Cosmological Solutions of Emergent Noncommutative Gravity

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Daniela Klammer

Joint work with
Harold Steinacker

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University of Vienna

outline

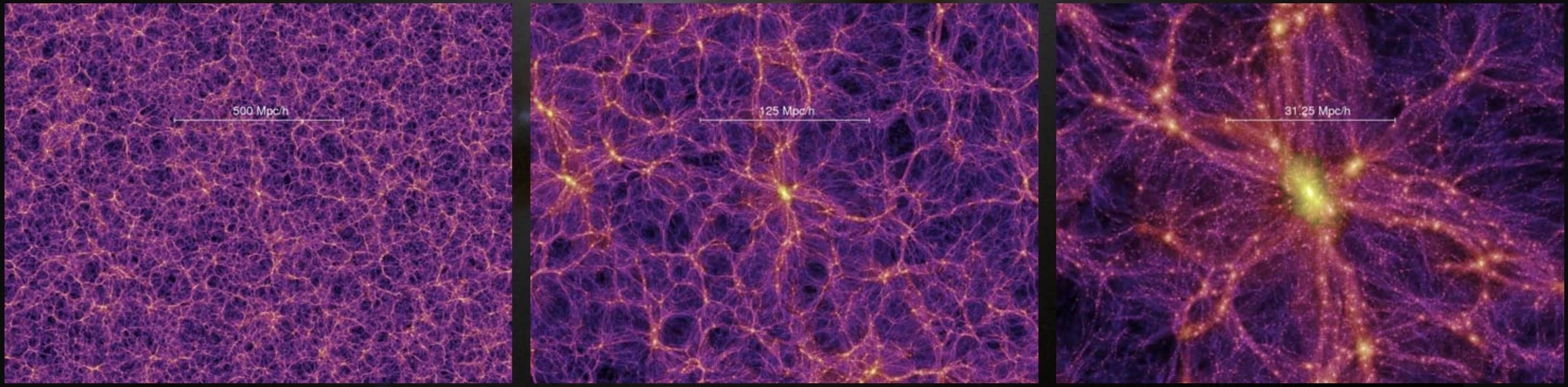
- Cosmology
- Cosmological Constant
- Emergent NC Gravity
- The cosmological solution
- Aspects of this universe

Cosmology

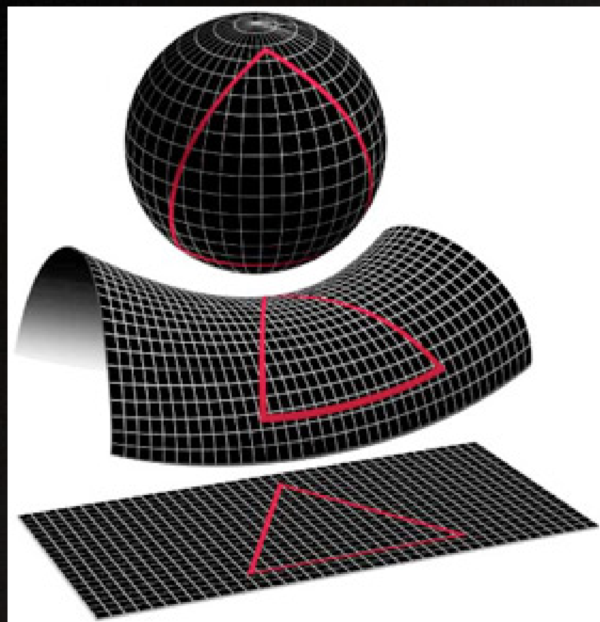
universe is homogenous & isotropic (if scale is large enough)

Friedmann-Robertson-Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{1}{1-kr^2}\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2 \right)$$



FRW: Geometry of the universe



$k = 1$	closed
$k = -1$	open
$k = 0$	flat

Concordance model – ΛCDM model

- FRW-metric, Friedmann eq., cosmological eq. of state (after inflation)
- cosmological constant Λ = dark energy term
- cold dark matter, non-baryonic
- Inflation: scale invariant spectrum of primeordial perturbations, flat curvature $k = 0$, universe is much larger than observable particle horizon

Friedmann Equations

Expansion of the universe governed by Einstein eq.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

