The background of the slide is a dark, deep blue or black space filled with faint, wispy clouds of light, resembling the Cosmic Microwave Background. A prominent, bright, white-yellow spot with a soft glow is located in the upper right quadrant, and another smaller, similar spot is visible in the lower left quadrant. The overall texture is grainy and ethereal.

# Cosmological Solutions of Emergent Noncommutative Gravity

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Joint work with  
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Bayrischzell 2009

University of Vienna

The background of the slide is a cosmic scene featuring a large, dark, irregularly shaped nebula or dust cloud. The cloud is set against a bright, glowing orange and yellow background, likely representing a star-forming region or a nebula. Several bright, point-like stars are scattered across the field, some appearing as small white dots and others as slightly larger, more prominent points of light. The overall color palette is dominated by warm, golden-yellow and orange tones, with deep blacks and dark greys in the nebula's structure.

outline

Cosmology

Cosmological Constant

Emergent NC Gravity

The cosmological solution

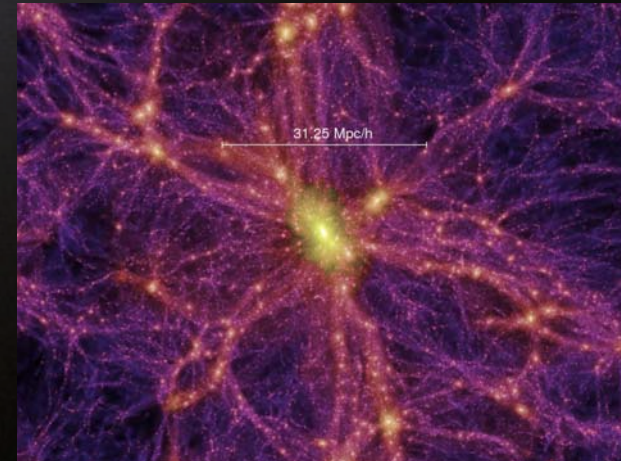
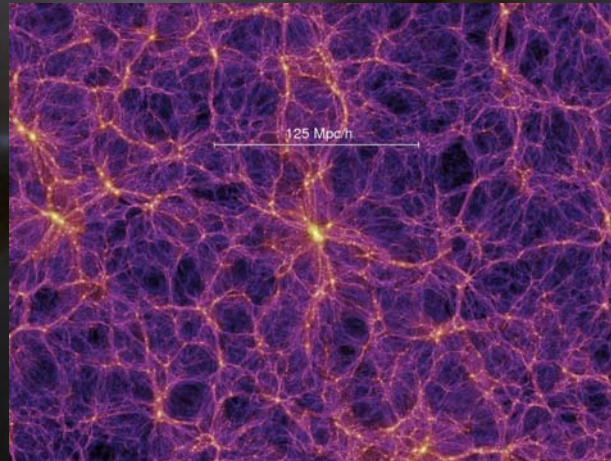
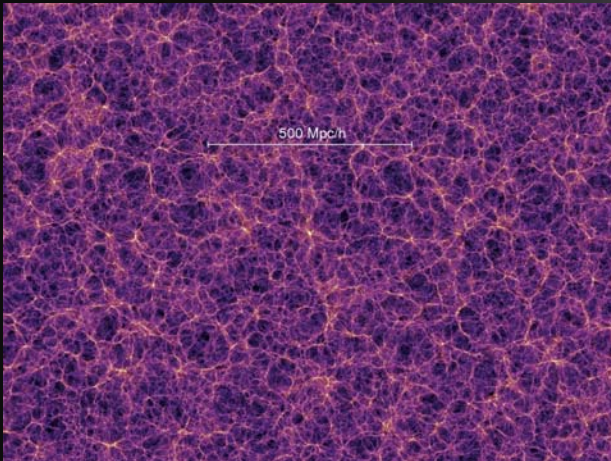
Aspects of this universe

