# Quantized cosmological spacetimes and higher spin in the IKKT model

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Quantum Structure of Spacetime

# Motivation

Matrix Models ... natural framework for fundamental theory

- pre-geometric, constructive
- dynamical “quantum” (NC) spaces, gauge theory
- stringy features

  max. SUSY $\rightarrow$ inherit good behavior of critical string (UV)
- avoid string compactifications

  $\rightarrow$ need different mechanism for gravity & chirality
- IKKT: allows to describe “beginning of time”!

outline:

- matrix models & matrix geometry
- *4D covariant quantum spaces*: fuzzy $S^4_N$, $H^4_n$
- cosmological space-times: $\mathcal{M}^{3,1}$ & BB!
- fluctuations $\rightarrow$ higher spin gauge theory
- metric, vielbein; gravity?

HS, arXiv:1606.00769
M. Sperling, HS arXiv:1707.00885
HS, arXiv:1709.10480, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907

# The IKKT model

**IKKT or IIB model** Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X,\Psi] = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \quad + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N,\mathbb{C})\,, \qquad a = 0,...,9, \qquad N \text{ large}$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY

proposed as non-perturbative definition of IIB string theory

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, $N$ large
- add $m^2 X^a X_a$ to set scale, IR regularization

$$Z = \int dX d\Psi\, e^{iS[X]}$$

Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff

different points of view:

- classical solutions $=$ "branes"

  justified by max. SUSY (cf. critical string thy)

  generically NC geometry, "matrix geometry"

  fluctuations $\rightarrow$ field theory, 3+1D physics, dynamical geometry
  UV/IR mixing $\rightarrow$ IKKT model $\rightarrow$ unique 4D NC gauge theory

**hypothesis**

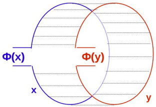
space-time = (near-) classical solution of IIB model

- 10 bulk physics:

  sugra arises in M.M. from quantum effects (loops)

  Kabat-Taylor, IKKT,...

  "holographic"




cf. HS arXiv:1606.00646

## "matrix geometry" (≈ NC geometry):

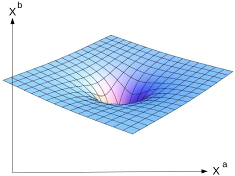
- $S_E \sim Tr[X^a, X^b]^2 \Rightarrow$ config's with small $[X^a, X^b] \neq 0$ dominate

  i.e. "almost-commutative" configurations

- $\exists$ quasi-coherent states $|x\rangle$, minimize $\sum_a \langle x|\Delta X_a^2|x\rangle$

  $X^a \approx$ diag., spectrum $=: \mathcal{M} \subset \mathbb{R}^{10}$

  $$\langle x|X^a|x'\rangle \approx \delta(x - x')x^a, \qquad x \in \mathcal{M}$$




- hypothesis: classical solutions dominate

  "condensation" of matrices, geometry

**NC branes embedded in target space $\mathbb{R}^{10}$**

$$X^a \sim x^a : \qquad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

cf. Q.M: replace functions $x^a \rightsquigarrow$ matrices / observables $X^a$

typical examples: quantized Poisson manifolds

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$$

quantized symplectic space $(\mathbb{R}^4, \omega)$

admits translations, no rotation invariance

- fuzzy 2-sphere $S^2_N$

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe; Madore)

**generically:**

fluctuations $\to$ NC gauge theory, & dynamical geometry issues for NC spaces / field theory:

- quantization $\rightarrow$ UV / IR mixing

  $\hookrightarrow$ max. SUSY model: IKKT, BFSS, BMN
- Lorentz / $SO(4)$ covariance in 4D ?