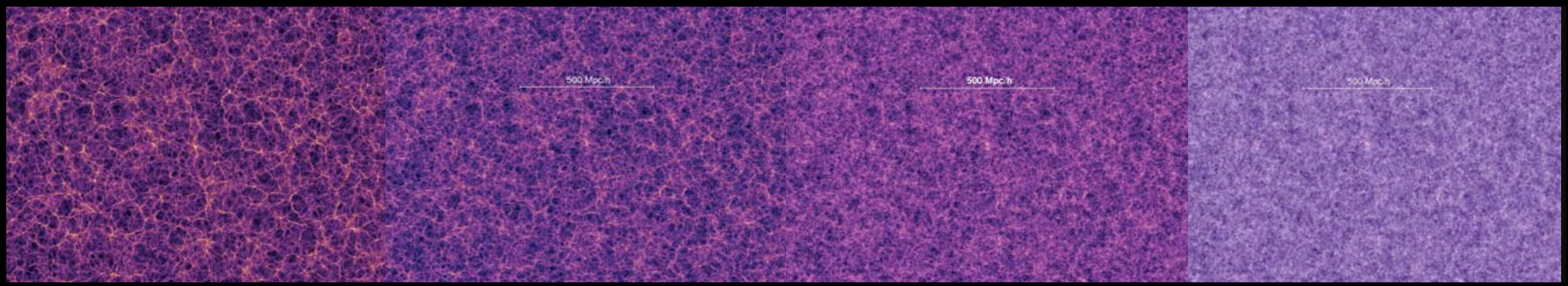
↓

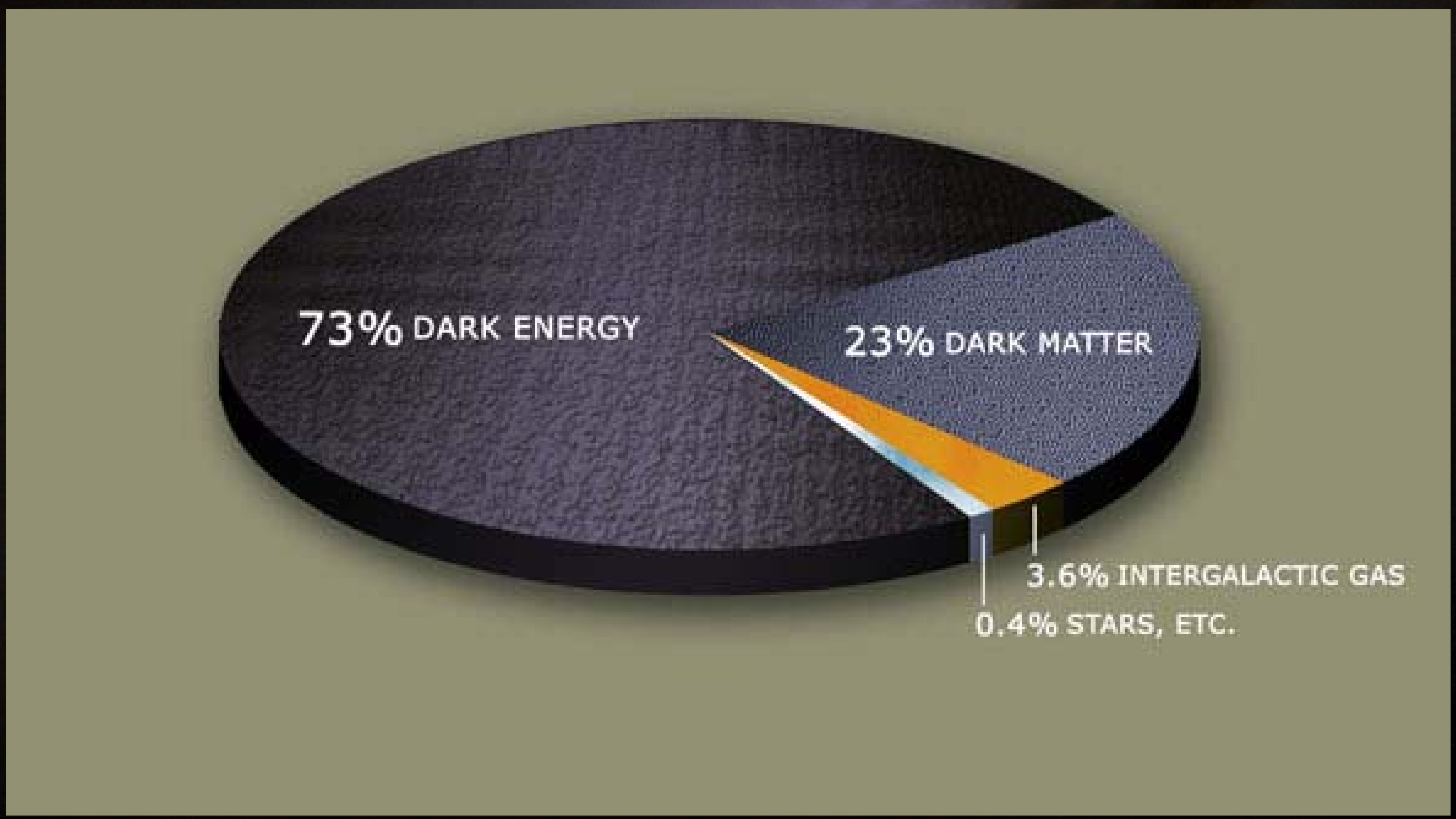
$$\dot{a}^2 + k = \frac{8\pi G \rho a^2}{3} + \frac{\Lambda}{3} a^2$$

