Phase structure of a defect field theory with a domain wall.

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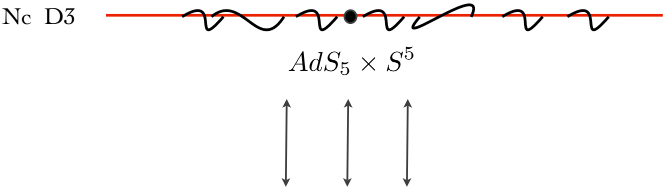
Outline

1. AdS/CFT correspondence
   - Adding flavours D3/D7 Karch & Katz
   - Meson melting phase transition
   - Critical behaviour
2. D0/D4 system
   - Lower dimensional correspondence
   - Berkooz-Douglas matrix model
   - Comparison
3. Defect field theory with a domain wall
   - Introducing probe D5-branes
   - Introducing the domain wall
   - Critical point

AdS/CFT correspondence

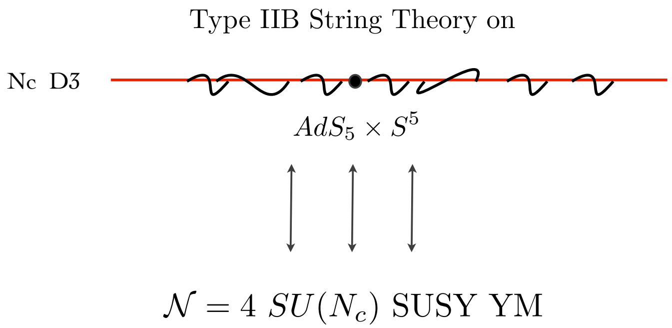
Type IIB String Theory on



$$\mathcal{N} = 4 \; SU(N_c) \text{ SUSY YM}$$

- Gubser-Klebanov-Polyakov-Witten formula:

$$\langle e^{\int d^d x \phi_0(x) \langle \mathcal{O}(x) \rangle} \rangle_{\text{CFT}} = \mathcal{Z}_{\text{string}}[\phi_0(x)]$$



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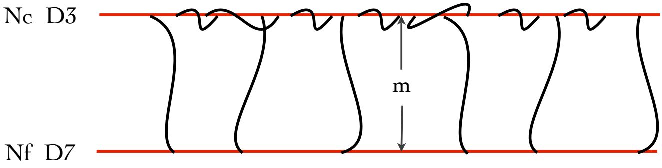
Adding flavours D3/D7 Karch & Katz

Generalizing the correspondence



	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	·	·	·	·	·	·
D7	-	-	-	-	-	-	-	-	·	·

- Adding N_f massive $\mathcal{N} = 2$ Hypermultiplets:

$$m_q \int d^2\theta\, \tilde{Q}\, Q \to \mathrm{SYM} \quad \text{with} \quad m_q = m/2\pi\alpha'$$

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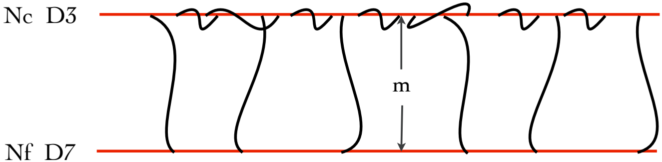

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String spectrum

3-3 strings	pure $\mathcal{N}$=4 SYM adjoint of $SU(N_c)$
3-7 strings	Q_i fundamental chiral field
7-3 strings	$\tilde{Q}^i$ anti-fundamental chiral field
7-7 strings	gauge field on the D7 brane frozen by infinite volume

Probe approximation $N_f \ll N_c$

- The probe is described by a Dirac-Born-Infeld action $S \propto \int d^7\xi \, e^{-\Phi} \sqrt{||G_{ab} - 2\pi\alpha' \mathcal{F}_{ab}||}$
- The profile of the D-brane encodes the fundamental condensate of theory. The semi-classical fluctuations correspond to meson-like excitations.
- The D-brane gauge field can describe: external electromagnetic field, chemical potential, electric current etc.
- Numerous applications: thermal and quantum phase transitions, chiral symmetry breaking, magnetic catalysis etc.

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$$ds^2 = -\frac{u^4 - u_0^4}{R^2\, u^2} dt^2 + \frac{u^2}{R^2}\, d\vec{x}^2 + \frac{u^2\, R^2}{u^4 - u_0^4} du^2 + u^2\, d\Omega_5^2\ .$$

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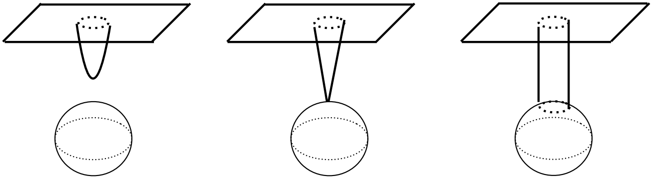

