# Quantum Gravity from Noncommutative Gauge Theory

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# Outline

# Lesson from Quantum Mechanics

Quantization $\Leftrightarrow$ Duality

## Quantum Mechanics: $\hbar$-deformation

- In the classical world with $\hbar = 0$, the wave and the particle are completely independent and exclusive entities.
- Formulation of mechanics in NC phase space
  $[x^i, p_k] = i\hbar\delta^i_k$
- Wave-particle duality in NC phase space: $\lambda = \frac{2\pi\hbar}{p}$

## Noncommutative Field Theory: $\theta$-deformation

- Gravity: Spacetime symmetry
  Electromagnetism: Internal symmetry
- Formulation of field theory in NC spacetime
  $[y^a, y^b]_\star = i\theta^{ab}$
- In NC spacetime, Internal symmetry $\Rightarrow$ Spacetime symmetry
- Gauge/Gravity duality in NC spacetime

# A Novel Form of Equivalence Principle

Consider a $U(1)$ bundle $E \longrightarrow M$, where $M$ supports a symplectic structure $B$, i.e., $(M, B)$ a symplectic manifold.

- Physically we are considering open strings on a D-brane $M$ in a $B$-field background.
- $\Lambda$-symmetry: $(B, A) \longrightarrow (B - d\Lambda, A + \Lambda)$
- $U(1)$ bundle on a symplectic manifold: $(M, B + F)$ where $F = dA$
- Darboux theorem (more precisely Moser lemma) says that there "always" exists a diffeomorphism $\phi : M \longrightarrow M$ such that $\phi^*(B + F) = B$ as long as $B$ is a symplectic 2-form.
- $B : TM \longrightarrow T^*M : X \mapsto A$ such that $\iota_X B = A$ or $\mathcal{L}_X B = F$
  up to symplectomorphisms:
  $A \sim A + d\lambda \;\Leftrightarrow\; X \sim X + X_\lambda$ where $\iota_{X_\lambda} B = d\lambda$.

**Riemannian geometry from symplectic geometry**

Electromagnetism on a symplectic manifold $(M, B)$ $\equiv$ Einstein gravity $(\mathcal{M}, g)$.

# A Beautiful Example

Self-dual electromagnetism on a symplectic manifold $(M, B)$

$\equiv$ Self-dual Einstein gravity

## (I) Poisson algebra $(C^\infty(M), \{\cdot,\cdot\}_\theta)$: $C^\infty(M) \Leftrightarrow \Gamma(TM)$

- Cosymplectic structure: $\theta \equiv B^{-1} : T^*M \to TM$ defines a Poisson bracket $\{f, g\}_\theta = \theta^{ab}\frac{\partial f}{\partial y^a}\frac{\partial g}{\partial y^b}$ for $f, g \in C^\infty(M)$.
- Poisson algebra admits the Lie algebra homomorphism $C^\infty(M) \to TM : f \mapsto X_f$ such that $X_f(g) = \{g, f\}_\theta$ and $X_{\{f,g\}_\theta} = -[X_f, X_g]$.
- Gauge fields in $C^\infty(M)$ $\Leftrightarrow$ Vielbeins in $\Gamma(TM)$

## (II) NC $U(1)$ instantons = Gravitational instantons

- $f := D_a = B_{ac}y^c + \widehat{A}_a(y), \quad g := D_b = B_{bd}y^d + \widehat{A}_b(y)$, then $\{D_a, D_b\}_\theta = -B_{ab} + \widehat{F}_{ab}$ where $\widehat{F}_{ab} = \partial_a\widehat{A}_b - \partial_a\widehat{A}_b + \{\widehat{A}_a, \widehat{A}_b\}_\theta$.
  $\Rightarrow X_{\widehat{F}_{ab}} = -[D_a, D_b]$
- NC $U(1)$ instantons: HSY, `hep-th/0608013`
  $\widehat{F}_{ab} = \pm\frac{1}{2}\varepsilon_{ab}{}^{cd}\widehat{F}_{cd} \Leftrightarrow [D_a, D_b] = \pm\frac{1}{2}\varepsilon_{ab}{}^{cd}[D_c, D_d]$
- Gravitational instantons: Ashtekar et al. (1988)
  $R_{mnab} = \pm\frac{1}{2}\varepsilon_{ab}{}^{cd}R_{mncd} \Leftrightarrow [D_a, D_b] = \pm\frac{1}{2}\varepsilon_{ab}{}^{cd}[D_c, D_d]$

# Emergent Gravity arXiv:0809.4728

## Basic Idea

Gravity is not a fundamental force but a collective or emergent phenomenon from NC or large N gauge fields:
$(1 \otimes 1)_S \rightleftarrows 2 \oplus 0$ or $\subset \otimes \supset \rightleftarrows \bigcirc$.