  - obstacle: NC spaces: $[X^\mu, X^\nu] =: i\theta^{\mu\nu} \neq 0$

    breaks Lorentz invariance
  - $\exists$ fully covariant fuzzy four-sphere $S^4_N$

  Grosse-Klimcik-Presnajder 1996; Castelino-Lee-Taylor; Ramgoolam; Kimura; Abe

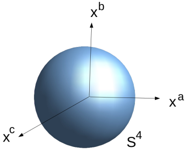
  Hasebe; Medina-O'Connor; Karabali-Nair; Zhang-Hu 2001 (QHE!) ...

  price to pay: "internal structure" $\rightarrow$ higher spin theory

# covariant fuzzy four-sphere $S^4_N$

5 hermitian matrices $X_a, \ a = 1, ..., 5$ acting on $\mathcal{H}_N$

$$\sum_a X_a^2 = R^2$$




covariance: $X_a \in End(\mathcal{H}_N)$ transform as vectors of $SO(5)$

$$\begin{aligned} [\mathcal{M}_{ab}, X_c] &= i(\delta_{ac}X_b - \delta_{bc}X_a), \\ [\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= i(\delta_{ac}\mathcal{M}_{bd} - \delta_{ad}\mathcal{M}_{bc} - \delta_{bc}\mathcal{M}_{ad} + \delta_{bd}\mathcal{M}_{ac}) \ . \end{aligned}$$

$\mathcal{M}_{ab}$ ... $so(5)$ generators acting on $\mathcal{H}_N$

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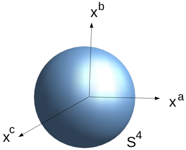


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$\mathcal{M}_{ab}$ ... $so(5)$ generators acting on $\mathcal{H}_N$

oscillator construction: Grosse-Klimcik-Presnajder 1996; ...

$$\begin{aligned} X_a &= \psi^\dagger \gamma_a \psi, \\ \mathcal{M}^{ab} &= \psi^\dagger \Sigma^{ab} \psi \end{aligned} \qquad [\psi^\beta, \psi^\dagger_\alpha] = \delta^\beta_\alpha$$

acting on $\mathcal{H}_N = \psi^\dagger_{\alpha_1} ... \psi^\dagger_{\alpha_N} |0\rangle \ \cong \ (\mathbb{C}^4)^{\otimes_S N} \ \cong \ (0, N)_{so(5)}$

relations:

$$X_a X_a = R^2 \sim \frac{1}{4} r^2 N^2$$

$$[X^a, X^b] = i r^2 \mathcal{M}^{ab} =: i\Theta^{ab}$$

$$\epsilon^{abcde} X_a X_b X_c X_d X_e = (N+2) R^2 r^3 \qquad \text{(volume quantiz.)}$$

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geometry from coherent states $|p\rangle$:

$$\boxed{\{p_a = \langle p|X_a|p\rangle\} = S^4}$$

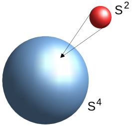
closer inspection:

degeneracy of coherent states, “internal” $S^2$ fiber

cf. Karczmarek, Yeh, arXiv:1506.07188

semi-classical picture: hidden bundle structure

$$\begin{array}{ccc} \mathbb{C}P^3 & \ni & \psi \\ \downarrow & & \downarrow \\ S^4 & \ni & x^a = \psi^+ \Gamma^a \psi \end{array}$$




Ho-Ramgoolam, Medina-O'Connor, Abe, ...

fuzzy case:
oscillator construction $[\Psi, \Psi^\dagger] = \delta \ \rightarrow$ functions on fuzzy $\mathbb{C}P^3_N$

fuzzy $S^4_N$ is really fuzzy $\mathbb{C}P^3_N$, hidden extra dimensions $S^2$ !

Poisson tensor

$$\theta^{\mu\nu}(x,\xi) \sim -i[X^\mu, X^\nu]$$

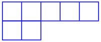
local $SO(4)_x$ rotates fiber $\xi \in S^2$

averaging over fiber $\rightarrow$ $[\theta^{\mu\nu}(x,\xi)]_0 = 0$ , local $SO(4)$ preserved!