Meson melting phase transition

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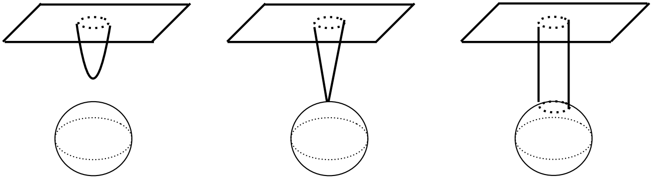

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Critical behaviour Dp/Dq

- Consider a general Dp/Dq system (D. Mateos, R. Myers, R. Thomson 2007) and the parametrisation:

$$d\Omega_{8-p}^2 = d\theta^2 + \sin^2\theta\, d\Omega_n^2 + \cos^2\theta d\Omega_{7-p-n}$$

- Next we zoom in at the near horizon geometry:

$$u = u_0 + \pi T z^2\,, \quad \theta = \frac{y}{L}\left(\frac{L}{u_0}\right)^{\frac{p-3}{4}}\,, \quad \vec{\tilde{x}} = \left(\frac{u_0}{L}\right)^{\frac{7-p}{4}} \vec{x}$$

- To obtain the metric:

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Critical behaviour Dp/Dq cont.

- The resulting EOM for Minkowski embeddings is:

$$z\,y\,\ddot{y} + (y\dot{y} - nz)(1+\dot{y}^2) = 0$$

- It has scaling symmetry: if $y(z)$ is a solution so is $y(\mu\, z)/\mu$. And a critical solution $y = \sqrt{n}\,z$, linearising we obtain:

$$y = \sqrt{n}\,z + \frac{T^{-1}}{(Tz)^{\frac{n}{2}}}\left[a\sin(\alpha\log Tz) + b\cos(\alpha\, Tz)\right] \; ,$$

- with $\alpha = \sqrt{4(n+1)-n^2}/2$. Under the scaling symmetry the constants a, b transform as:

$$\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \frac{1}{\mu^{\frac{n}{2}+1}} \begin{pmatrix} \cos(\alpha\log\mu) & \sin(\alpha\log\mu) \\ -\sin(\alpha\log\mu) & \cos(\alpha\log\mu) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

- This is a double spiral signalling a discrete self-similar structure, which forces the phase transition into a first order one.
- This behaviour is universal (depends only on n).

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What follows

- Use universality to study the D0/D4 system (same class of universality as the D3/D7 system) on a computer to test the gauge/gravity duality.
- Propose a somewhat general way to deform the transition into a second order one.

Lower dimensional correspondence

- The D3/D7 system is T-dual to the D0/D4 system, share many common properties (meson melting transition, meson spectra)
- The dual theory of the D0/D4 set-up is a flavoured version of the BFSS matrix model - the Berkooz-Douglas (BD) matrix model.
- The BD matrix model is 1D quantum mechanics and is super renormalisable, avoiding the fine tuning problem.
- Recall the metric of the D0-brane background:

$$ds^2 = -H^{-\frac{1}{2}} f\, dt^2 + H^{\frac{1}{2}} \left(\frac{du^2}{f} + u^2\, d\Omega_8^2 \right) ,$$

$$H = (L/u)^7 , \quad f(u) = 1 - (u_0/u)^7 , \quad L^7 = 15/2\, (2\pi\alpha')^5\, \lambda$$

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Holographic description: probe D4–branes

- we parametrise the unit S^8 as:

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\, d\Omega_3^2 + \sin^2\theta\, d\Omega_4^2\ ,$$

- D4 extends along t, u and Ω_3 and has a non-trivial profile $\theta(u)$.
- The profile of the D4-brane is determined by the DBI action:

$$S^E_{\rm DBI} = \frac{N_f\,\beta}{8\,\pi^2\,\alpha'^{5/2}\,g_s}\int du\, u^3 \cos^3\theta(u)\,\sqrt{1+u^2\,f(u)\,\theta'(u)^2}\ .$$

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Holographic description: probe D4–branes

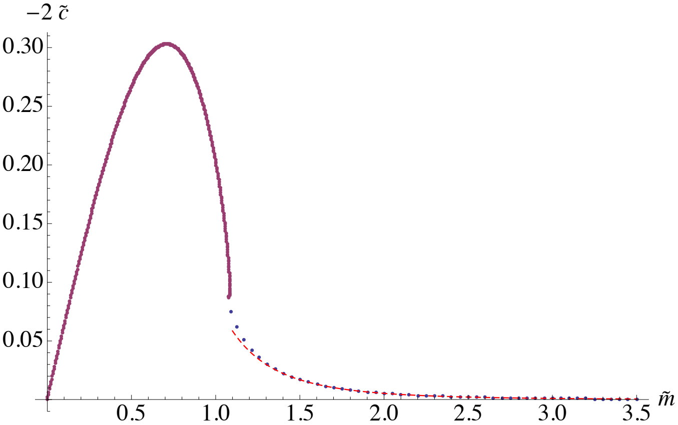
- The AdS/CFT dictionary relates the parameters $\tilde{m}$ and $\tilde{c}$ to the bare mass and fundamental condensate via:

$$
\begin{aligned}
m_q &= \left(\frac{120\,\pi^2}{49}\right)^{1/5} \left(\frac{T}{\lambda^{1/3}}\right)^{2/5} \lambda^{1/3}\,\tilde{m}\,, \\
\langle \mathcal{O}_m \rangle &= \left(\frac{2^4\,15^3\,\pi^6}{7^6}\right)^{1/5} N_f\,N_c \left(\frac{T}{\lambda^{1/3}}\right)^{6/5} (-2\,\tilde{c})\,.
\end{aligned}
$$

- It is this relation that we test on the lattice, with the precise numerical coefficients.