## In A Market

- $(\longrightarrow)$: AdS/CFT correspondence, Matrix models, open-closed string duality, KLT relation, etc.
- $(\longleftarrow)$: Kaluza-Klein theory, String compactifications.

## Basic Slogan

- The emergence of general relativity requires the emergence of spacetime itself (H. Elvang and J. Polchinski: hep-th/0209104)
- Spacetime is not given *a priori* but defined by more fundamental ingredients of the underlying theory

# Emergent Geometry

## Emergent geometry from NC gauge theory

- The relation between orthonormal frmaes $E_a \in \Gamma(TM)$, $E^a \in \Gamma(T^*M)$ in Einstein gravity and gauge theory bases $D_a \in \Gamma(TM)$, $V^a \in \Gamma(T^*M)$: $D_a = \lambda E_a$ and $E^a = \lambda V^a$ where $\lambda^2 = \det^{-1} V^a_b$.
- $ds^2 = E^a \otimes E^a = \lambda^2 \delta_{ab} V^a_c V^b_d dy^c dy^d$ where $V^a_c D^c_b = \delta^a_b$.

## Einstein equations from NC gauge fields

- $\{D_a, \{D_b, D_c\}_\theta\}_\theta = \widehat{D}_a \widehat{F}_{bc} \;\Leftrightarrow\; X_{\widehat{D}_a \widehat{F}_{bc}} = [D_a, [D_b, D_c]]$
- Bianchi identity $\widehat{D}_{[a} \widehat{F}_{bc]} = 0 \Leftrightarrow R_{[abc]d} = 0$.
- Equations of motion $\widehat{D}_a \widehat{F}^{ab} = 0 \Leftrightarrow R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}$. where $T_{ab} = T^{(M)}_{ab} + T^{(L)}_{ab}$.

- $\exists$ natural concept of Emergent Time: $(M, \omega = B)$ Hamiltonian vector field $X_H : \iota_{X_H} \omega = dH \;\Rightarrow\; \frac{df}{dt} = X_H(f) = \{f, H\}_{\omega^{-1}}$

# NC Spacetime and Quantum Gravity

- NC spacetime: Quantize a symplectic (or Poisson) manifold $(M, B)$ à la Dirac, i.e., $\langle B_{ab} \rangle_{\rm vac} = (\theta^{-1})_{ab} \quad \Leftrightarrow \quad [y^a, y^b]_\star = i\theta^{ab}$
  $\{f, g\}_\theta \to -i[\widehat{f}, \widehat{g}]_\star = \{f, g\}_\theta + \mathcal{O}(\theta^3)$ where $\widehat{f}, \widehat{g} \in \mathcal{A}_\theta$.
- $\exists$ 1-1 correspondence between $\mathcal{A}_\theta$ and $\Gamma_\theta(\widehat{TM})$, generalized vector fields:
  For $\widehat{D}_a(y) \equiv B_{ab} y^b + \widehat{A}_a(y) \in \mathcal{A}_\theta$, $ad_{\widehat{D}_a}[\widehat{f}] \equiv -i[\widehat{D}_a, \widehat{f}]_\star = D_a[f] + \mathcal{O}(\theta^3)$
- $-i[\widehat{D}_a(y), \widehat{D}_b(y)]_\star = -B_{ab} + \widehat{F}_{ab}(y)$ where $\widehat{F}_{ab} = \partial_a \widehat{A}_b - \partial_a \widehat{A}_b - i[\widehat{A}_a, \widehat{A}_b]_\star$
  $\Rightarrow \widehat{X}_{\widehat{F}_{ab}} = [ad_{\widehat{D}_a}, ad_{\widehat{D}_a}]_\star$
- NC gauge theory $=$ Matrix model or large $N$ gauge theory (H. Steinacker)

**NC field $=$ master field of large N matrix**

$\{\mathcal{A}_\theta, \mathcal{H}, [\cdot, \cdot]_\star\} = \{M_N, V_\mathcal{H}, [\cdot, \cdot]\}$ where $N = \dim \mathcal{H}$
Any field on NC space $\Leftrightarrow$ $N \times N$ matrix at $N \to \infty$

- Matrix theory $=$ Second quantized formulation of string/M theory
  $\Rightarrow$ NC gauge theory $=$ Background independent quantum gravity
- Quantum gravity exists !: Any Poisson manifold can always be quantized (M. Kontsevich)

