Assessment of traffic situations – dealing with braking distances and "remaining velocities"

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Abstract:
We present worksheets with which students of grade 11 can get important knowledge about brake applications, realize crucial phenomena, and develop formulas concerning "braking applications" primarily by individual work. In German literature there are also several other articles by other authors (see the references) dealing with the phenomenon of "remaining velocities". Here in our paper the focus lies on the autonomous work of students: How can we prepare worksheets so that students can come to deeper insights by working independently in groups?
We have already published a possible version in the last project publication ("planting mathematics", see [2]) but Jan Müller has changed some items so that we can present another version which is more related to "real data", "descriptive statistics", and "fitting of functions". He also tested this material in grade 11 in the last year quite successfully. The aim of the following learning environment is to sensitise students for problems in traffic education: assessing some given and specified or self constructed situations of danger in traffic.

INTRODUCTION

Beginning at the age of 15 students attend courses at driving schools in order to get their driving licence for mopeds, motorbikes and later also for cars. In these courses the problem of the dependence of the braking distance on the initial velocity is, of course, dealt with. But due to the heterogeneity of the participants only simple empirical formulas are used. E.g. $\left(\frac{v}{10}\right)^2$ gives approximately the braking distance in meters if v denotes the velocity in km/h. In most cases such formulas are not explained at driving schools. Time is not spent on answering the question where this formula comes from, how can one understand it thoroughly? It is a very useful formula because one can easily estimate the braking distance by looking at the speedometer: If you drive at a velocity of 50 km/h then a short head calculation shows that the braking distance is approx. $5^2 = 25$ meters. In the above form the formula also disregards the variety for the possible values of the so called "braking deceleration" b. The learning environment we want to present here should enable the students to develop such a formula by themselves, to understand it, to analyse and to assess traffic situations. In addition the students should be sensitised for the development of the velocity in terms of the distance covered (instead in terms of the time elapsed). This functional relation *velocity – distance* is a bit more complicated to analyse than the relation *velocity – time* but it leads to deep and important insights

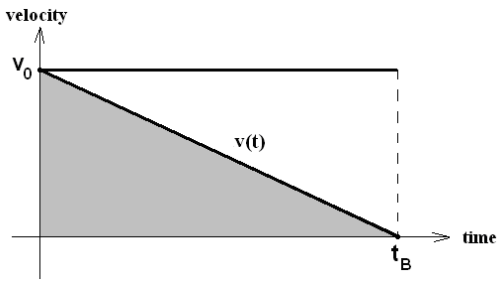
concerning "remaining velocities". You can face the dramatic and enormous consequences of too high speed in traffic much better when having a look at "remaining velocities" than by merely dealing with the formula for the braking distance[1] $s_B = \frac{v_0^2}{2b}$ and learning it by heart, even if one interprets the formula correctly: "double velocity causes four times the braking distance". Considering the "remaining velocity" in case of a collision is a proper way to show the striking consequences of too high speed, especially in case of accidents where people are involved (safety in traffic)!

The learning environment had three parts:

PART A: THE BRAKING TIME AND THE BRAKING DISTANCE IN TERMS OF THE INITIAL VELOCITY – SEE WORKSHEET A1

In this part the formula $s_B(v_0, b) = \frac{v_0^2}{2b}$ for the braking distance s_B should be derived (in terms of the initial velocity v_0 and the braking deceleration *b*). We assume a steady brake application that means a constant braking deceleration *b*, which is determined by the power of the brakes, by the road surface, by weather conditions, tyre profiles etc.

The modelling is oriented at a geometric illustration of the brake application in the *v*-*t*-diagram: The braking time (time between beginning of the brake application and stop) is denoted as t_B. It is clear that the car would cover the distance $v_0 \cdot t_B$ without braking. This product – i.e. the braking distance – can be interpreted as the area of a rectangle in the *v*-*t*-diagram. When braking steadily the velocity will decrease steadily during the braking time until it is 0. The *v*-*t*-diagram then is no more a constant function but a linear function with negative slope. The usual consideration with small time intervals explains plausible that also in case of a non constant velocity the *area* under the *v*-*t*-graph gives the distance covered (→ see the grey triangle in the figure).

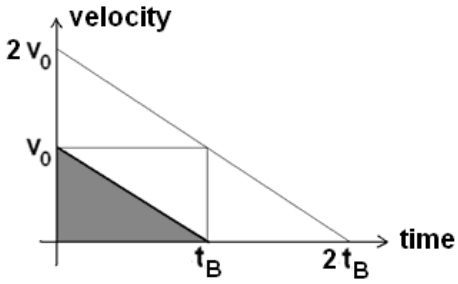


[1] The Index *B* for the distance *s* comes from "braking". v_0 denotes the initial velocity and *b* denotes the (constant) braking deceleration.

