

$$e^{2\lambda^{(\mathrm{OTOC})} t} = \frac{1}{N} \sum_{i,j} \langle \phi | \{\hat{\psi}_i(t), \hat{\psi}_j(0)\}^2 | \phi \rangle = \frac{1}{N} \sum_i e^{2\lambda_i t}$$

RMT behavior



$$s_i = \lambda_{i+1} - \lambda_i$$

RMT behavior

- $K > 0 \to$ chaotic at high energy, non-chaotic at low energy

(Garcia-Garcia, Loureiro, Romero-Bermudez, Tezuka, 2017)

- Our numerical data suggests:

Chaotic states → RMT

non-chaotic states → Poisson

- Brownian circuit version is consistent with this interpretation.

Spin chain (XXZ model)

$$\hat{H} = \sum_{i=1}^{N_{site}} \left(\frac{1}{4} \vec{\sigma}_i \vec{\sigma}_{i+1} + \frac{\omega_i}{2} \sigma_{z,i} \right)$$

XXX model

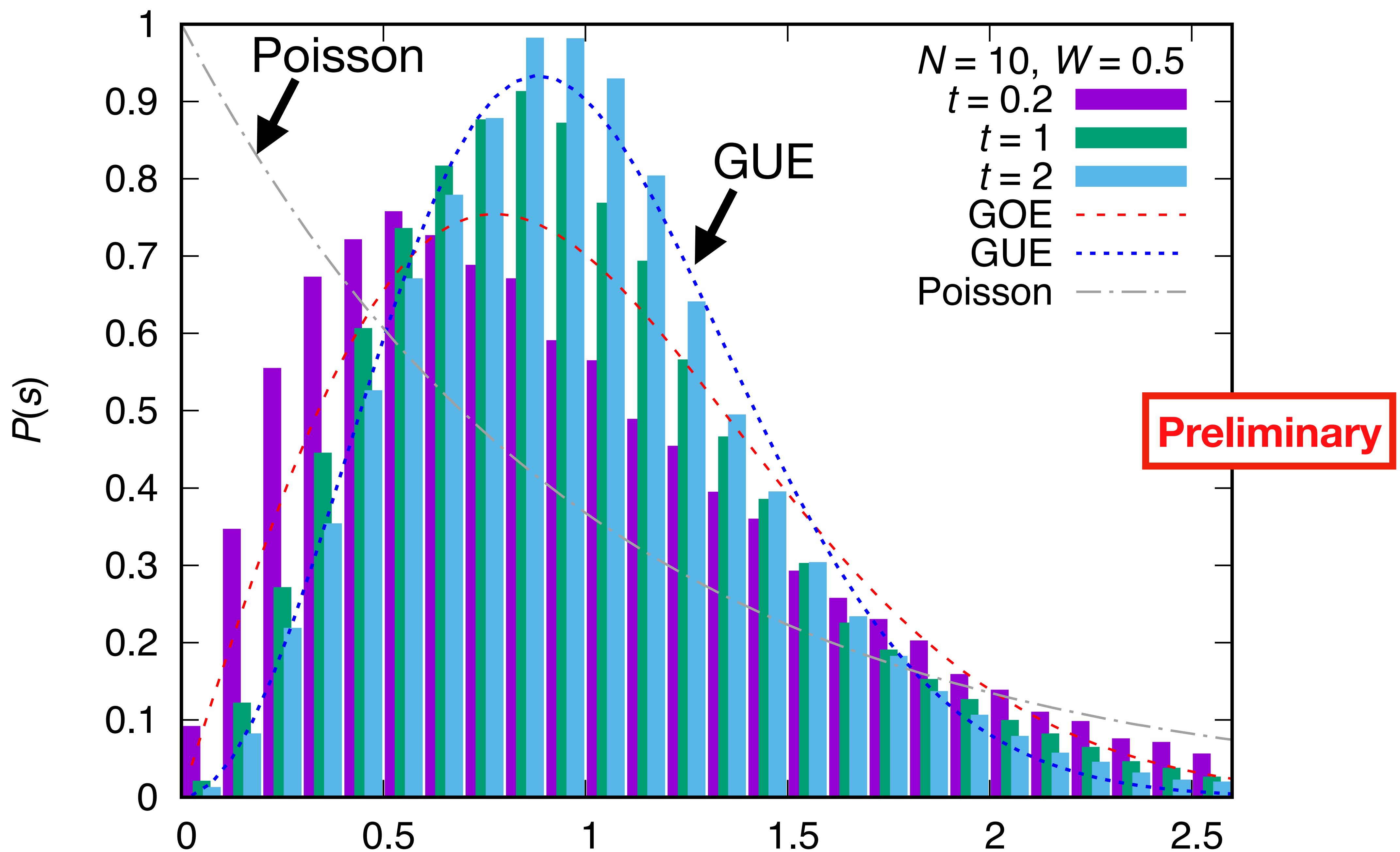
random magnetic field

$$-w \leq w_i \leq +w$$

- Ergodic at small w
- Many-body localized (MBL) at large w

$$\hat{M}_{ij} \equiv [\sigma_{+,i}(t), \sigma_{-,j}(0)]$$

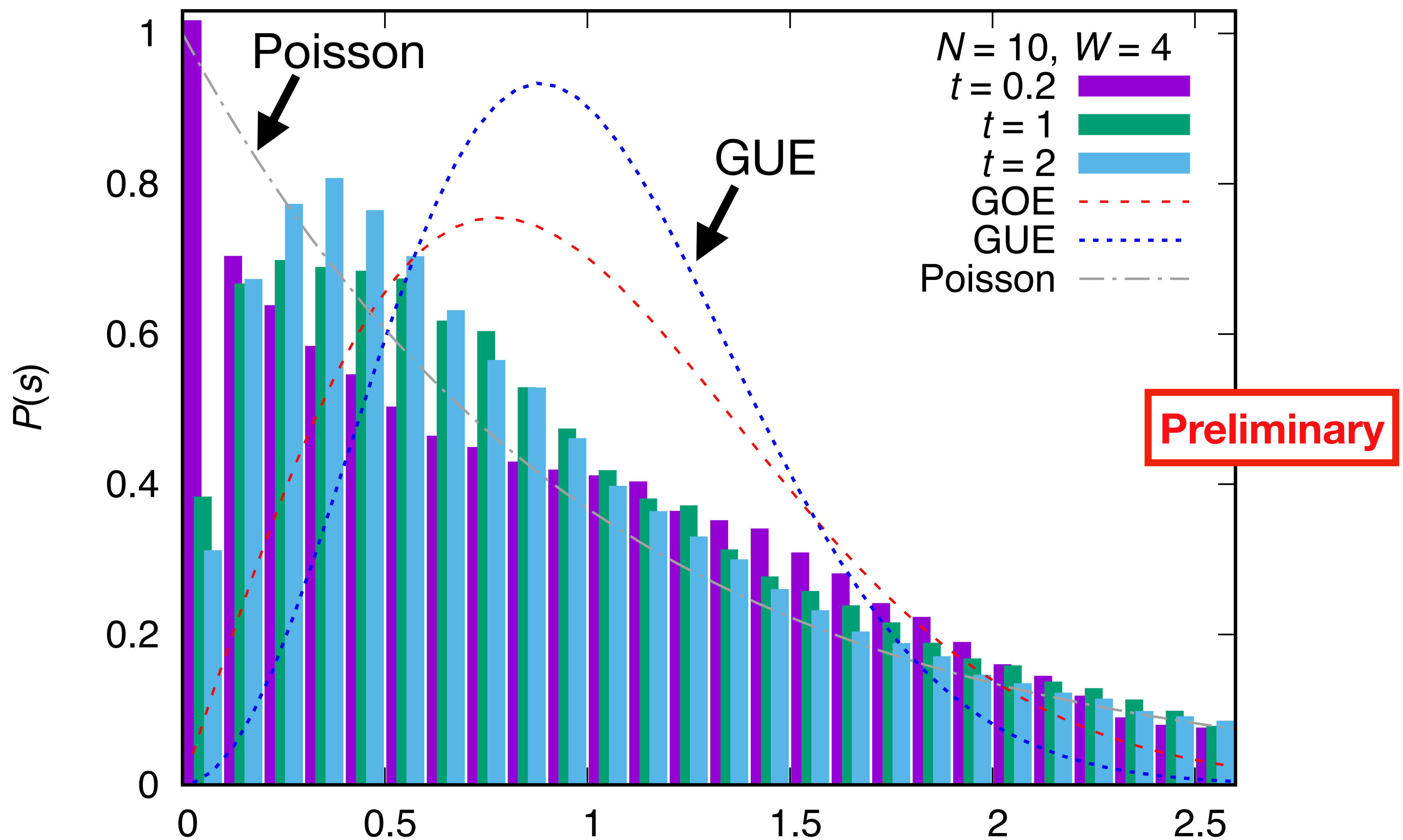
RMT vs Lyapunov spectrum in XXZ model



$$s_i = \lambda_{i+1} - \lambda_i$$

N=10, w=0.5 (ergodic phase)

RMT vs Lyapunov spectrum in XXZ model



$$s_i = \lambda_{i+1} - \lambda_i$$

N=10, w=4.0 (MBL phase)

Summary of numerical observations

- Classical chaos

  D0 matrix model — ‘strongly’ universal
