The Dear Old Undeformed Lorentz Action and NC Spacetime

Gherardo Piacitelli
SISSA - Trieste
e-mail: piacitel@sissa.it

Bayrischzell, May 17, 2009

The Problem

No unitary representation U of $\mathcal{L}$ can make the relations

$$[\hat{x}_\theta^\mu, \hat{x}_\theta^\nu] = i\theta^{\mu\nu} I$$

covariant. On one side,

$$U(\Lambda)^{-1}[\hat{x}_\theta^\mu, \hat{x}_\theta^\nu]U(\Lambda) = \Lambda^{\mu'}{}_\mu \Lambda^{\mu'}{}_\mu [\hat{x}_\theta^\mu, \hat{x}_\theta^\nu] = i\theta'^{\mu\nu} I = i\Lambda^\mu{}_{\mu'} \Lambda^\mu{}_{\mu'} \theta^{\mu'\nu'} I;$$

On the other

$$U(\Lambda)^{-1}(i\theta^{\mu\nu} I)U(\Lambda) = i\theta^{\mu\nu} U(\Lambda)^{-1} U(\Lambda) = i\theta^{\mu\nu} I$$

and

$$\theta \neq \theta'$$

unless Λ is in the stabiliser of θ.

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Approaches to The Problem

1. Give up and replace $\mathcal{L}$ with the stabiliser;
2. keep same θ in all frames, and deform (the action of) $\mathcal{L}$;
3. keep undeformed action of $\mathcal{L}$, and let θ transform as a tensor;
4. DFR model (actually the first proposal, 1994).

I wish to convince you that $2 = 3$, that 3 leads naturally to 4, and can be recovered from it up to an additional assumption I wish to criticise.

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Star Product

Conservative approach based on Weyl quantisation:

$$f \mapsto W_\theta(f) = \int dk \check{f}(k) e^{ik_\mu \hat{x}^\mu_\theta},$$

where

$$\check{f}(k) = \frac{1}{(2\pi)^4} \int dx f(x) e^{-ik_\mu x^\mu_\theta}.$$

Twisted product = auxiliary tool defined by:

$$W_\theta(f \star_\theta g) = W_\theta(f) W_\theta(g) \quad \text{(operator product)}$$

One finds

$$f \star_\theta g = (m \circ F_\theta)(f \otimes g),$$

where m = commutative product: $m(f \otimes g)(x) = f(x)g(x)$, and

$$F_\theta = e^{\frac{1}{2}\theta^{\mu\nu}\partial_\mu \otimes \partial_\nu}$$

on an appropriate class of symbols (HIC SUNT LEONES).

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The Problem - Again

With the Lorentz action

$$(\alpha_\Lambda f)(x) = f'(x) = f(\Lambda^{-1}x),$$

on symbols, we have

$$W_\theta(f')W_\theta(f') \neq W_\theta((\alpha_\Lambda f') \star_\theta (\alpha_\Lambda f'))$$

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Twisted Covariance

- keep action on functions of one variable: $f \mapsto \alpha_\Lambda f$
- deform action on functions of two variables[1]

$$f \otimes g \mapsto \Big(F_\theta^{-1} \circ (\alpha_\Lambda \otimes \alpha_\Lambda) \circ F_\theta\Big)(f \otimes g).$$

It is an action:

$$\Big(F_\theta^{-1} \circ (\alpha_\Lambda \otimes \alpha_\Lambda) \circ F_\theta\Big)\Big(F_\theta^{-1} \circ (\alpha_M \otimes \alpha_M) \circ F_\theta\Big) = \Big(F_\theta^{-1} \circ (\alpha_{\Lambda M} \otimes \alpha_{\Lambda M}) \circ F_\theta\Big)$$

It solves the problem:

$$m_\theta\Big(\Big(F_\theta^{-1} \circ (\alpha_\Lambda \otimes \alpha_\Lambda) \circ F_\theta\Big)(f \otimes g)\Big) = \alpha_\Lambda m_\theta(f \otimes g).$$

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[1] Notation:

$$(f \otimes g)(x, y) = f(x)g(y),$$
$$(\alpha_\Lambda \otimes \alpha_\Lambda)(f \otimes g)(x, y) : f(\Lambda^{-1}x)g(\Lambda^{-1}y).$$

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Concrete Actions vs Abstract algebra

Twisted action: computing

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means:

1. $f\otimes g\mapsto F_\theta f\otimes g = e^{\frac{1}{2}\theta^\mu{}_\nu\partial_\mu\otimes\partial\nu} f\otimes g$
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3. $\Big(m\circ F_{\theta'}(\alpha_\Lambda f)\otimes(\alpha_\Lambda g)\Big)(f\otimes g) = m_{\theta'}(f'\otimes g') = f'\star_\theta' g'.$