That means the distance covered during the brake application equals half the distance in the case of not braking during the time t_B: $s_B = \frac{1}{2} \cdot v_0 \cdot t_B$ (*)

The braking time is $t_B = \frac{v_0}{b}$. Within every second the velocity gets less by[2] *b* m/s. This can be inserted in (*) which leads to: $\boxed{s_B(v_0, b)} = \frac{1}{2} \cdot v_0 \cdot t_B = \frac{1}{2} \cdot v_0 \cdot \frac{v_0}{b} = \boxed{\frac{v_0^2}{2b}}$.

If the initial velocity is doubled ($v_0 \to 2v_0$) you can see in the *v*-*t*-diagram that also the braking time is doubled ($t_B \to 2t_B$). Geometrically spoken a central dilatation with factor 2 happens to the considered "braking triangle". Therefore the area (i.e. braking distance) is multiplied by the factor 4!



After working with worksheet A1 the students subsequently did some adequate exercises (see A2) which led to an assessment of given situations by using numerical and graphical methods developed before. Take for example task 3 of A2 and you will see: Due to the longer braking distances it is clear that the power of the brakes gets less. So it is possible to determine a limit beyond which the braking system of the car cannot be seen as safe.

It is also possible to deal more deeply with the above formula of driving schools $s_B \approx \left(\frac{v}{10}\right)^2$. Which value of *b* is used here[3]?

PART B: THE VELOCITY DURING A BRAKING PROCESS AS A FUNCTION OF THE DISTANCE COVERED – SEE WORKSHEET B1

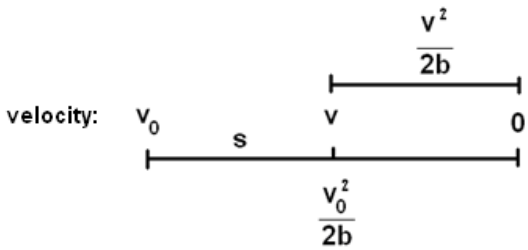
In the second part the students should – by individual work – derive the formula $v(s) = \sqrt{v_0^2 - 2bs}$ for calculating the velocity in terms of the

[2] In the first lessons the interpretation of *b* was "per second the velocity gets less by *b* m/s", we did not use the unit m/s² in the beginning.
[3] In the class of Jan H. Müller this was done at the end of the course.

distance s covered, the initial velocity v_0 and the braking deceleration b. This is done in the following way (see B1, task 1):
Students should draw a free hand sketch. They are asked to speculate on how the velocity develops in terms of the distance covered. The result was that many students were unsure whether they should take a linear function ("straight line") or a line slightly "curved upwards". To clarify this uncertainty the next step in this exercise is to deal with "real data". This shows that the non linear model (the curved upwards one) fits better here.

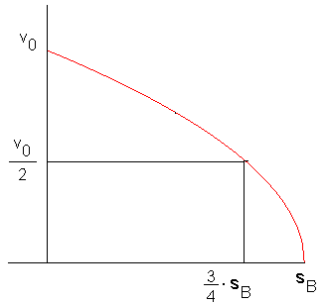
The term $v(s) = \sqrt{v_0^2 - 2bs}$ of the function – which is aimed at – is derived by interpreting the difference $\frac{v_0^2}{2b} - \frac{v^2}{2b}$ (with $v < v_0$). Both $\frac{v_0^2}{2b}$ and $\frac{v^2}{2b}$ give braking distances, on the one hand with initial velocity v_0 and on the other with initial velocity $v < v_0$.

Therefore the difference $\frac{v_0^2}{2b} - \frac{v^2}{2b}$ denotes the distance s which is necessary to reduce the velocity from v_0 to v: $s = \frac{v_0^2}{2b} - \frac{v^2}{2b}$.



Solving this equation for v yields $v(s) = \sqrt{v_0^2 - 2bs}$. This square root term may help to better understand the non linearity of the relation between the velocity and the distance covered (during a braking application) – see above: free hand sketch.

To point out the advantage or usefulness of this function the students should calculate (→B2, task 3). By which percentage has velocity decreased after 10%, 20%, . . ., 90%, 100% of the braking distance. Here you can see drastically that after half the braking distance only ¼ of the velocity is reduced, and that you need ¾ of the braking distance to reduce half the velocity. This shows that velocity is reduced primarily at the end of the braking distance (*only* in terms of the *time elapsed* the velocity slows down steadily).



PART C: ASSESSING A CERTAIN TRAFFIC SITUATION – SEE WORKSHEET C

In the third part the students worked together in groups and chose one traffic situation (out of two) to analyse, to describe mathematically and to assess with respect to the parameters considered (→ worksheet C). In our sample study the students had two weeks time, also for the parts 1 and 2 together, so that the whole learning environment lasted four weeks (3 lessons a week). Students planned for themselves which way of working they preferred, e.g. they worked partly alone and at home and discussed their results with the other members of the group during the lessons.
This kind of teaching organisation had the advantage of giving teachers the chance to help and encourage very well both high-performance students and low-performance students individually Result: high-performance students wanted to deal with very challenging problems and questions, the others wanted to be closer to questions of part 1 and 2. The aim for all was to produce a Power-Point presentation of about 10 minutes, which was presented in the last two lessons..