  Other chaotic systems — universal

- Quantum chaos

  SYK — ‘strongly’ universal
  Other chaotic systems — universal
  MBL — not universal (Poisson-like)

- Lyapunov growth can be seen precisely.

- The largest Lyapunov exponent is not enough.
- Lyapunov spectrum captures physics more precisely.
- New universality.
- Black hole is (probably) special.
- What is the mechanism?
- How can we formulate the spectrum in gravity side?
- Relation to the late time universality (energy spectrum)?
- ‘KS entropy’ vs EE growth rate?
- Generalization of the chaos bound to KS entropy?

Topics skipped today

- Classical simulation of 2d YM

  MH-Romatschke, in preparation

  → black hole/black string topology change

- Probing geometry from matrix model via Euclidean simulation

  → more realistic real time simulation with quantum effect?

  Rinaldi-Berkowitz-MH-Maltz-Vranas, 2017

- Physical realization of QFT on optical lattice

  → experimental study of BH via holography?

  Danshita-MH-Tezuka, 2016
  Danshita-MH-Nakajima-Sundborg-Tezuka-Wintergerst, in progress

- Universality of energy spectrum in quantum chaos and implication

  to BH information problem

  Cotler-Gur Ari-MH-Polchinski-Saad-Shenker-Stanford-Streicher-Tezuka, 2016