Holographic description: fundamental condensate

- Solving numerically the EOM for θ we extract the condensate curve:



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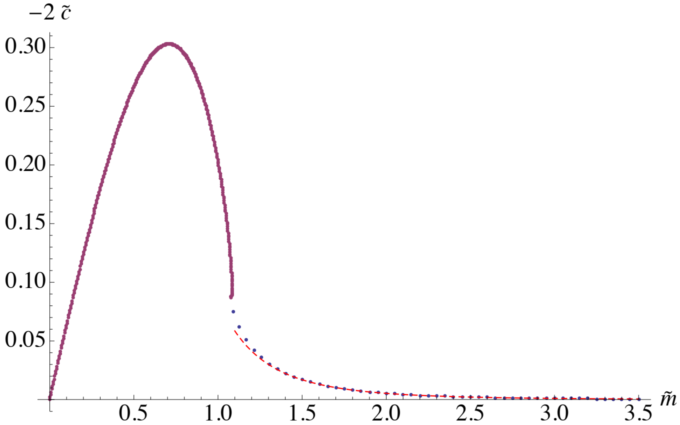

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Holographic description: Mass susceptibility

- It is the slope of the condensate curve at zero mass.
- Linearising the EOM at small mass we get:

$$\langle \mathcal{C}^m \rangle = (14\,15^2\,\pi^9)^{\frac{1}{5}} \left(\frac{\csc(\pi/7)\,\Gamma(\frac{3}{7})\,\Gamma(\frac{5}{7})}{\Gamma(\frac{1}{7})^2\,\Gamma(\frac{2}{7})\,\Gamma(\frac{4}{7})} \right) N_f\, N_c \left(\frac{T}{\lambda^{1/3}} \right)^{4/5}$$

$$\approx 1.136\, N_f\, N_c \left(\frac{T}{\lambda^{1/3}} \right)^{4/5} .$$

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Berkooz-Douglas matrix model

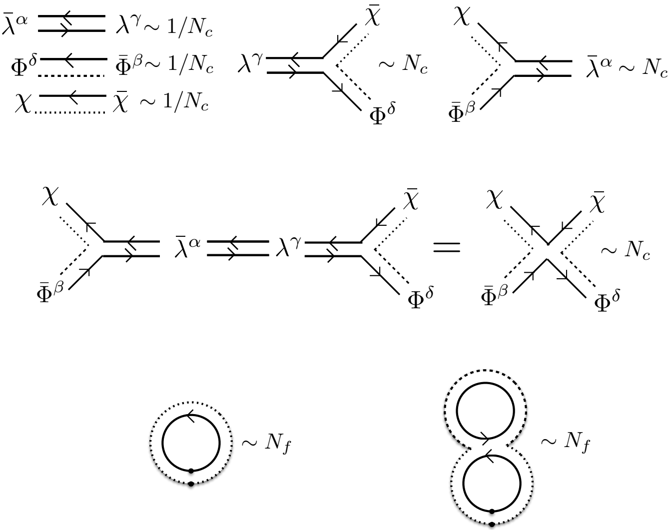
- Original motivation -M_5 brane density hep-th/9610236 (Berkooz & Douglas).
- Reducing the D5/D9 system (Van Raamsdonk, hep-th/0112081):

$$\begin{aligned} \mathcal{L} &= \frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}D_0X^aD_0X^a + \frac{i}{2}\lambda^{\dagger\,\rho}D_0\lambda_\rho + \frac{1}{2}D_0\bar{X}^{\rho\dot{\rho}}D_0X_{\rho\dot{\rho}} + \frac{i}{2}\theta^{\dagger\dot{\rho}}D_0\theta_{\dot{\rho}}\right) \\ &+\frac{1}{g^2}\mathrm{tr}\left(D_0\bar{\Phi}^\rho D_0\Phi_\rho + i\chi^\dagger D_0\chi\right) + \mathcal{L}_{\rm int} \end{aligned}$$

where:

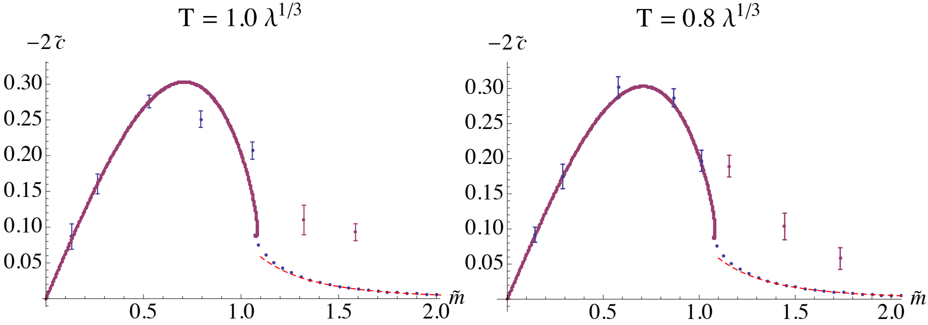
$$\begin{aligned} \mathcal{L}_{\rm int} &= \frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[X^a,X^b][X^a,X^b] + \frac{1}{2}[X^a,\bar{X}^{\rho\dot{\rho}}][X^a,X_{\rho\dot{\rho}}] - \frac{1}{4}[\bar{X}^{\alpha\dot{\alpha}},X_{\beta\dot{\alpha}}][\bar{X}^{\beta\dot{\beta}},X_{\alpha\dot{\beta}}]\right) \\ &-\frac{1}{g^2}\mathrm{tr}\left(\bar{\Phi}^\rho(X^a-m^a)(X^a-m^a)\Phi_\rho\right) \\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar{\Phi}^\alpha[\bar{X}^{\beta\dot{\alpha}},X_{\alpha\dot{\alpha}}]\Phi_\beta + \frac{1}{2}\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\alpha - \bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\right) \\ &+\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}\bar{\lambda}^\rho\gamma^a[X^a,\lambda_\rho] + \frac{1}{2}\bar{\theta}^{\dot{\alpha}}\gamma^a[X^a,\theta_{\dot{\alpha}}] - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\theta}^{\dot{\alpha}}[X_{\beta\dot{\alpha}},\lambda_\alpha]\right) \\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar{\chi}\gamma^a(X^a-m^a)\chi + \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\chi}\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\Phi}^\alpha\bar{\lambda}_\beta\chi\right) \end{aligned}$$

Quenched versus dynamical



Comparison with lattice: fundamental condensate

- We present condensate curves generated for $N = 10$, $\Lambda = 16$ and $T/\lambda^{1/3} = 0.8, 1.0$ (work with D. O'Connor JHEP 1605 (2016) 122)



- Excellent agreement at small $\tilde{m}$.
- For smaller T it extends to the whole black hole phase!
- Significant deviations in the confined phase.

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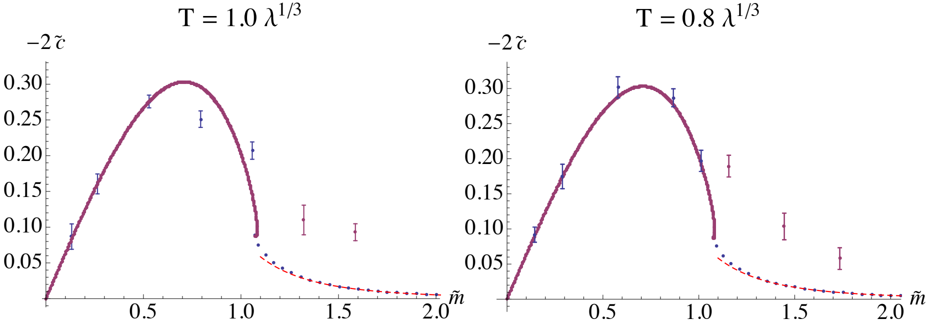

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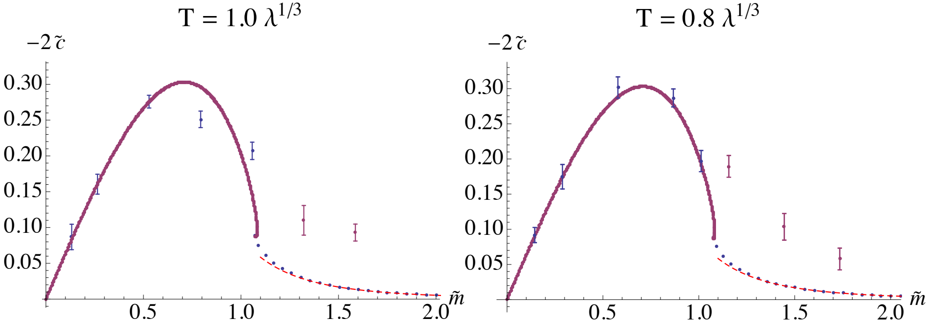

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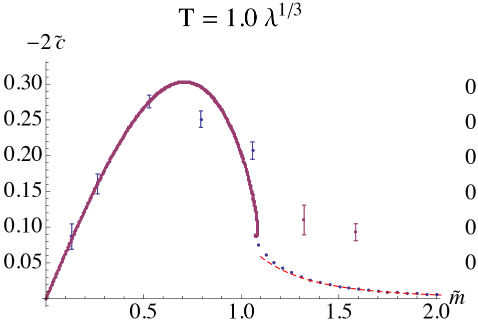