Due to the formula $v(s) = \sqrt{v_0^2 - 2bs}$ from part 2 it was possible to calculate so called "remaining velocities": At the very position at which a car driving initially at 30 km/h comes to stop[4], another car driving initially at a speed of 50 km/h – beginning the brake application at the same point – has a "remaining velocity of 40 km/h. Such a high collision velocity is in most cases lethal for a child! With respect to a zone where cars are allowed to drive 30 km/h at most (for instance in many residential areas in inner cities) this means: Someone driving only 20 km/h too fast has at the "crucial position"[5] a significantly higher velocity than someone who is obeying the speed limit and **not braking at all**!

If one takes into account the different reaction *distances* of these two drivers (let's assume equal reaction *times*: 1 s) and a braking deceleration of $b \approx 6$ m/s² (this is a highly realistic value) then we have the fact that the 50 km/h car has even its **full velocity (50 km/h)** at the "crucial position"! That means it even has not yet started to brake at this position!

Such (or similar) scenarios were meant when we wrote at the beginning "considering the 'remaining velocity' in case of a collision is a good way to point-out drastically the striking consequences of too high speed, especially in case of accidents where people are involved (safety in traffic)!" We are sure that these realisations have more effects than phrases like: "Double velocity means four times the braking distance". So mathematics can contribute to more responsibility and cautiousness in traffic.

[4] E.g. because of a careless child fetching a ball .
[5] The position at which the 30 km/h car comes to stop.

CONCLUSION

The described learning environment was profitable to the students for several reasons:

- Due to the way of organisation the teacher had a better chance to advise students individually, e.g. to discuss advantages/disadvantages of the working strategy chosen, or to help them in numerical and mathematical analysis. High-performance students were encouraged to help others.
- The working material (worksheets and exercises) made it necessary for the students to decide for themselves which parts had to be done at home. A feedback after dealing with this learning environment showed that many students had troubles with this way of working while supervising themselves.
- The whole topic was dealt with in grade 11. It was linked to the topic of "descriptive statistics". Therefore ideas like "linear regression", "fitting curves" could be repeated. In the physics lessons the students had dealt with *steadily accelerated (decelerated) motions* a few weeks before, so that this project was also a good repetition and addition.

If real data and descriptive statistics are not supposed to play such an important role there is also a version described in the last publication "planting mathematics" (→ [2]). In this form the topic was (or should be) dealt with in several other countries: Austria, Denmark, England, Hungary, Italy, Poland, Romania, Slovakia. We do hope that the experiment was (will be) successful!

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A1) Braking time and braking distance as a function of various velocities

Task 1: km/h and m/s are two typical units for velocity (speed)

a) Which advantages and/or disadvantages does each of these have?

b) Convert the velocities!

km/h	30	50	70	80	100	130					
m/s							5	10	20	30	45

c) Specify a general formula for each of the conversions.

Task 2: The following table shows the data of ONE braking process.

Time t [s] after beginning to brake	0.5	1.3	1.8	2.2	2.5	2.9
Velocity v [m/s]	15.3	12.3	9.4	7.2	4.0	1.6

a) Show the data in a graph.

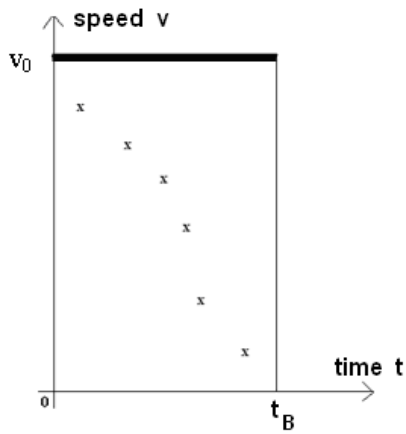
b) Calculate a value b which roughly indicates by how much the speed per second decreases on average in this braking process. How do you technically or colloquially call this value?

c) The following table shows the data of VARIOUS braking processes. Using the value b from part b) calculate each of the braking times t_B for each of the various initial velocities v_0.

v_0 [m/s]	10	15	18	22	28	35
t_B [s]						

d) Find a formula with which the braking time t_B can be calculated using the given initial velocities v_0 and the brake retardation b : $t_B(v_0, b)$

Task 3: If you neither brake nor accelerate, the velocity will remain constant v_0 (graph: thick line). If you brake, the velocity will decrease with time (graph: data points).



a) How can you calculate the distance covered by a car driver who is driving t_B seconds at a speed of v_0 m/s?

b) How could you compare the distance that a driver would cover during a braking process ("braking distance") with the distance he would cover in the same time without braking?

c) Which distance s_B did the driver with the data in 2a) cover while braking ("braking distance")?

d) How can you calculate this braking distance s_B in general using v_0 and t_B or b : $s_B(v_0, b)$?