# Background Independent Quantum Gravity

- 0-dimensional IKKT Matrix model: No *a priori* spacetime structure

$$S_{IKKT} = -\frac{1}{8\pi g_s} \mathrm{Tr}[\Phi^a, \Phi^b]^2 \tag{1}$$

- Algebraic relations:

$$[\Phi^{[a}, [\Phi^b, \Phi^{c]}]] = 0, \qquad [\Phi_a, [\Phi^a, \Phi^b]] = 0. \tag{2}$$

- Specify a vacuum: $\Phi^a_{\mathrm{vac}} = \frac{y^a}{\kappa}$ and expand $\Phi^a \equiv \frac{\theta^{ab}}{\kappa} \widehat{D}_b$ where $\widehat{D}_a = (B_{ab} y^b + \widehat{A}_a(y))$
- $\mathbf{R}^d_{NC} : \langle B_{ab} \rangle_{\mathrm{vac}} = (\theta^{-1})_{ab} \quad \Leftrightarrow \quad [y^a, y^b]_\star = i\theta^{ab}$ where $a, b = 1, \cdots, d$.
- IKKT Matrix model = NC U(1) gauge theory on $\mathbf{R}^d_{NC}$

$$S_{NC} = \frac{1}{4g^2_{YM}} \int d^d y (\widehat{F}_{ab} - B_{ab}) \star (\widehat{F}^{ab} - B^{ab}) \tag{3}$$

where $\widehat{F}_{ab} = \partial_a \widehat{A}_b - \partial_b \widehat{A}_a - i[\widehat{A}_a, \widehat{A}_b]_\star$.

# Dynamical origin of spacetime

- Flat spacetime is coming from the uniform vacuum $\langle B_{ab}\rangle_{vac} = \theta^{-1}_{ab}$.
- In four dimensions, $[G] = [\theta] = L^2$. In general, $\frac{G\hbar^2}{c^2} \sim g^2_{YM}|\theta|$.
- So $\rho_{vac} \sim B^2_{ab} \sim M^4_P$ where $M^2_P = 1/8\pi G$ in 4 dimensions.
- A flat spacetime is not free gratis but a result of the Planck energy condensation in a vacuum.
- $\mathcal{L}_B \to \mathcal{L}_{B'} = \mathcal{L}_B - 2\Lambda$: Shift symmetry = Seiberg-Witten equivalence
  $\Leftrightarrow$ Global Lorentz transformation
  No cosmological constant problem !
- The flat spacetime and Lorentz symmetry must be very robust against any perturbations and gravitational fields should be very weak.
- UV/IR mixing or holography for fluctuations around the vacuum $\langle B_{ab}\rangle_{vac} = \theta^{-1}_{ab}$
  $\Rightarrow$ $\exists$ entanglement energy for UV and IR fluctuations
  (H. Grosse, H. Steinacker, M. Wohlgenannt)

# Dark Energy

- Cosmological fluctuations around the vacuum $\langle B_{ab}\rangle_{vac}$:
  $\rho = \rho_{\rm vac} + \delta\rho \sim M_P^4(1 + L_P^2/L_H^2 + \cdots)$
  where $L_H$ is a typical IR scale paired with $L_P$ through UV/IR mixing, so vacuum fluctuation $\Delta\rho \sim 1/L_P^2 L_H^2$.
- Liouville (or Poisson) energy-momentum tensor
  (M. Burić, J. Madore, G. Zoupanos)

$$T_{ab}^{(L)} = \frac{1}{16\pi G\lambda^2}\Big(\rho_a\rho_b + \Psi_a\Psi_b - \frac{1}{2}g_{ab}(\rho_c^2 + \Psi_c^2)\Big) \qquad (4)$$

  where $\rho_a = 2\partial_a\lambda$ and $\Psi_a = E_a^\mu\Psi_\mu$.
- $T_{ab}^{(L)}$ exert gravitational repulsion (Raychauduri equation) and behaves likes a cosmological consant, i.e., $p = -\rho$ for spacelike perturbations $\rho_a$ and $\Psi_a$.
- $T_{00}^{(L)} \sim 1/8\pi G L_H^2$ where $L_H$ is the radius of 3-dim. spacelike hypersurface.

# Superconductivity vs. Emergent Gravity

| Theory | Superconductivity | Emergent gravity |
|---|---|---|
| Microscopic degree of freedom | electron | gauge field |
| Order parameter | Cooper pair | graviton |
| G | $U(1)$ | Diff(M) |
| H | $\mathbf{Z}_2$ | $U(1)_{NC}$ |
| Control parameter | $\frac{T_c}{T} - 1$ | $\theta^{ab}$ |
| Macroscopic description | Laudau-Ginzburg | Einstein gravity |
| Microscopic description | BCS | gauge theory |