Hence twisted covariance is equivalent to

$$(f\star_\theta g)' = f'\star_{\theta'} g'.$$

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The Situation Now

Notation: θ fixed once and for all by a privileged observer, σ dummy element of Σ. Then

- Σ = orbit of $(\theta^{\mu\nu})$ under Lorentz action;
- to each $(\sigma^{\mu\nu})$, there is a $\star_\sigma$ defining an algebra $\mathcal{A}_\sigma$;
- there is an action of Lorentz group α_Λ sending $f \in \mathcal{A}_\sigma \mapsto f \circ \Lambda^{-1} \in \mathcal{A}_{\sigma'}$, where $\sigma'^{\mu\nu} = \Lambda^{\mu}{}_{\mu'}\Lambda^{\mu}{}_{\mu'}\sigma^{\mu'\nu'}$;
- this map respects $\star$-products at the cost of using the right σ in every $\mathcal{A}_\sigma$ in the family $\{\mathcal{A}_\sigma : \sigma \in \Sigma\}$.

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Tensor or not? Back to Interpretation!

Untwisted form covariance + tensoriality of θ may seem appealing, but form covariance alone not a guidance, when equivalence of observers is broken at a fundamental level. Up to now the two formalisms have same dignity.

Problem is: above only formal remark. To decide, go back to interpretation of $i\theta$ as the commutator of the coordinates.

Assume Jack=preferred observer, Jane=observer connected to Jack by Λ.

Jane:

- $[\hat{y}^\mu, \hat{y}^\nu] = ?$ (no a priori assumption),
- $W'(f') = \int dk \, \check{f}'(k) e^{ik\hat{y}}$ (same physics),
- $W'(m_\theta(\alpha_\theta^{(2)}(f \otimes g)) = W'(f')W'(g')$ (twisted cov).

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Weyl quantisation requires θ tensor

We first compute)

$$W'(f')W(g') = \left(\int dh \check{f}'(h) e^{ih\hat{y}}\right)\left(\int dh \check{g}'(k) e^{ik\hat{y}}\right) =$$
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$$\check{f}'(h)\check{g}'(k-h) =$$
$$= \int dk \, e^{i(h+k)\hat{y}} \int dh \, \check{f}'(h)\check{g}'(k) e^{-\frac{i}{2}h\theta'(k+h)}$$

where $\theta'^{\mu\nu} = \Lambda^\mu{}_{\mu'}\Lambda^\nu{}_{\nu'}\theta^{\mu'\nu'}$. It follows

$$e^{ih\hat{y}} e^{ik\hat{y}} = e^{-\frac{i}{2}h\theta' k} e^{i(h+k)\hat{y}},$$

i.e. the Weyl form of $[\hat{y}^\mu, \hat{y}^\nu] = i\theta'^{\mu\nu}$. Conclusion: $\hat{y} = \hat{x}_{\theta'}$ and θ is a tensor!

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$$e^{ih\hat{y}}e^{ik\hat{y}} = e^{-\frac{i}{2}h\theta' k}e^{i(h+k)\hat{y}},$$

i.e. the Weyl form of $[\hat{y}^\mu, \hat{y}^\nu] = i\theta'^{\mu\nu}$. Conclusion: $\hat{y} = \hat{x}_{\theta'}$ and θ is a tensor!

On the way to the DFR model

Now the situation is that to each observer there is: (1) a σ, (2) coordinates with commutator $i\sigma$, (3) the corresponding Weyl quantisation W_σ...

...and there is an action of the Lorentz group on the family of functions $\mathcal{A} = \{\mathcal{A}_\sigma : \sigma \in \Sigma\}$.

This suggests to consider the family $\mathcal{A}$ as a bundle of algebras over Σ, where each $\mathcal{A}_\sigma$ is the fibre over $\sigma \in \Sigma$.
Sections can be thought as functions $f = f(\sigma; x)$, where $f(\sigma; \cdot) \in \mathcal{A}_\sigma$.
The product is taken fibrewise:

$$(f \star g)(\sigma; \cdot) = f(\sigma; \cdot) \star_\sigma f(\sigma; \cdot).$$

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DFR C*algebra

The fibrewise product $\star$ turns the bundle $\{\mathcal{A}_\sigma : \sigma \in \Sigma\}$ into a well defined algebra.
The action is

$$(\alpha_\Lambda f)(\sigma; x) = (\det \Lambda) f(\Lambda^{-1} \sigma {\Lambda^{-1}}^t, \Lambda^{-1} x).$$

Theorem [DFR 95]; If Σ contains the standard symplectic matrix, there is a unique C*-norm on $\mathcal{A}$; the corresponding C*-completion is isomorphic (as a continuous field of C*-algebras) to $\mathcal{C}_0(\Sigma, \mathcal{K})$, $\mathcal{K}$=compact operators. The Lorentz group acts by endomorphisms.