Possible solutions for A1:

1a) Km/h is closer to everyday-life and you can imagine the concrete velocity much better; m/s is suited better as a unit of measurement for calculating since the braking processes happen on the time scale of seconds and on the distance scale of metres.

b)

km/h	30	50	70	80	100	130	18	36	72	108	162
m/s	≈8.3	≈13.9	≈19.4	≈22.2	≈27.8	≈36.1	5	10	20	30	45

c) v in m/s = $\frac{v \text{ in km/h}}{3.6}$ or v in km/h = $3.6 \cdot (v \text{ in m/s})$

2a/b)

Linear regression provides a coefficient of correlation of r ≈ – 0.99. Thus the data indicate a linear correlation between t and v. The trendline linear equation is $v \approx -5.81t + 19.14$. According to this, the velocity per second is reduced on average by the absolute value of the slope of the straight line, thus $b \approx 6$. This value (– 6) is called brake retardation or braking deceleration.

[redacted]

c)

v_0 [m/s]	10	15	18	22	28	35
t_b [s]	≈1.7	2.5	3	≈3.7	≈4.7	≈5.8

d) $t_B = \frac{v_0}{b}$; be attentive to the units: if t_B should come out in seconds, then you have to use m/s for v_0 and m/s² for b.

3a) If you drive at a constant speed of v_0 for a long period of time t_B, then the distance covered is calculated using the product of "velocity times time", or in short: $v_0 \cdot t_B$. The distance covered is, geometrically explained, the area of the rectangle that is restricted by the coordinate axes and the thick line.

b) If you now conceive a straight line $v(t) = v_0 - b \cdot t$ through the data points, then the distance covered while braking is barely half as large as the one with constant velocity. The distance covered is, geometrically explained, the area of the triangle that is restricted by the coordinate axes and the line that runs through the data points.

c) From the trendline linear equation $v \approx -5.81t+19.14$ you can extract that v_0 amounts to: v_0 =19.14m/s. t_B is calculated according to task 2 as $\frac{19.14}{5.81} \approx 3.3$. The braking distance is thus roughly $\frac{1}{2} \cdot 19.14 \frac{\text{m}}{\text{s}} \cdot 3.3\text{s} \approx 32\,\text{m}$ long.

d) The distance covered while braking s_B ("braking distance") is the surface area of the described triangle: $s_B = \frac{1}{2} \cdot v_0 \cdot t_B = \frac{1}{2} \cdot v_0 \cdot \frac{v_0}{b} = \frac{v_0^2}{2b}$

A2) Exercises:

Task 1: The following table shows the data of one braking process.

Time t [s]	0.3	1.1	1.9	2.4	2.7
Velocity v [m/s]	21.7	15.2	10.2	5.3	1.4

Which distance s_B did the car driver approximately cover while braking?

Task 2: The value b for the retardation of a car while braking is roughly $7\frac{m}{s^2}$.

v_0 [km/h]	30	40	50	60	80	100
s_B [m]						

a) Complete the table.
b) Show the data in a graph.
c) Compare the values. Deduce statements that new drivers should be aware of.

Task 3: Assume that a car travelling at a velocity of $v_0 = 70\,\text{km/h}$ is suddenly braked by the driver and he needs the given braking distances.

s_B [m]	10	20	30	40	50	60
b [m/s²]						

a) Complete the table.
b) Show the data in a graph.
c) Compare the values. According to the Road Traffic Licensing Regulation in Germany (called StVZO) § 41 (4): brakes and wheel chocks, automobiles must on average become slower at a rate of at least 5 m/s. What would you say to that as an MOT official?

Task 4: Assume you could improve the retardation value of the brakes using technical measures. Would this help the braking distance?

Improvement of the retardation value b by	10%	20%	30%	40%	50%	100%
Reduction of braking distances s_B also in %						

a) Complete the table.
b) Show the data in a graph.
c) Compare the values. Deduce statements that engineers should be aware of.

Task 5: Using the length of the braking distance skid mark which is made by a car at an accident, explain whether you can draw any conclusions about the velocity at which the accident happened. If yes, specify e.g. a formula with which the velocity can be reconstructed. If no, give reasons why!

Possible solutions for A2:

1) $r \approx 0.99$, $v \approx -8.1273t + 24.414$. The time until the car comes to a halt is $t_B = \frac{24.414 \text{ m/s}}{8.127 \text{ m/s}^2} \approx 3\text{s}$. Thus the braking distance is $\frac{1}{2} \cdot 24.4 \frac{\text{m}}{\text{s}} \cdot 3\text{s} \approx 37\text{m}$ long.