Conservation law

$$\dot{\rho} = -\frac{3\dot{a}}{a}(\rho + p)$$

Scale invariance: large-scale structure at different times





The cosmological constant problem

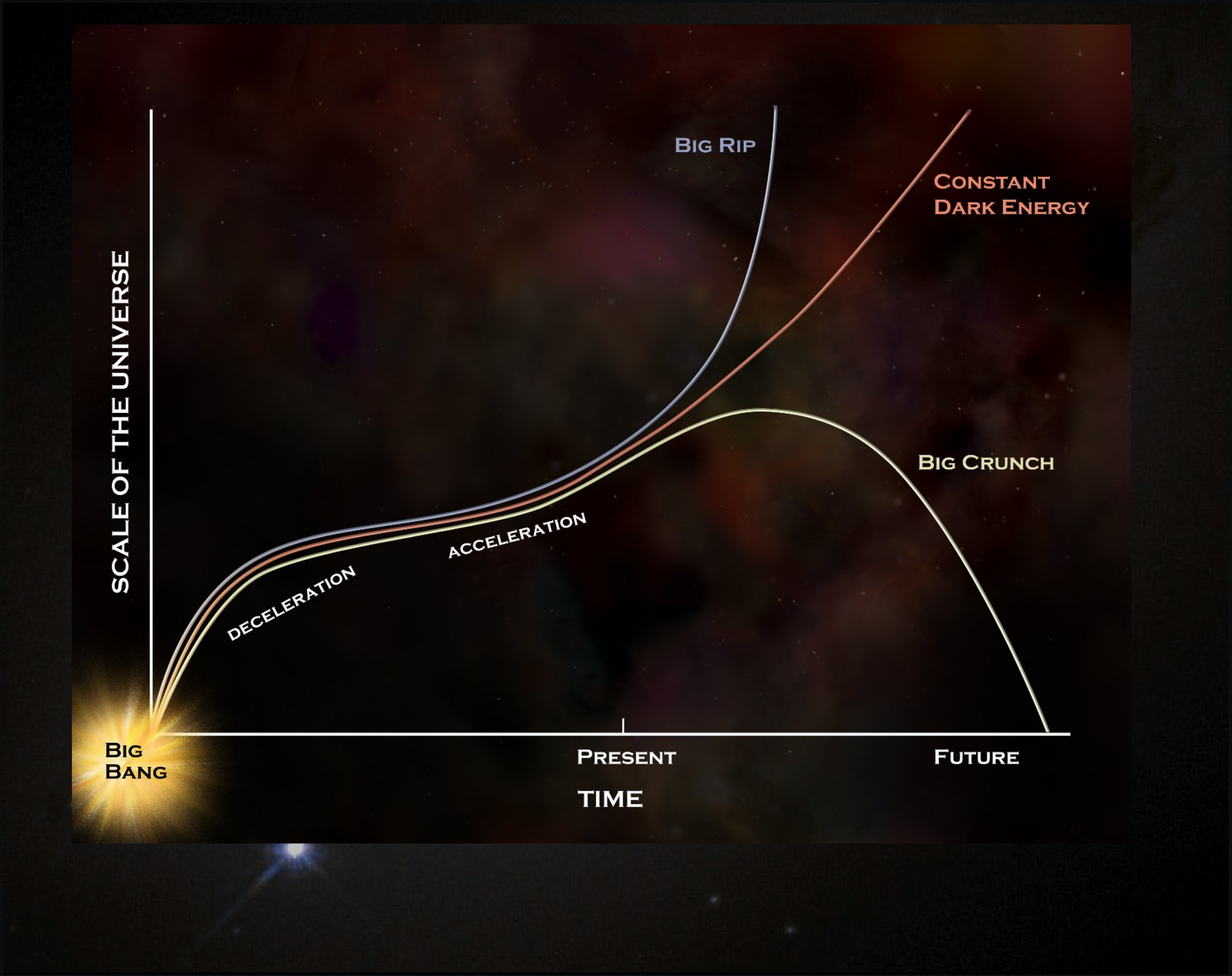
Why is the vacuum energy smaller than expected?