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A certain class of localisation states

A localisation state on DFR algebra is a linear functional formally written as

$$f \mapsto \iint d\sigma\, dk\, K(\sigma; x) f(\sigma; x)$$

with K such to ensure positivity (w.r.t. $\star$) and normalisation. We wish to select the states with kernel of the form

$$K(\sigma; x) = \delta(\sigma - \theta) w(x),$$

which give

$$f \mapsto \int dk\, w(x) f(\theta; x)$$

More cleanly: we define the projection on the fibre over θ:

$$\Pi_\theta[f](x) = f(\theta; x);$$

extend it by continuity to a map $\Pi_\theta : \mathcal{C}(\Sigma, \mathcal{K}) \to \mathcal{K}$. Then we are interested in the states of the form $\omega \circ \Pi_\theta$ with $\omega \in \mathcal{S}(\mathcal{K})$.

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θ-Universality

We now make an additional assumption: while in the DFR model all localisation states are available to each observer, we superpose on it the extra assumption of

θ-universality.

- There is a privileged class of observers;
- The privileged observers are connected by Λ's in the stabiliser of θ;
- The only available localisation states are those which, in the reference frame of a privileged observer, are of the form $\omega \circ \Pi_\theta$, where $\omega \in \mathcal{S}(\mathcal{K})$;

Unprivileged observers connected to privileged observers by some Λ only may localise with states of the form $\omega \circ \Pi_{\theta'}$, where $\theta' = \Lambda\theta\Lambda^t$.

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Twisted Covariance Recovered

The privileged observer can test the algebra only at θ; he only sees θ-twisted products:

$$\Pi_\theta f \star g = (\Pi_\theta f) \star_\theta (\Pi_\theta g)$$

Let

$$f'(\sigma; k) = (\det \Lambda) f(\Lambda^{-1} \sigma {\Lambda^{-1}}^t; \Lambda^{-1} x)$$

be the Lorentz transform of f, and analogously for g'; the (possibly) unprivileged primed observer only sees the fibre over $\theta' = \Lambda \theta \Lambda^t$:

$$(\Pi_{\theta'} f')(x) = f'(\theta'; x) = (\det \Lambda) f(\theta; \Lambda^{-1} x),$$

as expected.Finally the primed observer only sees θ'-twisted products:

$$\Pi_{\theta'}(f' \star g') = (\Pi_{\theta'} f') \star_{\theta'} (\Pi_{\theta'} g').$$

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Conclusions 1

We have shown that (twisted covariance + θ invariant) is equivalent to (untwisted covariance + θ covariant), and given an argument in favour of the latter, based on physical interpretation.

Moreover, we have seen that the latter is equivalent to (DFR model + θ-universality).

Now one may raise the question: which are the physical motivations for restricting the admissible localisation states? Namely why θ?

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Conclusions 2

Let me more precise by means of a trivial example: consider Newton laws for motions of a point mass in the 3-space. Let's say that we state "z-universality": the preferred observers only can see motions with $z(0) > 0$. Then we may distinguish the privileged observers from unprivileged ones; e.g. Jane, who is rotated by 180° around x axis, only sees $z'(0) < 0$.
The principle of relativity requires instead that, together with each admissible state, all the states which can be reached by a symmetry of the system must be available to all observers, including the privileged ones.
In the same way, on QST any transformed θ' should be available together with θ to a privileged (or not) observer.
To say it differently, it is not sufficient that the set of admissible localisation states is form-covariant; it must be invariant.

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Conclusions 3

In this class of models, the formalism does not at all forces θ-universality upon us.

More cogent physical motivations should be provided in order to take seriously the idea that θ is a universal datum breaking covariance, *within this class of models!!*

Although radical revisions of the concept of covariance might be necessary and welcome, *within this approach* deformations of Lorentz action would break the classification of Wigner particles, which could not be expected not to have consequences at "macroscopic" scale ($> 10^{-19}$cm).

Bibliography

This talk based on the following preprints:

- G.P., [arXiv:0901.3109] (short letter).
- G.P., [arXiv:0902.0575] (long, technical).

Other References:

- Doplicher et al, [arXiv:hep-th/0303037] (DFR model, 1994/5). See also less technical [arXiv:hep-th/0105251]).
- Chaichian et al, [arxiv:hep-th/0408069], Wess [arxiv:hep-th/0408080]. (on twisted covariance, 2004).
- Gracia–Bondia et al, [arXiv:hep-th/0604206].

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