2) The wanted values can be calculated quickly using $s_B = \frac{(v_0 / 3.6)^2}{14}$.

v_0 [km/h]	30	40	50	60	80	100
v_0 [m/s]	≈8.3	≈11.1	≈13.9	≈16.7	≈22.2	≈27.8
s_B [m]	≈5.0	≈8.8	≈13.8	≈19.8	≈35.3	≈55.1

In the last line, the intermediate data (line 2) was used for further calculation with full calculator accuracy.

In the diagram, the point (0/0) is additionally entered since, of course, no braking distance can occur at 0 km/h. You can clearly recognise (also by simply using the data) that the braking distance does not proportionally depend on the velocity. If you consider the diagram in connection with the data, then it even becomes evident that the braking distance roughly quadruples when the speed is doubled. This can also be easily recognised with the formula $s_B = \frac{v_0^2}{2b}$, since $s_B = \frac{(2v_0)^2}{2b} = \frac{4 \cdot v_0^2}{2b} = 4 \cdot \frac{v_0^2}{2b}$. New drivers should e.g. take into consideration that an increase in velocity considerably increases the length of the braking distance overproportionally.

3) The wanted values for the respective braking retardations can be calculated quickly using $b = \frac{v_0^2}{2s}$.

s [m]	10	20	30	40	50	60
b [m/s²]	≈18.9	≈9.5	≈6.3	≈4.7	≈3.8	≈3.2

When depicting the data points you can clearly see that with the reduction of the braking distance, the value of the retardation has to become considerably better overproportionally (you see this at the *x*-axis from right to left).

As an MOT official you should notice that braking distances of roughly 40 metres and more at a velocity of 70km/h are not allowed by the Road Traffic Licensing Regulation in Germany!

4) The wanted values for respective improvements in the length of the braking distance in % can be calculated by comparing the braking distance length with improved brakes with the old brakes. If you compare both values by e.g. dividing them, then you can read the improvement in % from the quotient. An example for a 10% improvement:

$\frac{s_{\text{improved}}}{s_{\text{old}}} = \frac{\frac{v_0^2}{2 \cdot 1.1 \cdot b}}{\frac{v_0^2}{2 \cdot b}} = \frac{2 \cdot b}{2 \cdot 1.1 \cdot b} = \frac{1}{1.1} \approx 0{,}91$, that means an improvement of the braking distance by 9%; for 20% the outcome is $\frac{1}{1.2} \approx 0.83$, thus, an improvement of 17%, etc.:

Improvement in the retardation value b [m/s²] by	10%	20%	30%	40%	50%	100%
Improvement in the braking distance length s_B [m] by	≈9%	≈17%	≈23%	≈29%	≈33%	≈50%

[redacted]

From the data and its depiction, you can clearly recognise that a certain relative improvement in braking efficiency (in %) does not cause the same relative improvement in braking distance.
If you improve e.g. the retardation value by 20% from 6m/s² to 7.2m/s², then the braking distance decreases by “only” 17%. On the other hand, every improvement that can save human lives is, of course, worth it.

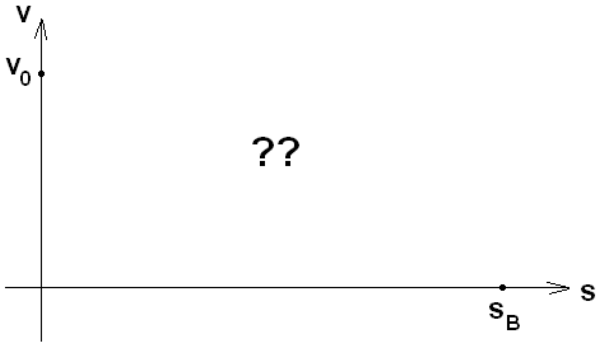
5) It can be very easily realised that in most cases, the length of a braking distance skid mark does not allow any conclusions to be drawn about the travelled velocity at the moment of an accident: at the moment we assume that two cars collide or a car hits a stationary obstacle, the car no longer has the chance to continue its skid mark. The skid mark is thus incomplete and does not allow any conclusions to be drawn about the velocity travelled.
But also in the case of bodily injury, that means a car hits e.g. a pedestrian and can continue its braking distance (almost) without interruption, thus the length of the skid mark is not only dependent upon the retardation of the brakes, but also on e.g. the road surface, moisture, leaves on the roadway, tyre tread, etc.
According to the statement from the police station in Olpe, Germany, in serious cases (e.g. accidents resulting in death), the deformation of the car body and the chassis is examined by experts in order to assess the velocity at the moment of the accident. This data is then compared to data achieved in a series of tests.

B1) The velocity during a braking process as a function of the distance covered

Assume you are driving a car with an approximately constant velocity v_0 and you suddenly have to slam on the brakes. Over the course of the distance s travelled while braking your velocity v will obviously continue to decrease until you ultimately come to a complete stop.