... 4D "covariant" quantum space

# fields and harmonics on $S^4_N$

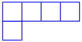
"functions" on $S^4_N$:

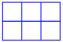
$$\boxed{End(\mathcal{H}_N) \cong \bigoplus_{s=0}^{N} \mathcal{C}^s} \qquad \mathcal{C}^s = \bigoplus_{n=0}^{N} (n, 2s) \ni$$ 

$(n, 0)$ modes = scalar functions on $S^4$:

$$\phi(X) = \phi_{a_1 \ldots a_n} X^{a_1} \ldots X^{a_n} \quad =$$ 

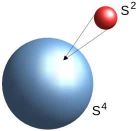
$(n, 2)$ modes = selfdual 2-forms on $S^4$

$$\phi_{bc}(X)\theta^{bc} = \phi_{a_1 \ldots a_n b;c} X^{a_1} \ldots X^{a_n} \theta^{bc} \quad =$$ 

$End(\mathcal{H}) \cong$ fields on $S^4$ taking values in $\mathfrak{hs} = \oplus$ 

**higher spin modes = would-be KK modes on $S^2$**

(local $SO(4)$ acts on $S^2$ fiber)



relation with spin $s$ fields: one-to-one map

$$\mathcal{C}^s \cong T^{*\otimes_S s} S^4$$

$$\phi^{(s)} = \phi^{(s)}_{b_1\ldots b_s;c_1\ldots c_s}(x)\,\theta^{b_1 c_1}\ldots\theta^{b_s c_s} \mapsto \phi^{(s)}_{c_1\ldots c_s}(x) = \phi^{(s)}_{b_1\ldots b_s;c_1\ldots c_s} x^{b_1}\ldots x^{b_s}$$

$$\{x^{c_1},\ldots,\{x^{c_s},\phi^{(s)}_{c_1\ldots c_s}(x)\}\ldots\} \leftarrow \phi^{(s)}_{c_1\ldots c_s}(x)$$

... "symbol" of $\phi \in \mathcal{C}^s$

M. Sperling & HS, arXiv:1707.00885

$\mathcal{C}^s \cong$ symm., traceless, tang., div.-free rank $s$ tensor field on $S^4$

$$\begin{aligned} \phi_{c_1\ldots c_s}(x) x^{c_i} &= 0\,, \\ \phi_{c_1\ldots c_s}(x) g^{c_1 c_2} &= 0\,, \\ \partial^{c_i}\phi_{c_1\ldots c_s}(x) &= 0\,. \end{aligned}$$

Poisson calculus: (semi-classical limit) M. Sperling & HS, 1806.05907

$\mathbb{C}P^3$ = symplectic manifold, $\{x^a, x^b\} = \theta^{ab}$

$$\eth^a \phi := -\frac{1}{r^2 R^2} \theta^{ab} \{x_b, \phi\}, \qquad \{x^a, \cdot\} = \theta^{ab} \eth_b$$

satisfy

$$\eth^a x^c \quad = P_T^{ac} = g^{ac} - \frac{1}{R^2} x^a x^c$$

matrix Laplacian:

$$\Box = [x^a, [x_a, .]] \sim -\{x^a, \{x_a, .\}\} = -r^2 R^2 \, \eth^a \eth_a$$

covariant derivative:

$$\nabla = P_T \circ \eth, \qquad \nabla \theta^{ab} = 0$$

curvature

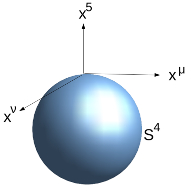
$$\mathcal{R}_{ab} := \mathcal{R}[\eth_a, \eth_b] = [\nabla_a, \nabla_b] - \nabla_{[\eth_a, \eth_b]}$$

... Levi-Civita connection on $S^4$

local description: pick north pole $p \in S^4$

$\to$ tangential & radial generators

$$X^a = \begin{pmatrix} X^\mu \\ X^5 \end{pmatrix}, \qquad \mu = 1, ..., 4 \text{...tangential coords at } p$$




separate $SO(5)$ into $SO(4)$ & translations

$$\mathcal{M}^{ab} = \begin{pmatrix} \mathcal{M}^{\mu\nu} & \mathcal{P}^\mu \\ -\mathcal{P}^\mu & 0 \end{pmatrix} \qquad \text{where} \quad \mathcal{P}^\mu = \mathcal{M}^{\mu 5}$$