# Cosmology

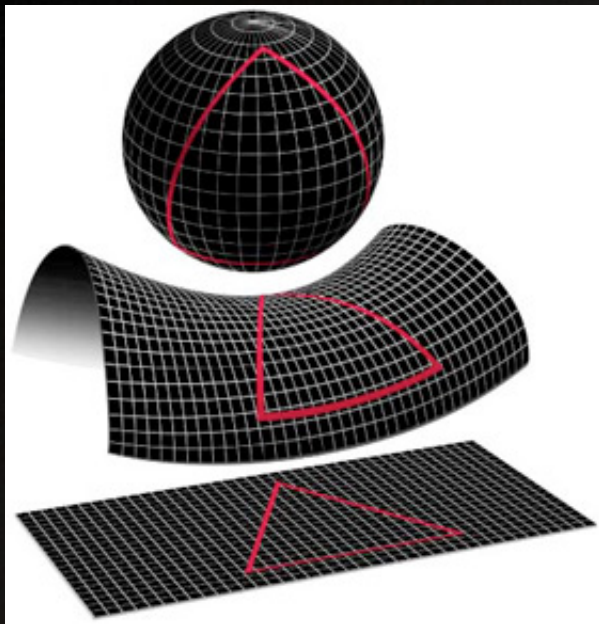
universe is homogenous & isotropic (if scale is large enough)

Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{1}{1 - kr^2} dr^2 + r^2 d\Omega^2 \right)$$



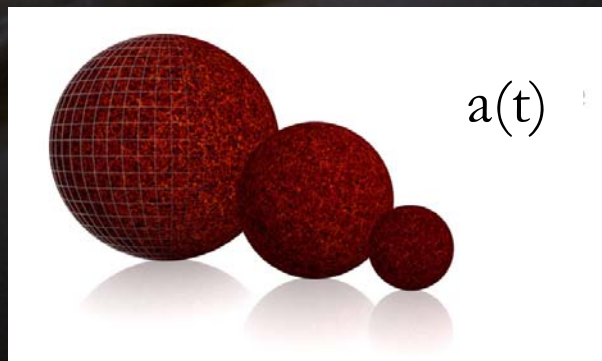
# FRW: Geometry of the universe



$k = 1$  closed

$k = -1$  open

$k = 0$  flat



# Concordance model – $\Lambda$ CDM model

- FRW-metric, Friedmann eq., cosmological eq. of state (after inflation)
- cosmological constant  $\underline{\Lambda}$  = dark energy term
- cold dark matter, non-baryonic
- Inflation: scale invariant spectrum of primeordial perturbations, flat curvature  $k = 0$ , universe is much larger than observable particle horizon

# Friedmann Equations

Expansion of the universe governed by Einstein eq.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi GT_{\mu\nu}$$

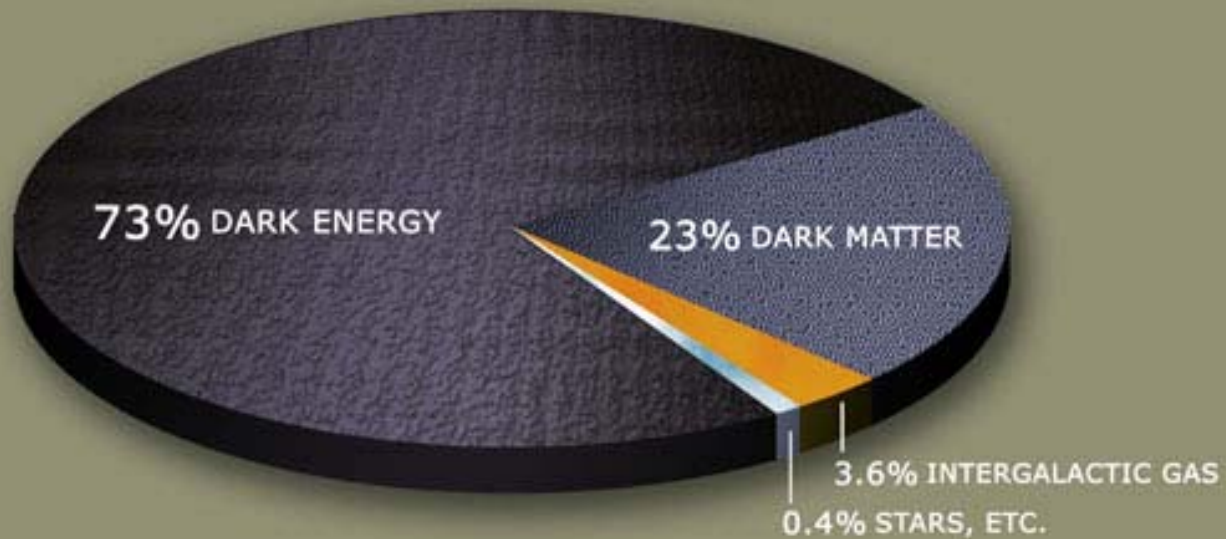
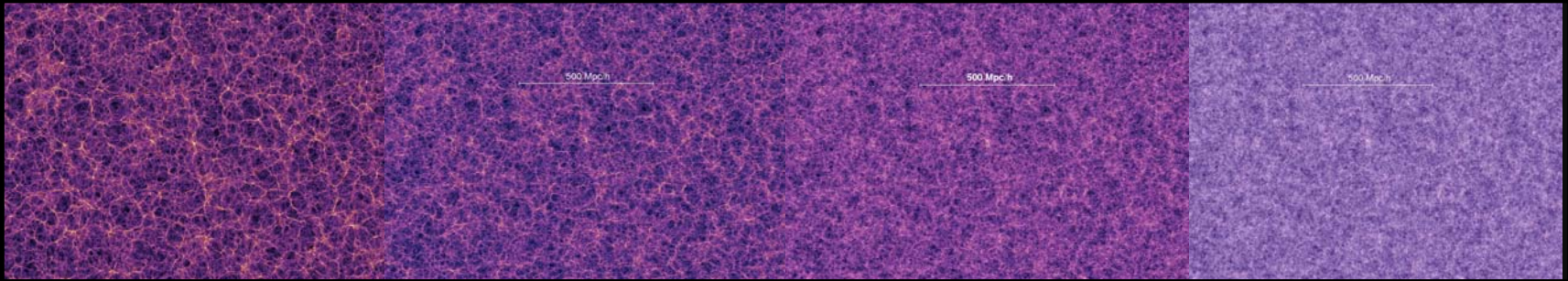