Task 1:

a) But HOW do you think the velocity $v(s)$ will decrease? At first, sketch the graph ("freehand") of the development of the velocity as you think it could go without calculating. Note in bullet point form why you think that the velocity develops the way in which you drew it.



b) The following table shows the data of such a braking process:

Distance s [m]	0	3.1	7.4	9.8	13.4	15.3	17.5	20
Velocity v [m/s]	15	14.3	12.8	11.5	10.1	8.1	5.2	0

Enter the data carefully into the diagram. Scale the axes for this data sensibly/appropriately. Redraw the diagram if needed, so that it is tidier.

c) Compare your graph with the data. Describe in abbreviated form what you notice here.

Task 2:

Consider a braking process with the velocity of v_0 → stop; what does $\frac{v_0^2}{2b}$ mean here? What does $\frac{v^2}{2b}$ mean during a braking process $v\ (< v_0)$ → stop? What does the difference $s = \frac{v_0^2}{2b} - \frac{v^2}{2b}$ mean?

If you solve this equation for v, you receive the function for the velocity v (initial velocity v_0 as a function of the travelled distance s: $v = v(s)$ – explain why!

Task 3:

a) How do you have to choose v_0 and b in the formula from task 2 in order for the graph of the function $v(s)$ to run exactly through the points (0/15) and (20/0)?

b) Draw the function graphs for $v(s)$ for these values.

Possible solutions for B1:

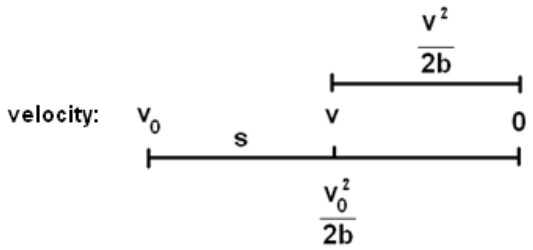
1)

Here only the data are presented. It is interesting that the velocity v does not sink "uniformly" (linear): at the beginning, the velocity actually slightly decreases and then (beginning at roughly 15 metres) it decreases astonishingly very rapidly!

[redacted]

2) The meaning of $\frac{v_0^2}{2b}$ is "braking distance at an initial velocity of v_0".

The meaning of $\frac{v^2}{2b}$ is "braking distance at an initial velocity of $v < v_0$".



The meaning of the difference $s = \frac{v_0^2}{2b} - \frac{v^2}{2b}$ is the necessary distance for the reduction of the velocity $v_0 \to v$. Solving for v results in $v = v(s) = \sqrt{v_0{}^2 - 2bs}$.

With this formula/function, you can calculate the residual velocity v of a braking car as a function of the distance s travelled while doing so (if you know v_0 and b).

3a/b) You learn from the data that v_0=15 must be applied. So we can conclude that $v(s) = \sqrt{225 - 2bs}$. Therewith it is ensured that the graph passes through the point (0/15). For (20/0) the ansatz follows $0 = \sqrt{225 - 2b \cdot 20}$. Solving this equation for b results in b = 5.625. Thus it follows $v(s) = \sqrt{225 - 11.25 \cdot s}$.

[redacted]

Tip: With Excel you can also click on other types of regressions instead of the trendlines for the data, in order to realise a data regression – please try it and find a reasonable answer!

B2) Exercises:

Task 1: The following table shows the data of a braking process.

Distance s [m]	0	10	14	19	22
Velocity v [m/s]	25	17.4	16.1	10.2	0

a) Describe (at least) 2 possibilities of how you can determine a function $v(s)$. Explain the advantages and disadvantages of your possibilities.

b) Determine a function $v(s)$ and draw its graph.

Task 2: Assume a car is suddenly braked by the driver at a velocity of $v_0 = 100\,\text{km/h}$. The value b for the retardation of the car while braking amounts to roughly $7\frac{\text{m}}{\text{s}^2}$.

a) Sketch the graph of the development of the velocity while braking as you think it could go. Enter at least 3 data points in your sketch which you think the graph passes through.

b) Complete the table.

Distance s [m]	0	10	20	30	40	50	
Velocity v [m/s]							0

c) Show the data point and the entire function $v(s)$ in a graph.

Task 3: Assume a car is suddenly braked by the driver at a velocity of $v_0 = 80\,\text{km/h}$. The value b for the retardation of the car while braking amounts to roughly $8\frac{\text{m}}{\text{s}^2}$.

a) What percentage of the initial velocity is reduced after 10%, 20%, ... of the braking distance? Complete the table.

Braking distance travelled s (%)	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Reduced velocity (%)	0%										100%

b) Show the data in a graph.

c) Reflect on your results: What should a new driver be aware of in this respect?

Task 4: The following table shows the data of a braking process.