Contributions from quantum fluctuations in known fields →
vacuum energy density ~ $(300\ \text{GeV})^4 \approx 10^{27}\ \text{g/cm}^3$

Observed value: $10^{-29}\ \text{g/cm}^3$

¿ $\sim 10^{60}$?

Coincidence problem: Why is the dark energy beginning to dominate now?


Big Rip
Constant
Dark Energy
Big Crunch
Scale of the Universe
acceleration
deceleration
Big
Bang
Present
Future
Time

NC Emergent Gravity

$$S_{YM} = -\mathrm{Tr}\left[X^a, X^b\right]\left[X^{a'}, X^{b'}\right] g_{aa'} g_{bb'}$$

$X^a \in L(\mathcal{H})$ … matrices/operators $a = 0, \ldots, D-1$

$$\boxed{[X^a, X^b] = i\,\theta^{ab}}$$

- θ^{ab} not costant
- X^a interpreted as quantization of coordinate function x^a on Poisson manifold M with poisson structure $\theta(x)$
- Semi-classical limit:

$$[X^a, \varphi] \sim i\,\theta^{ab}(x)\frac{\partial}{\partial x^b}\varphi$$

Extra dimensions & embedding

$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}$$

$$a, b = 0, \ldots, D-1$$

Embedding through scalar fields $\phi^i(x)$:

$$X^a = (X^\mu, \phi^i)$$

$$\phi^i = \phi^i(x^\mu)$$

Induced metric $g_{\mu\nu}(x)$:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + (\partial_\mu \phi^i)(\partial_\nu \phi^j)\delta_{ij}$$

Effective geometry

Couple test particle/ scalar field to matrix model

$$S[\varphi] = \mathrm{Tr}\,[X^a,\varphi]\,\left[X^b,\varphi\right]\,g_{ab}$$
$$\sim \int \mathrm{d}^4x\,\sqrt{|G_{\mu\nu}|}G^{\mu\nu}(x)(\partial_\mu\varphi)(\partial_\nu\varphi)$$

Effective metric:

$$G^{\mu\nu}(x) = e^{-\sigma}\theta^{\mu\alpha}(x)\theta^{\nu\beta}(x)g_{\alpha\beta}(x)$$

All fields couple to this effective metric.

eom

$$[X^a,[X^b,X^{a'}]]\eta_{aa'}=0$$

Semi-classical limit

$$\Delta_G\varphi^i=0$$

harmonic embedding

$$\nabla^\mu(e^\sigma\theta^{-1}_{\mu\nu})=e^{-\sigma}G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta$$

$$\eta=e^\sigma G^{\mu\nu}g_{\mu\nu}$$

covariant formulation

Harmonic embedding - minimal surfaces

Harmonic embedding & Λ

$$Z = \int \mathrm{d}X^a e^{-S_{YM}[X]}$$

$$\Gamma_{1-loop} = \frac{1}{16\pi^2} \int \mathrm{d}^4x \sqrt{|G|} \left(c_1 \Lambda_1^4 + c_4 R[G] \Lambda_4^2 + O(\ln \Lambda) \right)$$