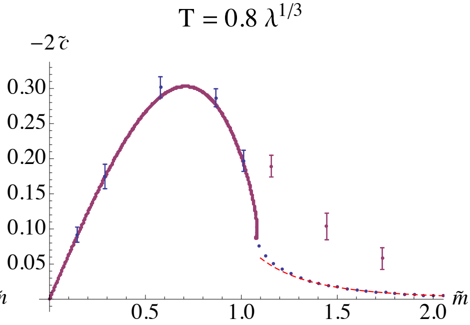

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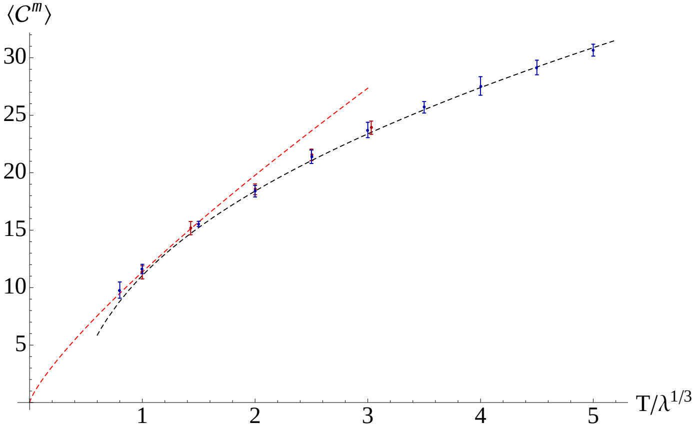
Comparison with lattice: Mass susceptibility

- Work with D. O'Connor, S.Kovacik and Y. Asano JHEP 1701 (2017) 113, JHEP 1803 (2018) 055
- Lattice data for $N = 10$, $\Lambda = 16$ and $0.8 \leq T/\lambda^{1/3} \leq 5$



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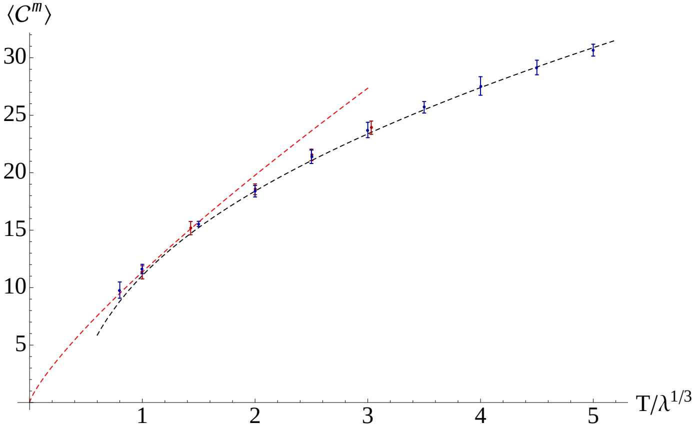

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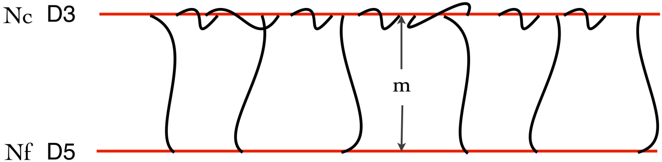
Generalizing the correspondence



	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	·	·	·	·	·	·
D5	-	-	-	·	-	-	-	·	·	·

- Adding N_f massive $\mathcal{N}=2$ Hypermultiplets:

$$m_q \int d^2\theta\, \tilde{Q}\, Q \rightarrow \mathrm{SYM} \quad \text{with} \quad m_q = m/2\pi\alpha'$$

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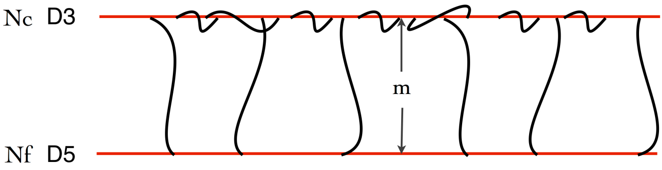

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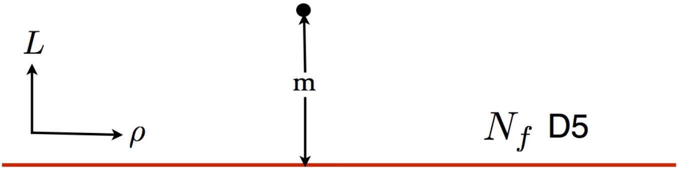
Probe D5 branes

- In the context of the AdS/CFT we “substitute" the D3-branes with an $AdS_5 \times S^5$ background:

$$ds^2 = \frac{\rho^2 + l^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + l^2}\left(d\rho^2 + \rho^2 d\Omega_2^2 + dl^2 + l^2 d\tilde{\Omega}_2^2\right) \ .$$

- The D5-brane is extended along $(x^0, x^1, x^2, \rho, \Omega_2)$. The separation along l corresponds to the bare mass m.