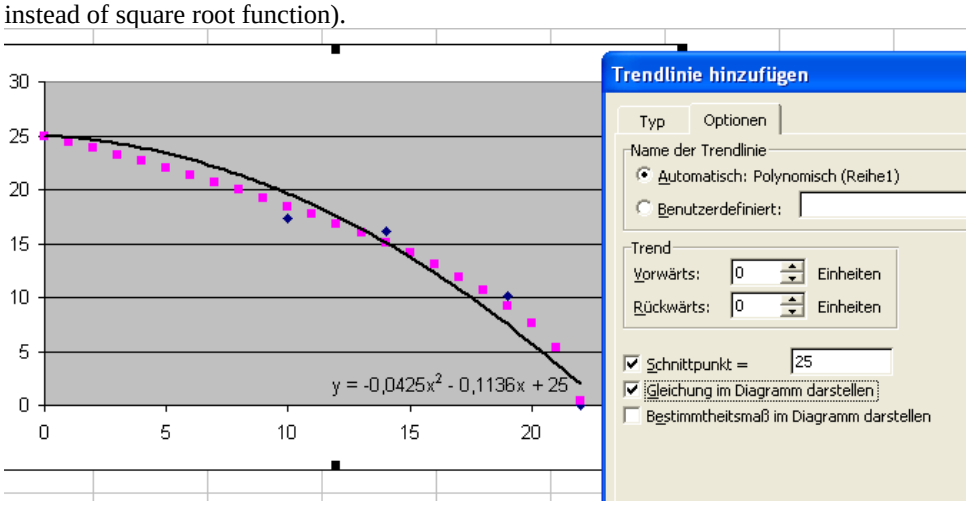
Time t [s]	0.4	1.9	2.3	3.2	3.9
Velocity v [m/s]	29.7	16.4	13.2	5.3	1.4

Determine and draw $v(s)$. How long is the braking distance s_B?

Possible solutions to B2:

1a) 1st possibility: As in task 2 you select 2 points of B1 and you carry out the algorithm analogously. Advantage: quick algorithm; Disadvantage: you only consider 2 data points.

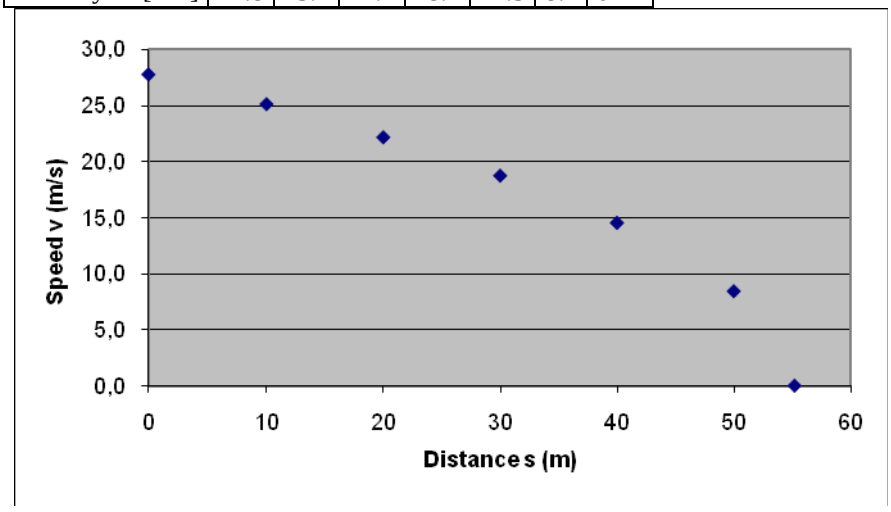
2nd possibility: With Excel you can depict the data as a diagram. If you click on diagram → type: e.g. "polynomial", then you get the equation as a regression function $v \approx -0.0425s^2 - 0.1136s + 25$. The coefficient of correlation is $r \approx 0.98$.– thus this is also not a bad choice. However, this depiction has the disadvantage that the curve definitely does not pass through (22/0). It is also the wrong type of function (parabola instead of square root function).



b) You learn from the data that $v_0 = 25$ must be applied. Hence: $v(s) = \sqrt{625 - 2bs}$. Therewith it is ensured that the graph passes through the point (0/25). For (22/0) we get $0 = \sqrt{625 - 2b \cdot 22}$. Solving this equation for b results in $b \approx 14.2$. Hence: $v(s) = \sqrt{225 - 28.4s}$.

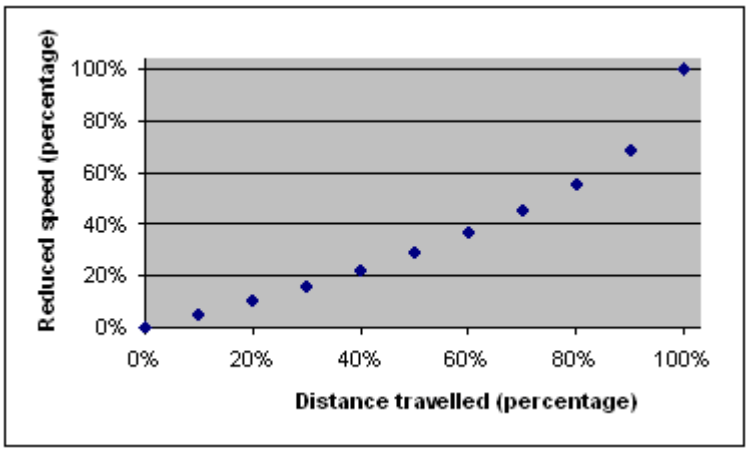
2) 100km/h ≈ 27.8m/s, from this follows the function $v(s) \approx \sqrt{771 - 14s}$