Poisson algebra $\{P_\mu, X^\nu\} \approx \delta^\nu_\mu$ locally local form of spin s harmonics: e.g. spin 2:

$$\phi^{(2)} = \phi_{\mu\nu}(x) P^\mu P^\nu + \omega_{\mu:\alpha\beta}(x) P^\mu \mathcal{M}^{\alpha\beta} + \Omega_{\alpha\beta;\mu\nu}(x) \mathcal{M}^{\alpha\beta} \mathcal{M}^{\mu\nu}$$

recall $End(\mathcal{H}) = \oplus \mathcal{C}^s$, $\mathcal{C}^s \cong$ rank $s$ tensor fields $\phi_{a_1 \dots a_s}(x)$

unique irrep $(n, 2s) \in End(\mathcal{H}) \Rightarrow$ constraints!

$$\begin{aligned} \omega_{\mu;\alpha\beta} &\propto \partial_\alpha \phi_{\mu\beta} - \partial_\beta \phi_{\mu\alpha} \\ \Omega_{\alpha\beta;\mu\nu} &\propto \mathcal{R}_{\alpha\beta\mu\nu}[\phi] \end{aligned}$$

... linearized spin connection and curvature determined by $\phi_{\mu\nu}$

similarly:

cosmological quantum space-times $\mathcal{M}^{3,1}_n$:

- exactly homogeneous & isotropic
- finite density of microstates
- mechanism for Big Bang
- starting point: fuzzy hyperboloid $H^4_n$

# Euclidean fuzzy hyperboloid $H^4_n$

Hasebe arXiv:1207.1968

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace of $\mathcal{M}^{05}$ is $n+1$-dim. irrep of $SU(2)_L$: fuzzy $S^2_n$

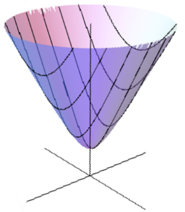
## fuzzy hyperboloid $H^4_n$

def.

$$\begin{aligned} X^a &:= r\mathcal{M}^{a5}, \qquad a = 0, ..., 4 \\ [X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab} \end{aligned}$$

5 hermitian generators $X^a = (X^a)^\dagger$ satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2 - 4)$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric: Euclidean $AdS^4$

oscillator construction: 4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta^\beta_\alpha$

$\mathcal{H}_n$ = suitable irrep in Fock space

Then

$$\mathcal{M}_{ab} = \bar\psi\Sigma_{ab}\psi, \qquad \gamma_0 = diag(1,1,-1,-1)$$
$$X^a = r\bar\psi\gamma^a\psi$$

$H^4_n$ = quantized $\mathbb{C}P^{1,2} = S^2$ bundle over $H^4$, selfdual $\theta^{\mu\nu}$

analogous to $S^4_N$, finite density of microstates fuzzy "functions" on $H^4_n$:

$$End(\mathcal{H}_n) \cong \bigoplus_{s=0}^{n} \mathcal{C}^s = \int_{\mathbb{C}P^{1,2}} d\mu\, f(m)\, |m\rangle\langle m|$$

= fields on $H^4$ taking values in $\mathfrak{hs} = \oplus_s \begin{array}{|c|c|c|}\hline & & \\ \hline & & \\ \hline\end{array} \ni \mathcal{M}^{a_1 b_1} \ldots \mathcal{M}^{a_s b_s}$

spin $s$ sectors $\mathcal{C}^s$ selected by spin Casimir

$$\mathcal{S}^2 = \sum_{a<b\leq 4} [\mathcal{M}^{ab}, [\mathcal{M}_{ab}, \cdot]] + r^{-2}[X_a, [X^a, \cdot]] \ ,$$

can show:

$$\mathcal{S}^2|_{\mathcal{C}^s} = 2s(s+1), \qquad s = 0, 1, ..., n$$

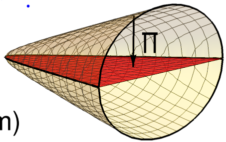
M. Sperling & H.S. 1806.05907

# open FRW universe from $H^4_n$

$$\boxed{Y^\mu := X^\mu, \quad \text{for } \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{ !)}}$$

$\mathcal{M}^{3,1}_n$ = projected $H^4_n$ embedded in $\mathbb{R}^{1,3}$ via projection

$$Y^\mu \sim y^\mu : \mathbb{C}P^{1,2} \to H^4 \stackrel{\Pi}{\longrightarrow} \mathbb{R}^{1,3} .$$




satisfies

$$[Y^\mu,[Y^\mu,Y^\nu]] = ir^2[Y^\mu,\mathcal{M}^{\mu\nu}] \qquad \text{(no sum)}$$
$$= r^2 \begin{cases} Y^\nu, & \nu \neq \mu \neq 0 \\ -Y^\nu, & \nu \neq \mu = 0 \\ 0, & \nu = \mu \end{cases}$$