$$\dot{a}^2 + k = \frac{8\pi G \rho a^2}{3} + \frac{\Lambda}{3}a^2$$

Conservation law

$$\dot{\rho} = -\frac{3\dot{a}}{a}(\rho + p)$$

# Scale invariance: large-scale structure at different times



# The cosmological constant problem

Why is the vacuum energy smaller than expected?

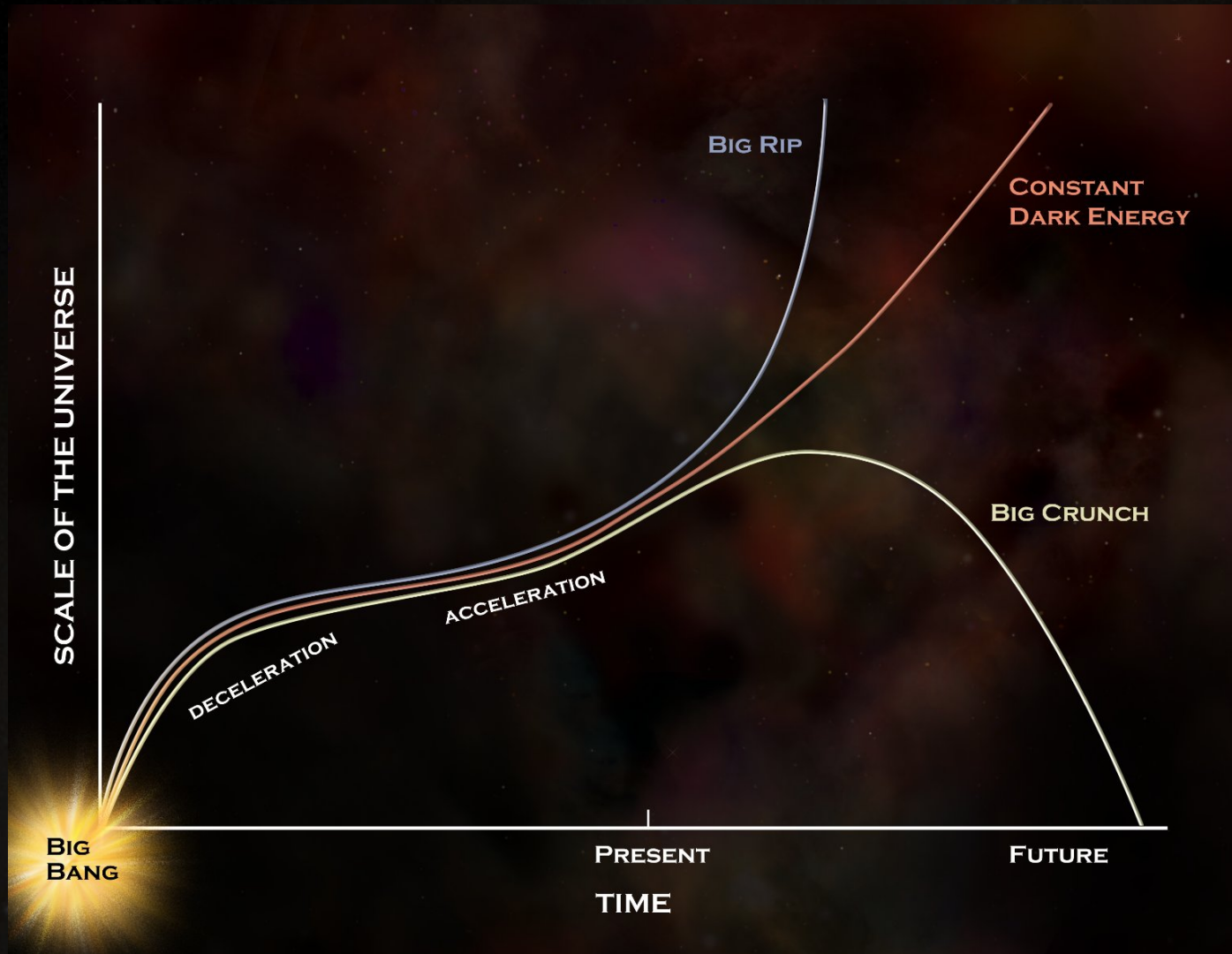
Contributions from quantum fluctuations in known fields  $\rightarrow$   
vacuum energy density  $\sim (300 \text{ GeV})^4 \approx 10^{27} \text{ g/cm}^3$