usually cosmological constant term

large because Λ_1 is large

omitting fermions for simplicity

Harmonic embedding & Λ

$$Z = \int \mathrm{d}X^a e^{-S_{YM}[X]}$$

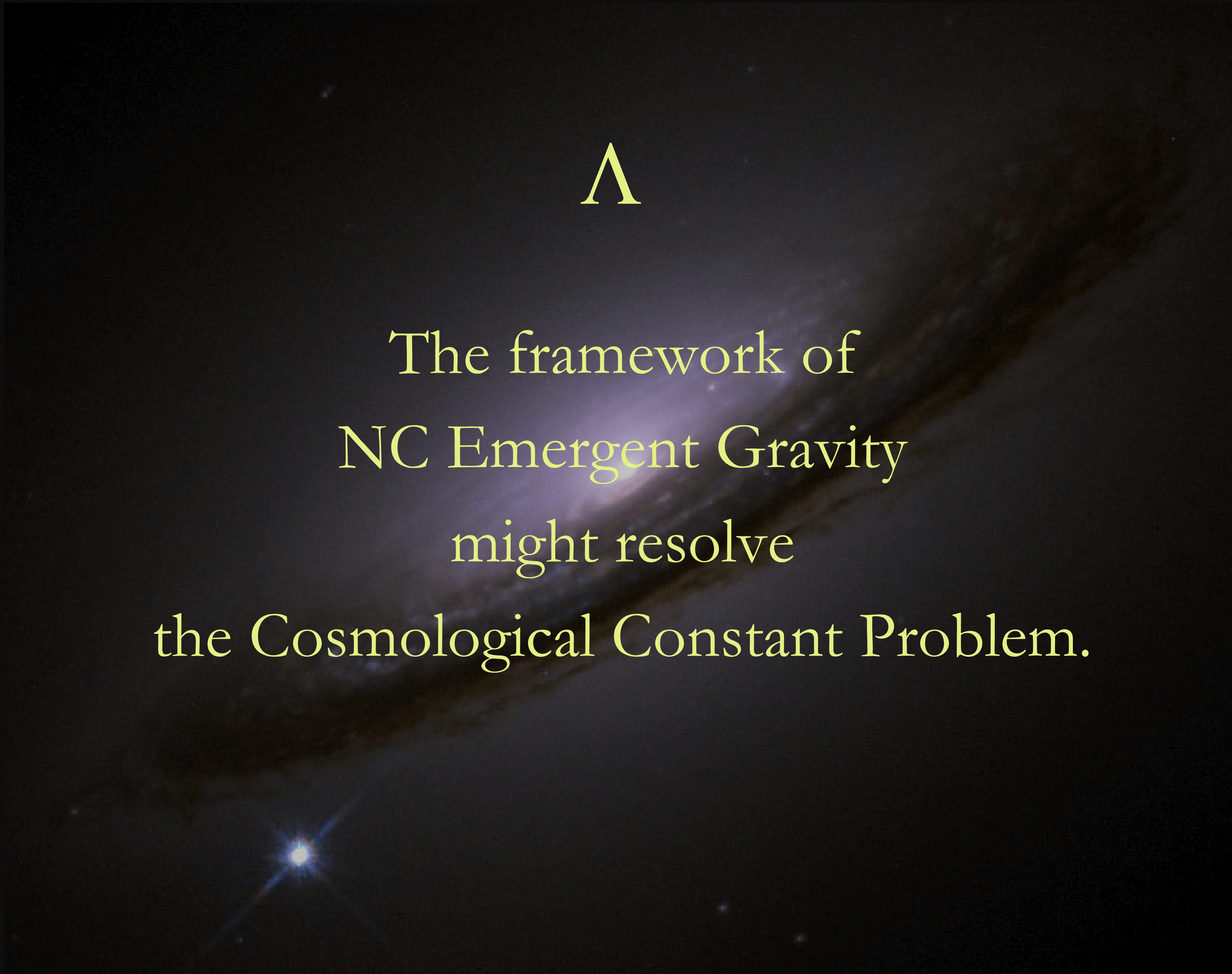
$$\Gamma_{1-loop} = \frac{1}{16\pi^2} \int d^4x \sqrt{|G|} \left(c_1 \Lambda_1^4 + c_4 R[G] \Lambda_4^2 + O(\ln \Lambda) \right)$$

in NC emergent gravity: different meaning

$$\delta \int d^4x \sqrt{|G|} = \int d^4x \sqrt{|g|} g^{\mu\nu} \delta g_{\mu\nu} = \int \sqrt{|g|} \delta\phi^i (\Delta_g \phi^j) \delta_{ij}$$

$$|G_{\mu\nu}(x)| = |g_{\mu\nu}(x)|$$

<u>vanishes</u>


Λ
The framework of
NC Emergent Gravity
might resolve
the Cosmological Constant Problem.

The cosmological solution

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + S(\chi)^2 d\Omega^2)$$

$$S(\chi) = (\sin\chi, \chi, \sinh\chi) = r$$
$$C(\chi) = (\cos\chi, \cosh\chi)$$

Embedding for k = ± 1

$$\vec{x}(t,\chi,\theta,\varphi) = \begin{pmatrix} \mathcal{R}(t) \begin{pmatrix} S(\chi)\sin\theta\cos\varphi \\ S(\chi)\sin\theta\sin\varphi \\ S(\chi)\cos\theta \\ C(\chi) \end{pmatrix} \\ 0 \\ x_c(t) \end{pmatrix} \in \mathbb{R}^{10}$$

$$\mathcal{R}(t) = a(t) \begin{pmatrix} \cos\psi(t) \\ \sin\psi(t) \end{pmatrix}$$