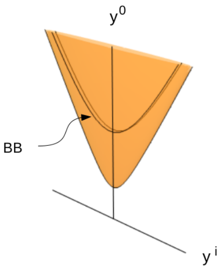
hence

$$\boxed{\Box_Y Y^\mu = [Y^\nu,[Y_\nu,Y^\mu]] = 3r^2 Y^\mu .}$$

.... solution of IKKT with $m^2 = 3r^2$.

HS arXiv:1710.11495

properties:




- $SO(3,1)$ manifest $\Rightarrow$ foliation into $SO(3,1)$-invariant space-like 3-hyperboloids $H^3_t$
- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

$$ds^2 \quad = dt^2 - a(t)^2 d\Sigma^2,$$

$d\Sigma^2$ ... $SO(3,1)$-invariant metric on space-like $H^3$

# metric properties

reference point $p \in H^4 \subset \mathbb{R}^{1,4}$

$$p^a = R(\cosh(\eta), \sinh(\eta), 0, 0, 0)$$

induced metric:

$$g_{\mu\nu} = (-1, 1, 1, 1) = \eta_{\mu\nu}, \qquad \mu, \nu = 0, 1, 2, 3 \qquad \text{(Minkowski!)}$$

$\rightarrow$ Milne metric:

$$ds^2_g = -dt^2 + t^2\, d\Sigma^2$$

however: induced metric $\neq$ effective (“open string”) metric

## effective metric (for scalar fields)

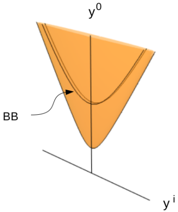
H.S. arXiv:1003.4134

encoded in $\Box_Y = [Y_\mu, [Y^\mu, .]] \sim \frac{1}{\sqrt{|G|}} \partial_\mu(\sqrt{|G|} G^{\mu\nu} \partial_\nu .)$:

$$G^{\mu\nu} = \alpha\, \gamma^{\mu\nu}, \qquad \alpha = \sqrt{\frac{|\theta^{\mu\nu}|}{|\gamma^{\mu\nu}|}},$$
$$\gamma^{\mu\nu} = g_{\mu'\nu'} [\theta^{\mu'\mu} \theta^{\nu'\nu}]_{S^2}$$

where $[.]_{S^2}$ ... averaging over the internal $S^2$.

$$\boxed{\gamma^{\mu\nu} = \frac{\Delta^4}{4} \mathrm{diag}(c_0(\eta), c(\eta), c(\eta), c(\eta))}$$




at $p$, where

$$c(\eta) = 1 - \tfrac{1}{3}\cosh^2(\eta)$$
$$c_0(\eta) = \cosh^2(\eta) - 1 \geq 0$$

### signature change at $c(\eta) = 0$

$\cosh^2(\eta_0) = 3$ ...Big Bang!

Euclidean for $\eta < \eta_0$, Minkowski $(+ - - -)$ for $\eta > \eta_0$

conformal factor $\alpha = \sqrt{\frac{|\theta^{\mu\nu}|}{|\gamma^{\mu\nu}|}} = \frac{4}{\Delta^4}|c(\eta)|^{-\frac{3}{2}}$

from $SO(4,2)$-inv. (Kirillov-Kostant) symplectic $\omega$ on $\mathbb{C}P^{1,2}$

$\rightarrow$ effective metric at $p$

$$G_{\mu\nu} = \mathrm{diag}\Big(\frac{|c(\eta)|^{\frac{3}{2}}}{c_0(\eta)}, -|c(\eta)|^{\frac{1}{2}}, -|c(\eta)|^{\frac{1}{2}}, -|c(\eta)|^{\frac{1}{2}}\Big)$$