Observed value:  $10^{-29} \text{ g/cm}^3$

$\zeta \sim 10^{60} \quad ?$

Coincidence problem: Why is the dark energy beginning to dominate now?





# NC Emergent Gravity

$$S_{YM} = -\text{Tr} [X^a, X^b] [X^{a'}, X^{b'}] g_{aa'} g_{bb'}$$

$X^a \in L(\mathcal{H})$  ... matrices/operators  $a = 0, \dots, D - 1$

$$[X^a, X^b] = i\theta^{ab}$$

- $\theta^{ab}$  not constant
- $X^a$  interpreted as quantization of coordinate function  $x^a$  on Poisson manifold  $M$  with poisson structure  $\theta(x)$
- Semi-classical limit:

$$[X^a, \varphi] \sim i\theta^{ab}(x) \frac{\partial}{\partial x^b} \varphi$$

# Extra dimensions & embedding

$$S_{YM} = -\text{Tr}[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}$$

$$a, b = 0, \dots, D - 1$$

Embedding through scalar fields  $\phi^i(x)$ :

$$X^a = (X^\mu, \phi^i)$$

$$\phi^i = \phi^i(x^\mu)$$

Induced metric  $g_{\mu\nu}(x)$ :

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + (\partial_\mu \phi^i)(\partial_\nu \phi^j)\delta_{ij}$$

# Effective geometry

Couple test particle/ scalar field to matrix model

$$S[\varphi] = \text{Tr} [X^a, \varphi] [X^b, \varphi] g_{ab} \\ \sim \int d^4x \sqrt{|G_{\mu\nu}|} G^{\mu\nu}(x) (\partial_\mu \varphi) (\partial_\nu \varphi)$$

Effective metric:

$$G^{\mu\nu}(x) = e^{-\sigma} \theta^{\mu\alpha}(x) \theta^{\nu\beta}(x) g_{\alpha\beta}(x)$$

All fields couple to this effective metric.