Cosmological solution: harmonic embedding

We want FRW-metric: $\dot{x}_c^2 - a^2\dot{\psi}^2 - \dot{a}^2 = k$

We need harmonic embedding: $\Delta_g x^a = 0$

$$\Delta_g(\mathcal{R}(t)S(\chi)\cos(\theta)) \stackrel{!}{=} 0$$

$$\Delta_g x_c \stackrel{!}{=} 0$$

New Evolution Equations

$$\frac{3}{a}(\dot{a}^2 + k) + \ddot{a} - \dot{\psi}^2 a = 0$$

$$5\dot{\psi}\dot{a} + \ddot{\psi} a = 0$$

$$\frac{3}{a}\dot{a}\dot{x}_c + \ddot{x}_c = 0$$

can be integrated

$$(\dot{a}^2 + k)a^6 + b^2 a^{-2} = m = const$$

$$\dot{\psi} = b\, a^{-5}, \qquad b = const > 0$$

$$a^3 \dot{x}_c = d = \sqrt{m} = const, \qquad m > 0$$

New Evolution Equations

$$H^2 = \frac{\dot{a}^2}{a^2} = -b^2 a^{-10} + m\, a^{-8} - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -3\, m\, a^{-8} + 4 b^2 a^{-10}$$

These equations follow from harmonic embedding condition.

Have not coupled to matter yet.

Corrections of order ρ/Λ^4 expected.

Evolution Eq. different from Friedmann Equation.

What determines the constants m & b?

Physical aspects – Milne universe

k = 0, +1: Lifetime of the universe too short. Excluded.

k = -1

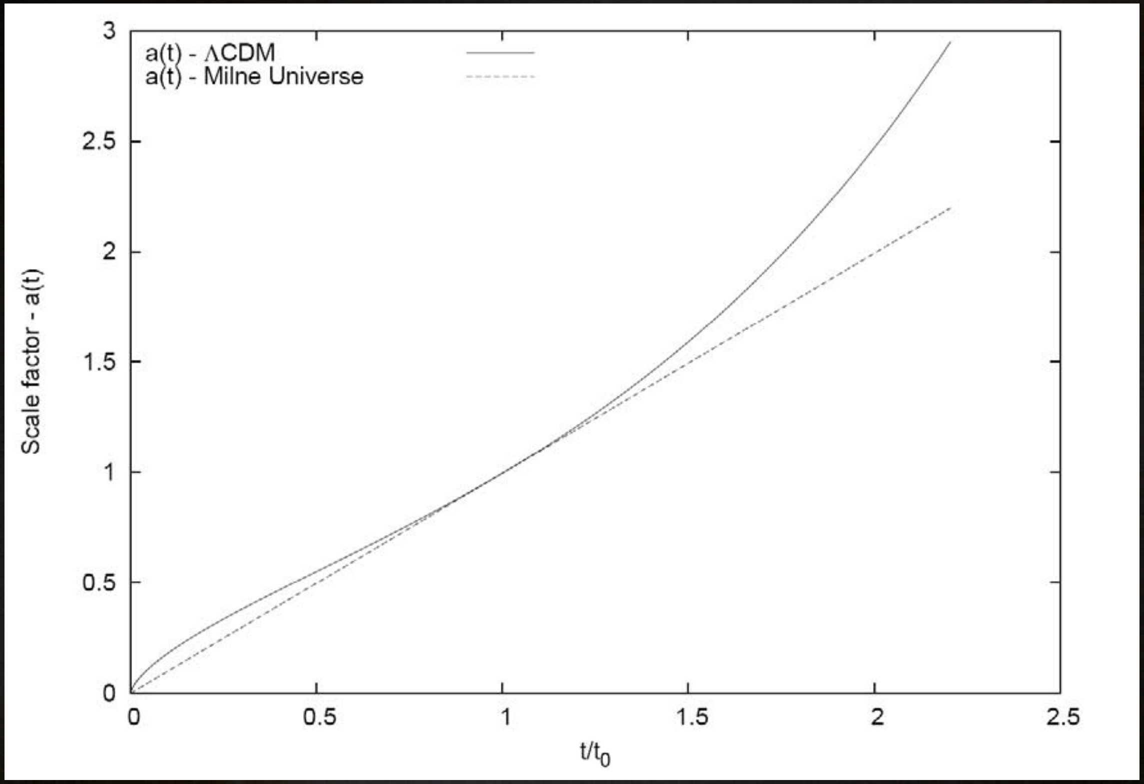
For large a

$$\dot{a} \to 1 \qquad \therefore \qquad a(t) \propto t$$

$$t_0 \sim 1/H_0 \sim 13.9 \cdot 10^9 \text{ y}$$

Natural correct result for the age of the universe, unlike in the ΛCDM model (where this is strange coincidence).