FLRW metric and scale factor (after BB)

$$ds^2_G = dt^2 - a^2(t)d\Sigma^2$$

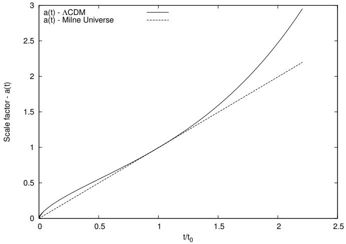
late times:

**linear coasting cosmology**

$$a(t) \approx \frac{3\sqrt{3}}{2}t\ .$$

$a(t) \sim t$ is remarkably close to observation:

- age of univ. $13.9 \times 10^9 y$ from present Hubble parameter




  artificial within GR,
  natural in M.M., provided gravity emerges below cosm. scales

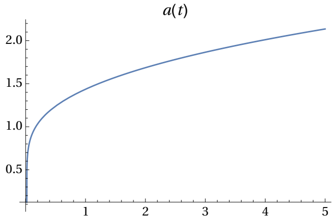
- can reasonably reproduce SN1a (without acceleration)

cf. Nielsen, Guffanti, Sarkar Sci.Rep. 6 (2016)

## Big Bang:

shortly after the BB $\eta \gtrsim \eta_0$:

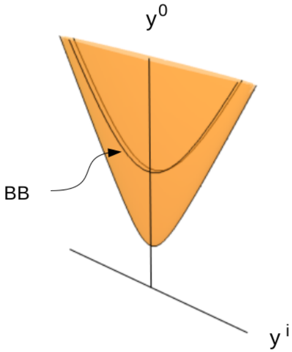
$$a(t) \propto c(t)^{\frac{1}{4}} \propto t^{1/7}$$




conformal factor & 4-volume form $|\theta^{\mu\nu}|$ responsible for singular expansion!

## other features:

- $\exists$ Euclidean pre-BB era
- 2 sheets with opposite intrinsic "chirality"
  (i.e. $\theta^{\mu\nu}$ (A)SD )




- $\exists$ higher-spin fluctuation modes

  $\rightarrow$ higher-spin gauge theory
- small $n$ possible (even $n = 0$)

## other cosmological solutions

- "momentum embedding" (same $\mathcal{M}^{3,1}_n$, different metric) $k = -1$

  M. Sperling & H.S. 1806.05907

- expanding closed universe $k = 1$
- recollapsing universe $k = 1$ HS arXiv:1709.10480

# momentum embedding:

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

$$\Box_T T^\mu = -3\frac{1}{R^2}\, T^\mu \ .$$

... solution of IKKT model with mass

- $[T^\mu, X^\nu] = if(t)\eta^{\mu\nu}$, *momentum* generator

  (cf. Hanada,Kawai, Kimura hep-th/0508211])
- similar expansion of functions $f(X) + f_\mu(X)T^\mu + ...$, higher-spin modes on $\mathcal{M}^{3,1}$
- similar eff. $SO(3,1)$ -invariant FRW metric, $k = -1$
- similar late-time behavior
- BB, initial $a(t) \sim t^{1/5}$, no signature change

... work in progress M. Sperling & HS

$\exists$ further FRW solutions with $k = +1$,

in presence of $SO(4,1)$-breaking mass $-m^2 Y^i Y^i + m_0^2 Y^0 Y^0$

- expanding closed universe
  from projection of fuzzy $H^4_n$
- recollapsing closed universe
  from projection of fuzzy $S^4_N$

HS, arXiv:1709.10480

# fluctuations & higher spin gauge theory on $H^4_n$

$$S[Y] = Tr(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S^4_N,\ H^4_n$

add fluctuations $Y^a = X^a + \mathcal{A}^a$

expand action to second oder in $\mathcal{A}^a$

$$S[Y] = S[X] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_a \underbrace{\Big((\Box + \frac{1}{2}\mu^2)\delta^a_b + 2[[X^a, X^b], .] - [X^a, [X^b, .]]\Big)}_{\mathcal{D}^2} \mathcal{A}_b$$

$$\Box = [X^a, [X_a, .]]$$

- fluctuations $\mathcal{A}_a$ describe gauge theory (NCFT) on $\mathcal{M}$ ("open strings" ending on $\mathcal{M}$)
- for $S^4_N,\ H^4_n$ : $\mathcal{A}_a$ ... $\mathfrak{hs}$-valued gauge field, incl. spin 2