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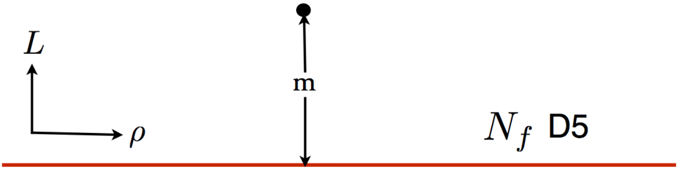

Introducing the domain wall

- The full action of the D5-brane is:

$$S_{D5} = -\frac{\mu_5}{g_s}\int d^6\xi\, e^{-\Phi}\sqrt{|G_{ab}+\mathcal{F}_{ab}|} + \mu_5\int \mathcal{P}\left[\sum_p C_p \wedge e^{\mathcal{F}}\right]$$

- We will show that fixing the gauge field on the internal S^2 will introduce a domain wall. Consider the (consistent anzats):

$$B_{(2)} = H\,R^2\,\Omega_{(2)}\ .$$

- The flux through the S^2 is equal to:

$$\int_{S^2} B_{(2)} = 4\pi\,H\,R^2 = \text{const}$$

- Consider any ball B^3 in the $\mathbb{R}^3$ parametrised by ρ, S^2:

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- Therefore we have that:

$$dB_{(2)} = 4\pi\, R^2\, H\, \delta^{(3)}(\vec{\rho}\,)d\rho \wedge d\Omega_{(2)}$$

- Now consider the main contribution to the WZ term:

$$S_{WZ} = \mu_5 \int\limits_{\mathcal{M}_6} C_{(4)} \wedge B_{(2)}$$

- Gauge invariance (charge conservation) for the $C_{(4)}$ RR form demands invariance under $C_{(4)} \to C_{(4)} + d\Lambda_{(3)}$:
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$$\frac{\delta S_{WZ}}{\mu_5} = \int\limits_{\mathcal{M}_6} d\Lambda_{(3)} \wedge B_{(2)} = -\int\limits_{\mathcal{M}_6} \Lambda_{(3)} \wedge dB_{(2)} = -(4\pi R^2 H)\int\limits_{\mathcal{M}_3} \Lambda_{(3)}$$

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- To cancel this contribution we need to consider D3 brane charges ending at $\mathcal{M}_3$

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- We need:

$$\Phi_H = 4\pi\, R^2 \left(\frac{H}{2\pi\alpha'}\right) = 2\pi\, k\ ,$$

- The other end of the extra D3 ends on the background D3 branes and introduces a boundary. To preserve the $C_{(4)}$ charge we have different number of D3-branes on both sides of the boundary.

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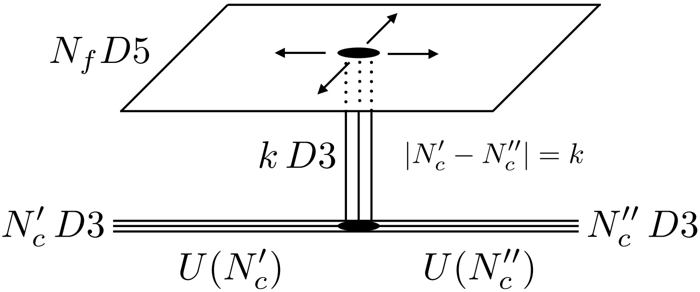

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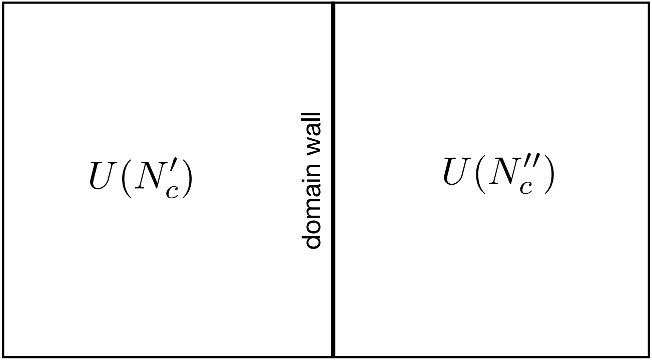

Probe D5 branes

- To introduce temperature we consider the AdS-black hole solution:

$$ds^2 = -\frac{u^4 - u_0^4}{u^2 R^2} dt^2 + \frac{u^2}{R^2} d\vec{x}^2 + \frac{u^2 R^2}{u^4 - u_0^4} du^2 + R^2 d\Omega_5^2$$

$$d\Omega_5^2 = d\theta^2 + \cos^2\theta d\Omega_2^2 + \sin^2\theta d\tilde{\Omega}_2^2$$

- The D5-brane wraps t, x_1, x_2, u, Ω_2 and has a profile in θ and x_3. The corresponding Lagrangian is

$$R^2 \mathcal{L}_{tot} = -H u^4 x_3'(u) + u \sqrt{H^2 + \cos^4\theta(u)} \times$$
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- It is instructive to consider the case of trivial profile along $\theta(u) \equiv 0$.

$$R^2\,\mathcal{L}_{red} = -H\,u^4\,x_3'(u) + u\,H\,\sqrt{(u^6 - u^2\,u_0^4)x_3'(u)^2 + R^4\,u^2}$$

- The EOM for x_3 can be solved in closed form but at it is instructive to solve it perturbatively at large u:

$$x_3(u) = x_{3,\infty} - \frac{H\,R^2}{u} + \frac{c_{x3}}{u^5} + O\left(\frac{1}{u^9}\right)$$

- $x_{3,\infty}$ is related to source and c_{x_3} is related to the VEV of the dual operator. Their powers in the expansion should differ by $2\Delta - d$ and hence $\Delta = 4$.