eom

$$[X^a, [X^b, X^{a'}]] \eta_{aa'} = 0$$

Semi-classical limit

$$\Delta_G \varphi^i = 0$$

harmonic embedding

$$\nabla^\mu (e^\sigma \theta_{\mu\nu}^{-1}) = e^{-\sigma} G_{\rho\nu} \theta^{\rho\mu} \partial_\mu \eta$$

$$\eta = e^\sigma G^{\mu\nu} g_{\mu\nu}$$

covariant formulation

# Harmonic embedding - minimal surfaces



# Harmonic embedding & $\Lambda$

$$Z = \int dX^a e^{-S_{YM}[X]}$$

$$\Gamma_{1-loop} = \frac{1}{16\pi^2} \int d^4x \sqrt{|G|} (c_1 \Lambda_1^4 + c_4 R[G] \Lambda_4^2 + O(\ln \Lambda))$$



usually cosmological constant term

large because  $\Lambda_1$  is large

omitting fermions for simplicity

# Harmonic embedding & $\Lambda$

$$Z = \int dX^a e^{-S_{YM}[X]}$$

$$\Gamma_{1-loop} = \frac{1}{16\pi^2} \int d^4x \sqrt{|G|} (c_1 \Lambda_1^4 + c_4 R[G] \Lambda_4^2 + O(\ln \Lambda))$$

in NC emergent gravity: different meaning

$$\delta \int d^4x \sqrt{|G|} = \int d^4x \sqrt{|g|} g^{\mu\nu} \delta g_{\mu\nu} = \int \sqrt{|g|} \delta \phi^i (\Delta_g \phi^j) \delta_{ij}$$

$$|G_{\mu\nu}(x)| = |g_{\mu\nu}(x)|$$

vanishes



A dark cosmic background featuring a bright star with a blue-white glow and diffraction pattern in the lower-left quadrant, and a diagonal streak of light representing a galaxy or nebula in shades of purple and blue across the center and right side.

$\Lambda$

The framework of  
NC Emergent Gravity  
might resolve  
the Cosmological Constant Problem.

# The cosmological solution

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + S(\chi)^2 d\Omega^2)$$

$$S(\chi) = (\sin \chi, \chi, \sinh \chi) = r$$

$$C(\chi) = (\cos \chi, \cosh \chi)$$

Embedding for  $k = \pm 1$

$$\vec{x}(t, \chi, \theta, \varphi) = \begin{pmatrix} \mathcal{R}(t) \begin{pmatrix} S(\chi) \sin \theta \cos \varphi \\ S(\chi) \sin \theta \sin \varphi \\ S(\chi) \cos \theta \\ C(\chi) \\ 0 \\ x_c(t) \end{pmatrix} \end{pmatrix} \in \mathbb{R}^{10}$$

$$\mathcal{R}(t) = a(t) \begin{pmatrix} \cos \psi(t) \\ \sin \psi(t) \end{pmatrix}$$

# Cosmological solution: harmonic embedding

We want FRW-metric:  $\dot{x}_c^2 - a^2 \dot{\psi}^2 - \dot{a}^2 = k$

We need harmonic embedding:  $\Delta_g x^a = 0$

$$\Delta_g (\mathcal{R}(t) S(\chi) \cos(\theta)) \stackrel{!}{=} 0$$

$$\Delta_g x_c \stackrel{!}{=} 0$$

# New Evolution Equations

$$\frac{3}{a}(\dot{a}^2 + k) + \ddot{a} - \dot{\psi}^2 a = 0$$

$$5\dot{\psi}\dot{a} + \ddot{\psi}a = 0$$

$$\frac{3}{a}\dot{a}\dot{x}_c + \ddot{x}_c = 0$$

can be integrated

$$(\dot{a}^2 + k)a^6 + b^2 a^{-2} = m = \text{const}$$

$$\dot{\psi} = b a^{-5}, \quad b = \text{const} > 0$$

$$a^3 \dot{x}_c = d = \sqrt{m} = \text{const}, \quad m > 0$$

# New Evolution Equations

$$H^2 = \frac{\dot{a}^2}{a^2} = -b^2 a^{-10} + m a^{-8} - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = -3m a^{-8} + 4b^2 a^{-10}$$

These equations follow from harmonic embedding condition.

Have not coupled to matter yet.

Corrections of order  $\rho/\Lambda^4$  expected.

Evolution Eq. different from Friedmann Equation.

What determines the constants  $m$  &  $b$ ?