Physical aspects – Milne universe



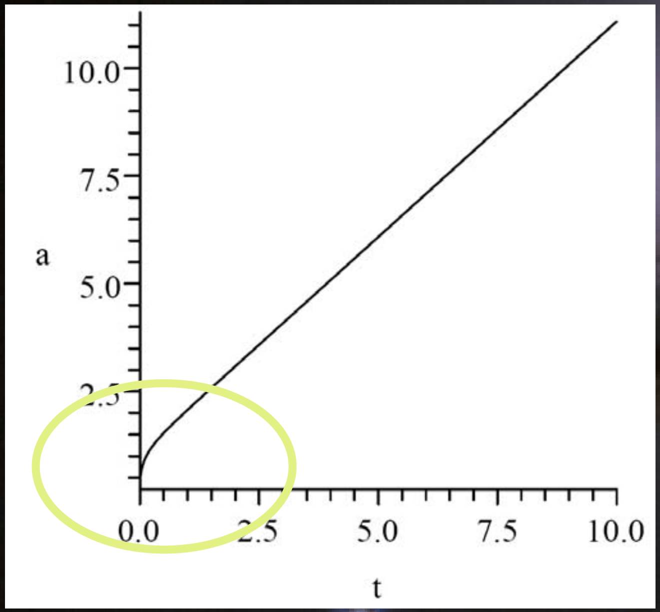
Milne universe is compatible with observation.

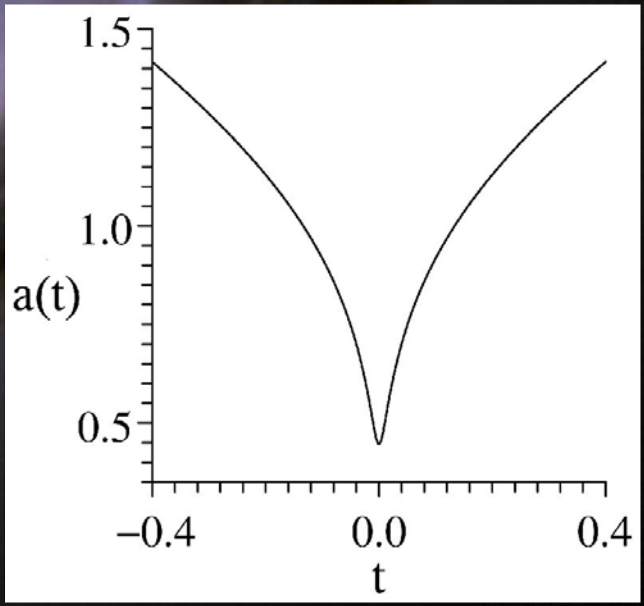
A. Benoit-Levy, G. Chardin arXiv:0811.2149 [astro-ph]

Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m\, a^{-6} + 1}$$

Minimal size of the universe: positive root





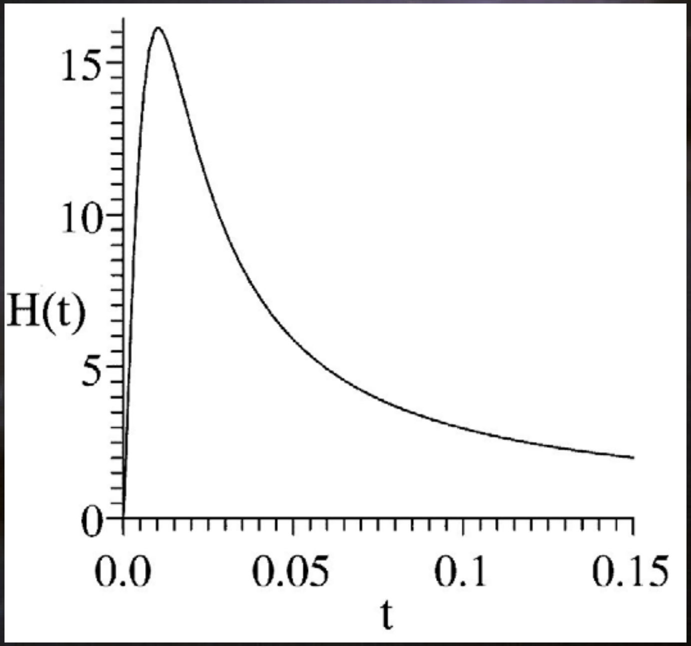
Big Bounce: Time evolution symmetric under t ↔ -t

m=5, b=1 in these qualitative charts.

Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m\,a^{-6} + 1}$$

Minimal size of the universe: positive root



Big Bounce: Time evolution symmetric under t ↔ -t

m=5, b=1 in these qualitative charts.