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Probe D5 branes

- Going back to the full Lagrangian and using that x_3 is cyclic:

$$\frac{\partial \mathcal{L}_{tot}}{\partial x_3'(u)} = \mathrm{const} = -\frac{H\, u_0^2}{L^2} \propto \langle \mathcal{O}_{x3} \rangle$$

- The dual operator $\mathcal{O}_{x3}$ is

$$\mathcal{O}_{x3} \equiv \frac{\delta S_{\text{fund}}}{\delta x_3} \propto \bar{q}^m \partial_{x_3} \left(X_V^A X_V^A \right) q^m + \dots$$

- Now we Legendre transform along x_3

$$\tilde{\mathcal{L}} = \mathcal{L}_{\text{tot}} - \frac{\partial \mathcal{L}_{tot}}{\partial x_3'} x_3' = \sqrt{u^4 \cos^4\theta(u) + H^2 u_0^4} \sqrt{1 + \left(\frac{u^4 - u_0^4}{u^2} \right) \theta'(u)^2}$$

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Probe D5 branes

- The EOM for θ has the asymptotic solution:

$$\theta(u) = \frac{m}{u} + \frac{c}{u^2} + \dots$$

- The on-shell action can be regularised adding the following counter terms:

$$\begin{aligned} \mathcal{L}_1 &\propto -\frac{1}{3}\sqrt{-\gamma} = -\frac{1}{3}u^3 + \dots \\ \mathcal{L}_2 &\propto +\frac{1}{2}\sqrt{-\gamma}\theta^2 = \frac{1}{2}m^2\, u + \dots \end{aligned}$$

AdS-black hole

- One can then show that:

$$\langle \mathcal{O}_m \rangle = \langle \frac{\delta S_{\text{fund}}}{\delta \theta} \rangle \propto -c$$

- This allows us to explore the phase structure of the theory by studying the condensate of the theory as a function of the bare mass
- There are two classes of embeddings with different topologies:
  - Minowski - closing by a shrinking S^2 above the horizon of the BH. Representing confined phase.
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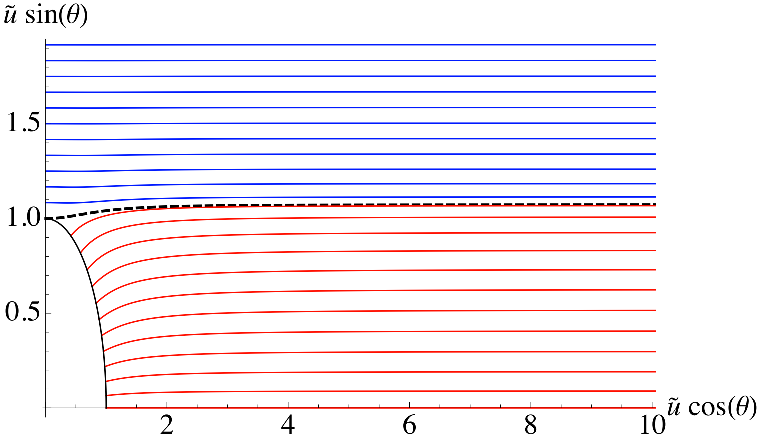
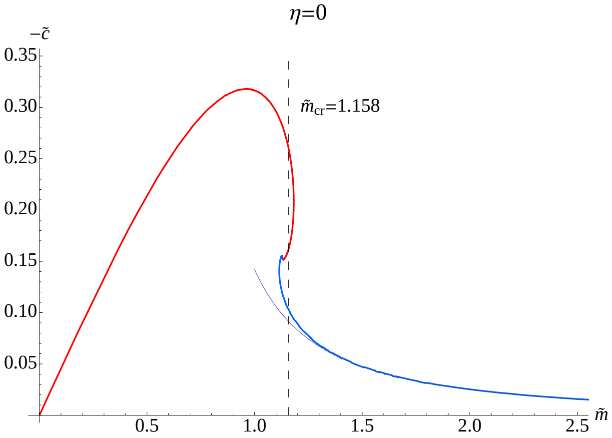

$\tilde{u}\,\sin(\theta)$
1.5
1.0
0.5
2
4
6
8
10
$\tilde{u}\,\cos(\theta)$

Phase Transition

There is a first order phase transition:



Effect of magnetic monopole

- When the flux on the internal S^2 is turned on. Minkowski embeddings are incomplete - a magnetic monopole is needed.
- Remarkably the BH embeddings develop a D3-brane throat and mimic Minkowski embeddings.
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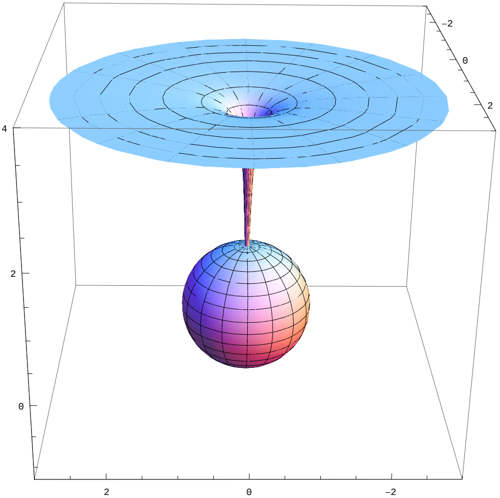
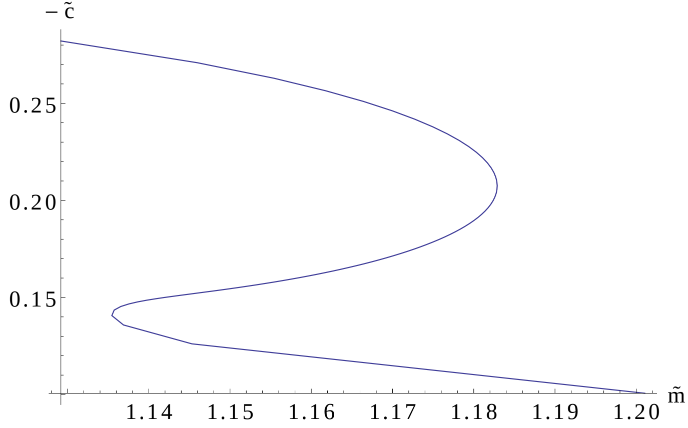
D3-brane throat



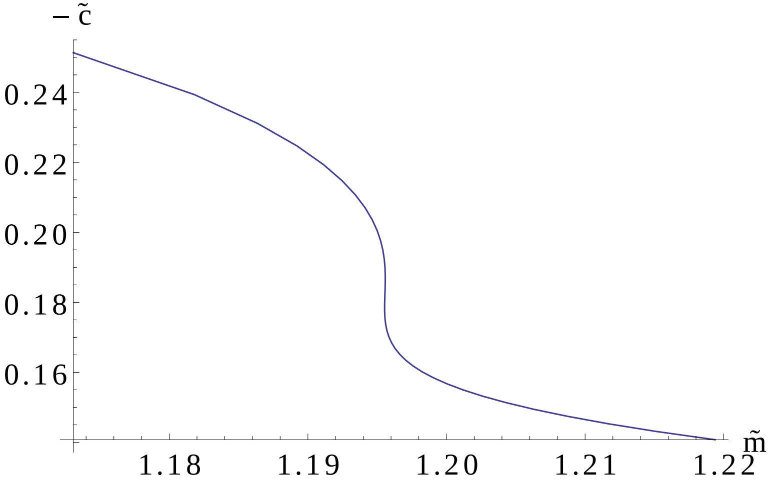
Critical point

- For small $H < H_{cr} \approx 0.044$:



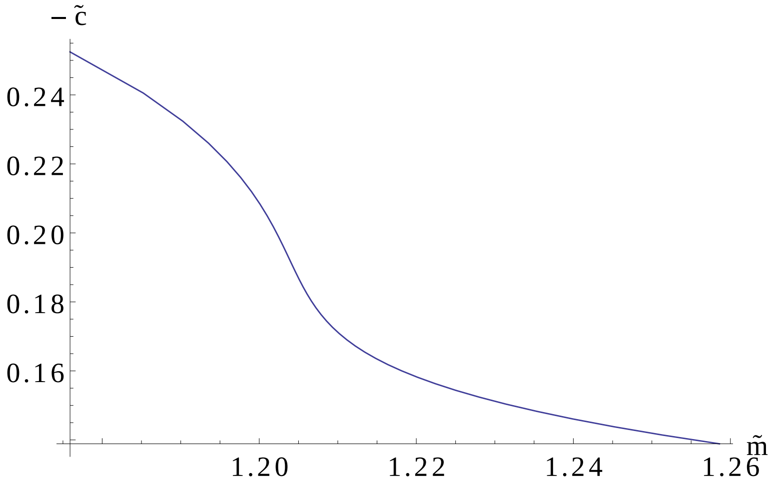
Critical point

- At the critical point $H = H_{cr} \approx 0.044$:



Critical point

- And crossover for $H > H_{cr} \approx 0.044$:



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- We reviewed checks of the AdS/CFT duality with flavour.
- We studied the D3/D5 holographic set-up with a transverse flux on the internal S^2.
- The resulting dual theory has domain wall separating areas with different gauge groups.
- The set-up renders Minkowski embeddings incomplete but they are realised as BH embeddings with a D3-brane throat.
- The phase diagram features critical line of first order phase transition ending on a critical point of a second order phase transition.
- Future work: obtain the critical exponents of the transition at criticality, study stability.

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Thank you!