# Physical aspects – Milne universe

$k = 0, +1$ : Lifetime of the universe too short. Excluded.

$$k = -1$$

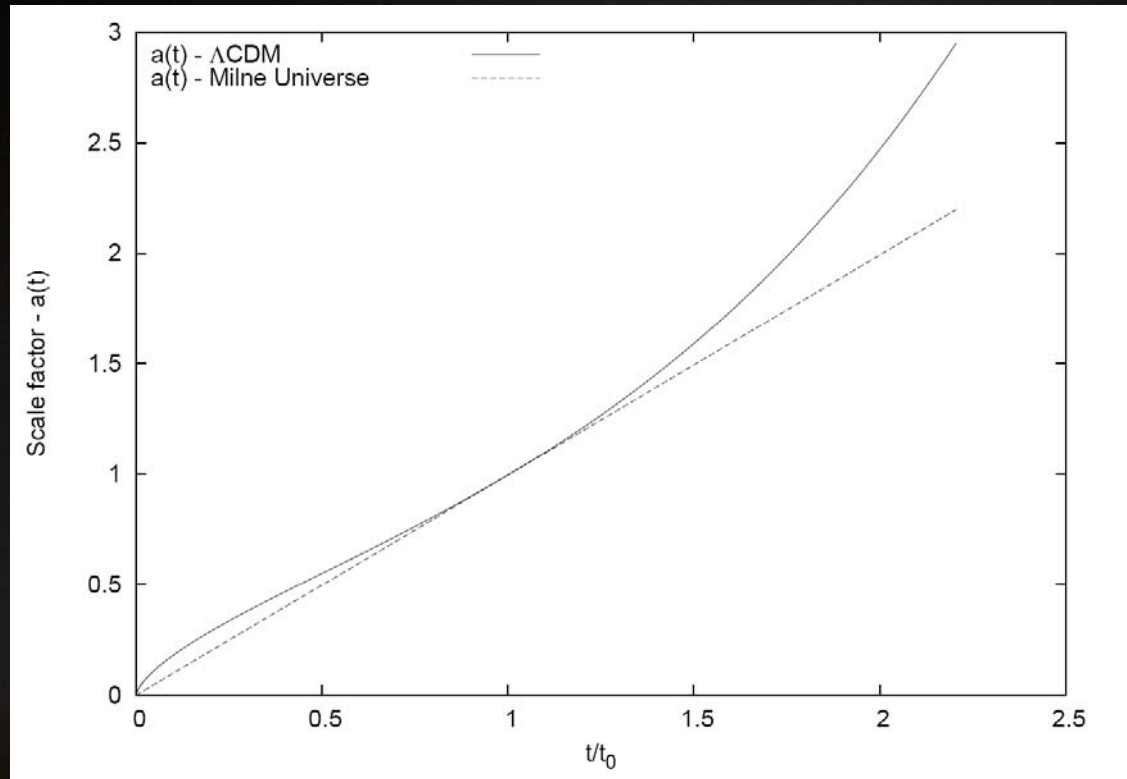
For large  $a$

$$\dot{a} \rightarrow 1 \quad \therefore \quad a(t) \propto t$$

$$t_0 \sim 1/H_0 \sim 13.9 \cdot 10^9 \text{ y}$$

Natural correct result for the age of the universe, unlike in the  $\Lambda$ CDM model (where this is strange coincidence).