4 indep. tangential fluctuation modes $\mathcal{A}_a \in End(\mathcal{H}) \otimes (5)$

$$
\begin{aligned}
\mathcal{A}_a^{(1)} &= \eth_a \phi^{(s)} , \\
\mathcal{A}_a^{(2)} &= \theta^{ab} \eth_b \phi^{(s)} = \{x^a, \phi^{(s)}\} \\
\mathcal{A}_a^{(3)} &= \phi_a^{(s)} \\
\mathcal{A}_a^{(4)} &= \theta^{ab} \phi_b^{(s)} .
\end{aligned}
$$

where $\phi^{(s)} \in End(\mathcal{H})$ ... spin $s$ mode, $\phi_a^{(s)} \propto \{x_a, \phi^{(s)}\}_{s-1}$

eigenmodes of $\mathcal{D}^2$:

$$
\begin{aligned}
\mathcal{B}_a^{(1)} &= \mathcal{A}_a^{(1)} - \frac{\alpha_s}{R^2 r^2}(\Box - 2r^2)\mathcal{A}_b^{(4)} , \\
\mathcal{B}_a^{(2)} &= \mathcal{A}_a^{(2)} + \alpha_s(\Box - 2r^2)\mathcal{A}_a^{(3)} , \\
\mathcal{B}_a^{(3)} &= \mathcal{A}_a^{(3)} \\
\mathcal{B}_a^{(4)} &= \mathcal{A}_a^{(4)}
\end{aligned}
$$

can diagonalize $\mathcal{D}^2$
all tangential modes are stable !

+ radial modes (unstable)

M. Sperling & H.S. 1806.05907

# metric and vielbein

consider scalar field $\phi = \phi(X)$ (= transversal fluctuation)

kinetic term

$$-Tr[X^a,\phi][X_a,\phi] \sim \int e^a\phi e_a\phi = \int \gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

vielbein

$$\begin{aligned} e^a &:= \{X^a,.\} = e^{a\mu}\partial_\mu \\ e^{a\mu} &= \theta^{a\mu} \end{aligned}$$

metric

$$\gamma^{\mu\nu} = \eta_{\alpha\beta}e^{\alpha\mu}\, e^{\beta\nu} = \frac{1}{4}\Delta^4\, g^{\mu\nu}$$

Poisson structure $\rightarrow$ frame bundle!

perturbed vielbein: $Y^a = X^a + \mathcal{A}^a$

$$e^a := \{Y^a, .\} \sim e^{a\mu}[\mathcal{A}]\partial_\mu \qquad \text{... vielbein}$$
$$\delta_{\mathcal{A}}\gamma^{ab} =: H^{ab}[\mathcal{A}] = \theta^{ca}\{\mathcal{A}_c, x^b\} + (a \leftrightarrow b)$$

linearize & average over fiber $\rightarrow$

$$G^{ab} = \gamma^{ab} + h^{ab}\,, \qquad h^{ab} \sim [H^{ab}]_0$$

spin 2 graviton:

$$\boxed{h_{ab}[\mathcal{B}^{(4)}] = 2\alpha_1(\Box - 2r^2)\phi_{ab}, \qquad \nabla^a h_{ab} = 0}$$

all other modes drop out: $h_{ab}[\mathcal{B}^{(i)}] = 0$

quadratic action for spin 2 graviton $h_{ab}[\mathcal{B}] = 2\alpha_1(\Box - 2r^2)\phi_{ab}$:

$$S_2[h_{ab}] \quad \propto \int \mathcal{B}_a \mathcal{D}^2 \mathcal{B}^a \propto \int h_{ab}[\mathcal{B}] h_{ab}[\mathcal{B}]$$

$h_{ab}$ doesn't propagate in classical model

due to field redefinitions via $(\Box - 2r^2)$

coupling to matter:

$$S[\text{matter}] \sim \int_{\mathcal{M}} d^4x\, h^{ab} T_{ab}$$

$\to$ auxiliary field $h_{ab} \sim T_{ab}$ !