Inflation & Big Bounce

$$\dot{a} = \sqrt{-b^2 a^{-8} + m\, a^{-6} + 1} = f(a)$$

Minimal size of the universe: positive root a_0

Expansion around a_0

$$\dot{a} \sim \sqrt{\frac{\mathrm{d}f(a)}{\mathrm{d}a}\Big|_{a=a_0}(a - a_0)}$$

shows inflation-like phase with a graceful exit

$$a(t) \propto t^2 \qquad a(t_{\mathrm{exit}}) = \sqrt{\frac{4b^2}{3m}} \leftrightarrow \ddot{a} = 0$$

Conclusions

we have found a cosmological solution of emergent NC gravity of FRW type.

no-fine tuning of the cosmological constant

purely geometric mechanism leads to evolution equations for the universe:

Milne-type universe

inflationary-like phase

big bounce

so far in agreement with observation

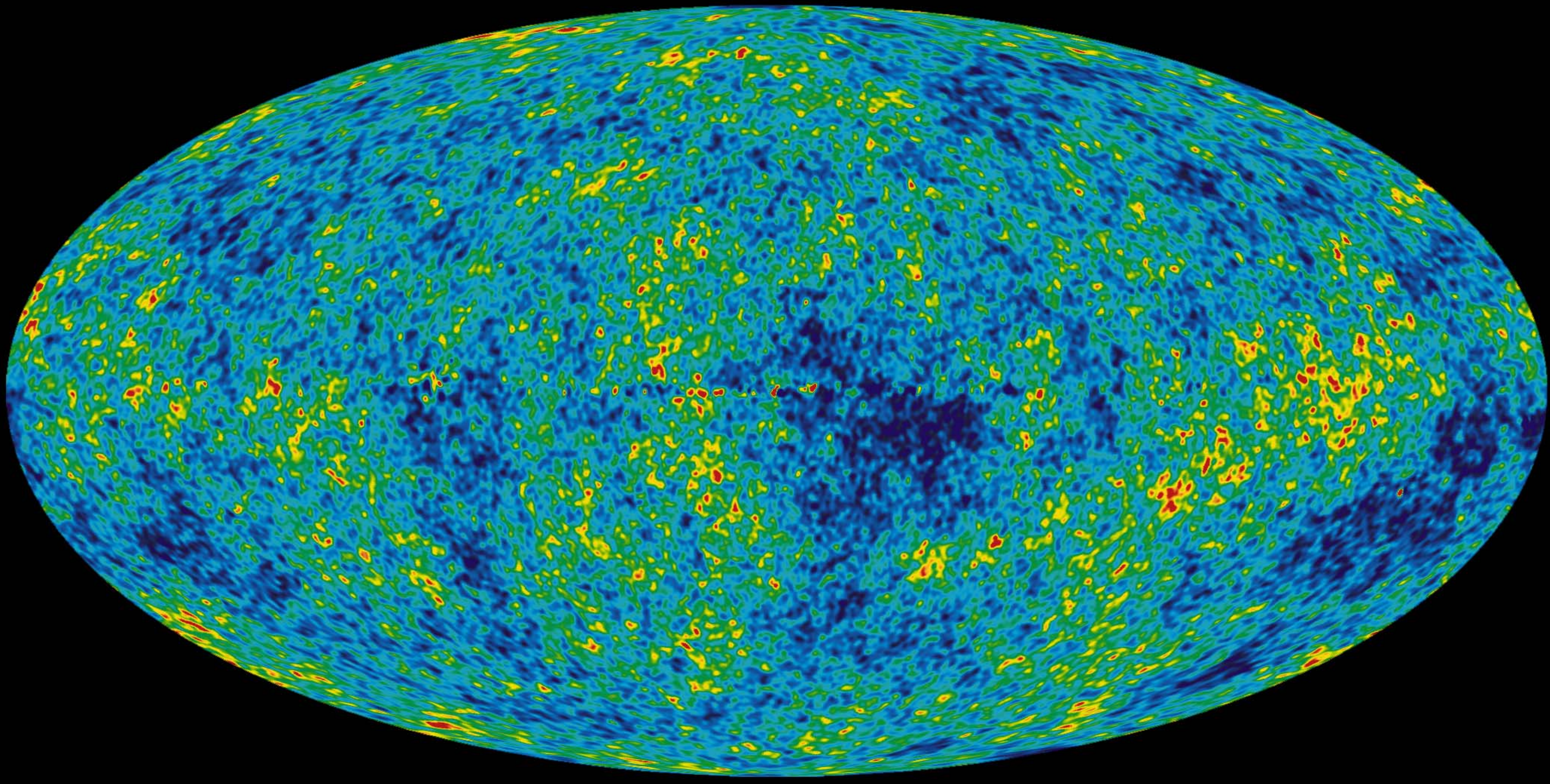
What remains to be done?

Add matter content to this universe

¿ m,b ?

Detailed study of the early universe necessary

¿ compatible with CMB ?




Thank you