# Physical aspects – Milne universe



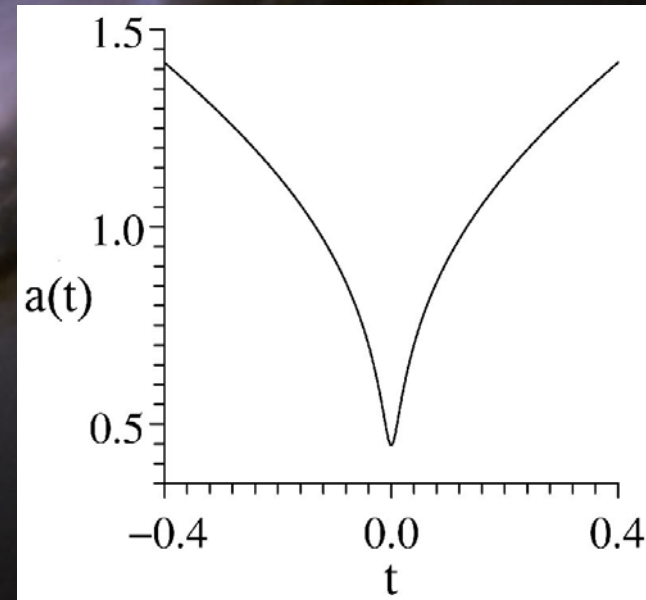
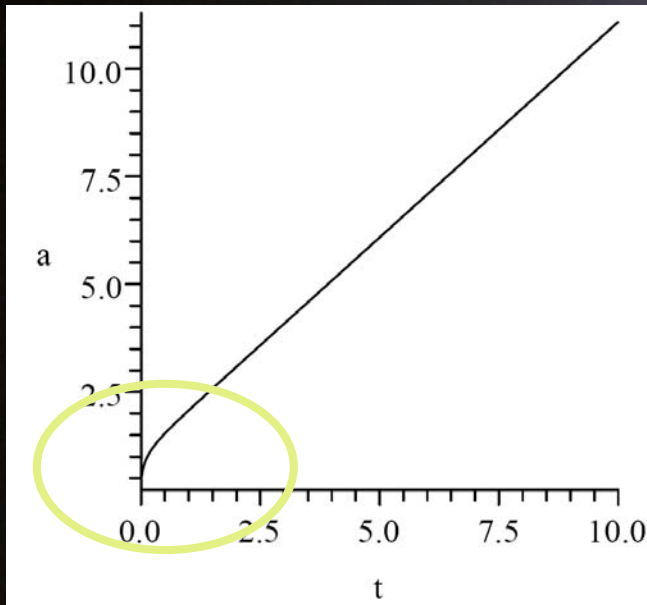
Milne universe is compatible with observation.

A. Benoit-Levy, G. Chardin arXiv:0811.2149 [astro-ph]

# Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m a^{-6} + 1}$$

Minimal size of the universe: positive root



Big Bounce: Time evolution symmetric under  $t \leftrightarrow -t$

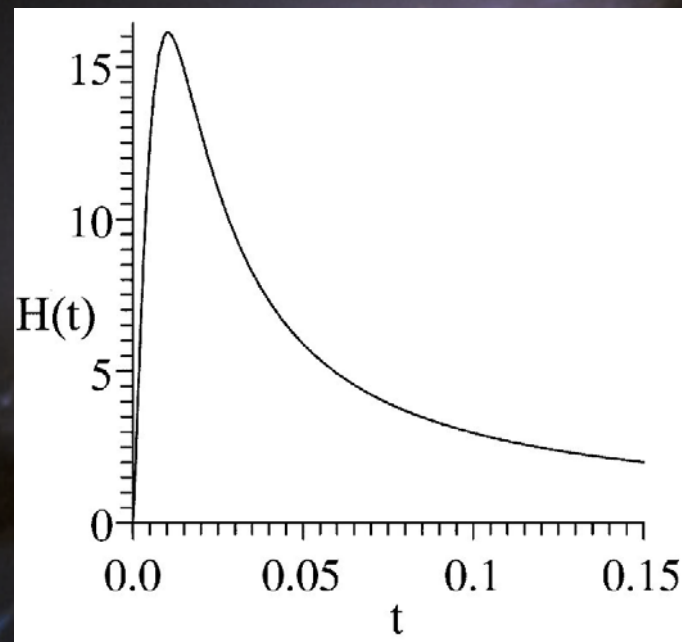
$m=5, b=1$  in these qualitative charts.



# Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m a^{-6} + 1}$$

Minimal size of the universe: positive root



Big Bounce: Time evolution symmetric under  $t \leftrightarrow -t$

$m=5, b=1$  in these qualitative charts.

# Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m a^{-6} + 1} = f(a)$$

Minimal size of the universe: positive root  $a_0$

Expansion around  $a_0$

$$\dot{a} \sim \sqrt{\left. \frac{df(a)}{da} \right|_{a=a_0} (a - a_0)}$$

shows inflation-like phase with a graceful exit

$$a(t) \propto t^2$$

$$a(t_{\text{exit}}) = \sqrt{\frac{4b^2}{3m}} \leftrightarrow \ddot{a} = 0$$

# Conclusions

we have found a cosmological solution of emergent NC gravity of FRW type.

no-fine tuning of the cosmological constant

purely geometric mechanism leads to evolution equations for the universe:

Milne-type universe

inflationary-like phase

big bounce

so far in agreement with observation

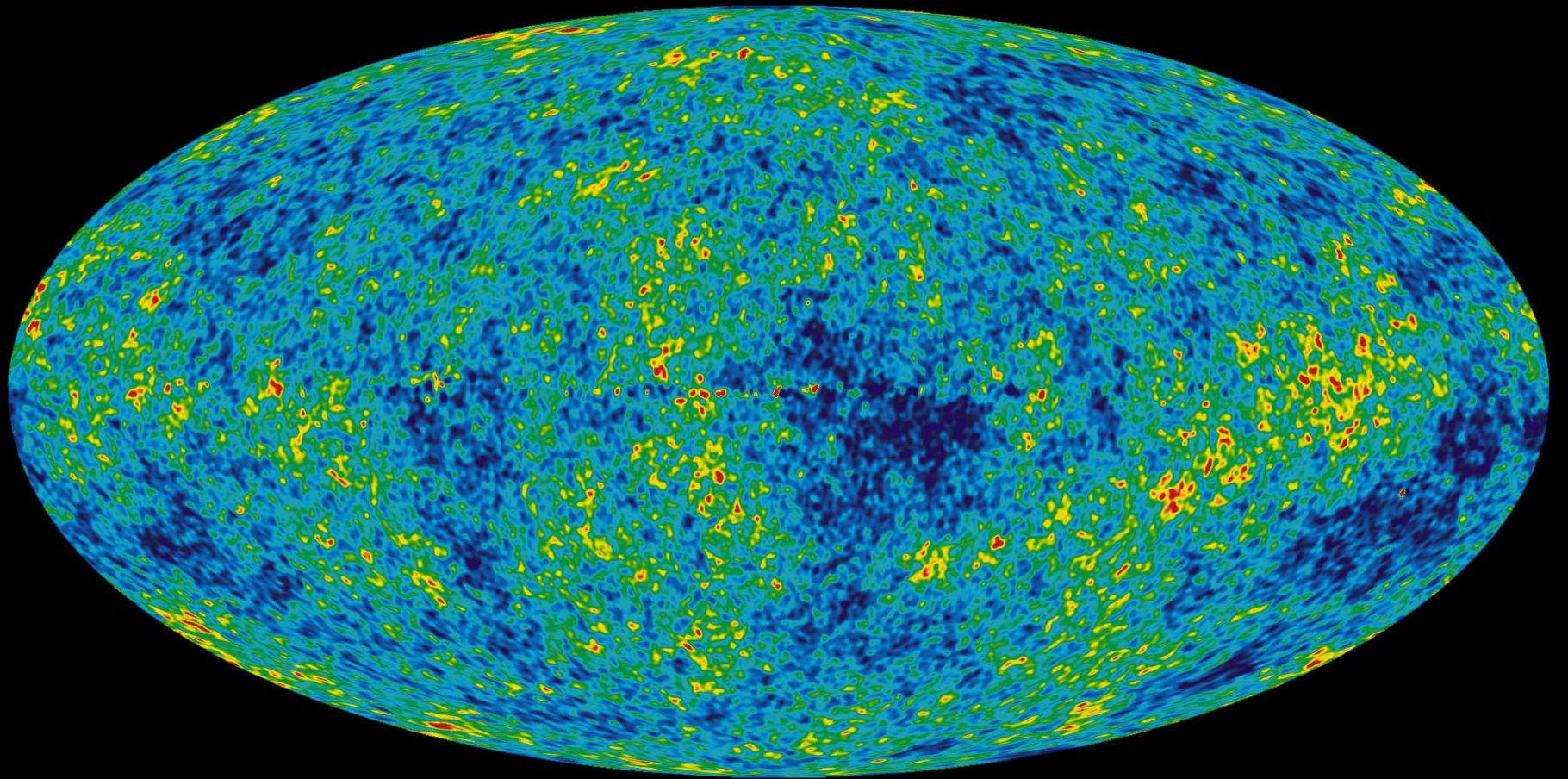
# What remains to be done?

Add matter content to this universe

¿ m,b ?

Detailed study of the early universe necessary

¿ compatible with CMB ?





Thank you