HS, arXiv:1606.00769, M. Sperling, HS arXiv:1707.00885 however:

1. quantum effects $\rightarrow$ induced gravity action $\sim \int h_{\mu\nu} \Box h^{\mu\nu}$
   $\rightarrow$ (lin.) Einstein equations (+ possibly c.c. and/or mass)
2. consider different action (however: UV/IR mixing)
3. for cosmological space-times:
   ... to be worked out

---

GR not renormalizable $\Rightarrow$ need different starting point

$\rightarrow$ emergent gravity ?

present model might be healthy candidate

# towards higher-spin gravity on $\mathcal{M}^{3,1}$

momentum embedding $Y^a = T^a$ best suited

- space of modes = tangential modes on $H^4$, similar structure
  clean separation of higher spin modes
- manifest $SO(3,1)$, local Lorentz-invar. not guaranteed
  (could be bi-metric...)
- conjecture: no ghosts
- compute mass spectrum (to exclude tachyons, instabilities)

work in progress M. Sperling, HS

# summary

- matrix models: promising framework for quantum theory of space-time & matter
- $\exists$ nice cosmological FRW space-time solutions
  - reg. BB, finite density of microstates
  - IKKT allows to address origin of time !
- all ingredients for gravity, good UV behavior (SUSY)
- $\rightarrow$ regularized higher spin theory, cf. Vasiliev
- may not lead to gravity at classical level; emergent gravity?

  more work required for cosm. space-times

stay tuned!

gauge transformations:

$Y^a \to UY^aU^{-1} = U(X^a + \mathcal{A}^a)U^{-1}$ leads to $(U = e^{i\Lambda})$

$$\delta\mathcal{A}^a = i[\Lambda, X^a] + i[\Lambda, \mathcal{A}^a]$$

expand

$$\Lambda = \Lambda_0 + \frac{1}{2}\Lambda_{ab}\mathcal{M}^{ab} + \ldots$$

... $U(1) \times SO(5) \times \ldots$ - valued gauge trafos

diffeos from $\delta_v := i[v_\rho P^\rho, .]$

$$\begin{aligned} \delta h_{\mu\nu} &= (\partial_\mu v_\nu + \partial_\nu v_\mu) - v^\rho\partial_\rho h_{\mu\nu} + (\Lambda \cdot h)_{\mu\nu} \\ \delta A_{\mu\rho\sigma} &= \tfrac{1}{2}\partial_\mu\Lambda_{\sigma\rho}(x) - v^\rho\partial_\rho A_{\mu\rho\sigma} + (\Lambda \cdot A)_{\mu\rho\sigma} \end{aligned}$$

etc.

further solutions: expanding closed universe

$$S[Y] = \frac{1}{g^2}\mathrm{Tr}\Big([Y^a, Y^b][Y^{a'}, Y^{b'}]\eta_{aa'}\eta_{bb'} - m^2 Y^i Y^i + m_0^2 Y^0 Y^0\Big) .$$

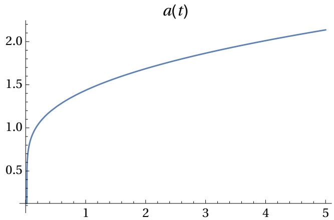
∃ solution:

$$\boxed{Y^i = X^i, \quad \text{for } \; i = 1, ..., 4, \qquad Y^0 = \kappa X^5}$$

for $X^a$... fuzzy $H_n^4$

FRW cosmology with spatial $S^3$, $k = 1$

cosm. scale factor: late time $a(t) \sim t^{1/3}$, BB $a(t) \sim t^{1/7}$



further solutions: recollapsing closed universe

$$S[Y] = \frac{1}{g^2}\mathrm{Tr}\Big([Y^a, Y^b][Y^{a'}, Y^{b'}]\eta_{aa'}\eta_{bb'} - m^2 Y^i Y^i + m_0^2 Y^0 Y^0\Big) .$$

$\exists$ solution

$$\boxed{Y^i = X^i, \quad \text{for } \; i = 1, ..., 4, \qquad Y^0 = \kappa X^5}$$

for $X^a$... fuzzy $S^4_N$

FRW cosmology with spatial $S^3$, $k = 1$

cosm. scale factor:

BB $a(t) \sim t^{1/7}$