Distance s [m]	0	10	20	30	40	50	55.2
Velocity v [m/s]	27.8	25.1	22.2	18.7	14.5	8.4	0



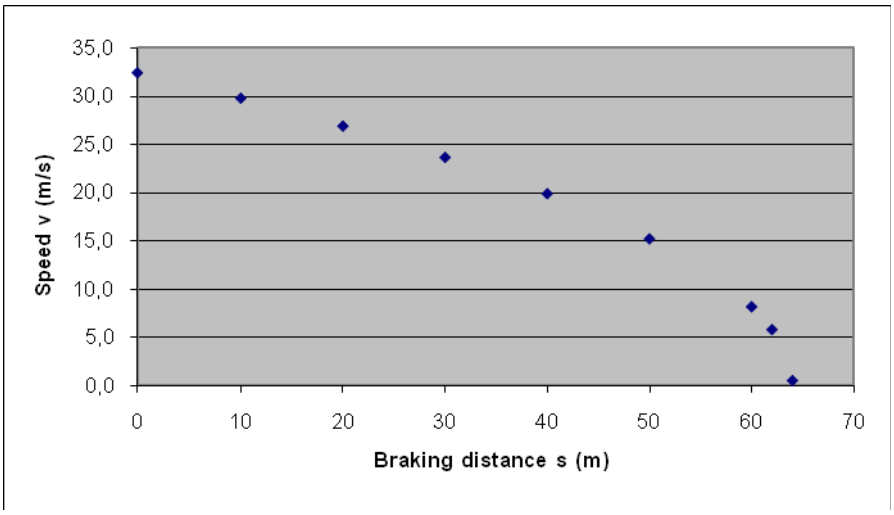
3) $v_0 = 80\,\text{km/h} \approx 22.2\,\text{m/s}$, from this it follows $s_b \approx 31\,\text{m}$

Braking distance travelled s (percentage)	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Braking distance travelled s (m)	0.0	3.1	6.2	9.3	12.3	15.4	18.5	21.6	24.7	27.8	31
$v(s)$ (m/s)	22.2	21.1	19.9	18.6	17.2	15.7	14.1	12.2	9.9	7.0	0
Reduced velocity (percentage)	0%	5%	11%	16%	23%	29%	37%	45%	55%	68%	100%



It is easy to recognise in the diagram that after more than 80% of the travelled braking distance only about 50% of the original velocity is reduced. Therefore the reduction of the remaining 50% of the velocity happens in the last 6 metres (from a total of 30!). That underlines once again the plea of the driving schools and road patrols for careful, cautious and anticipatory driving!!!!

4) The trendline linear equation is $v \approx -8.2193t + 32.433$ (with a coefficient of correlation of $r \approx 0.998$!). As a result, there is a braking retardation of $b \approx 8.2\,\text{m/s}^2$ and a beginning velocity of $v_0 \approx 32.4\,\text{m/s}$. From this follows the formula $v(s) \approx \sqrt{1049.8 - 16.4s}$.



The braking distance length is calculated using $s_B \approx \dfrac{32.4^2}{2 \cdot 8.2} \approx 64\,\text{m}$.

C) Two typical situations in road traffic

Situation 1: A car is driving in a 30km/h zone and is overtaken by another car. As both cars are roughly on level with each other, children run onto the street without looking. The slower car barely comes to a halt in front of the children.

Situation 2: A car is overtaken on a country road by another car. As both cars are roughly on level with each other, another car appears in the oncoming traffic.

Task:
Choose a situation and analyse it based on your knowledge. That means:

- Find schoolmates who have also chosen the same situation. Tip: Not too many!!! Otherwise, speaking from experience, the teamwork does not work as well.
- Think of as many questions as possible that seem relevant to your situation (brainstorming – collecting ideas!).
- Decide on at least one of your questions and try to analyse it. The expectations of the analysis are:

- ➢ Analyse the question based on a concrete situation (e.g. two assumed velocities for both cars).
- ➢ Analyse the question using different velocities for ONE car.
- ➢ Generalise the different velocities into a function and examine its properties in order to make conclusions for your issues/questions.
- ➢ Prepare a ca. 10-20 minute presentation (advantageous: Power-Point!)

Tip:
In all of the worksheets, the reaction time of the driver or the so-called moment of shock was not accounted for! If you consider the reaction time relevant, then you should also consider this in your analysis.

Eine Schülergruppe präsentierte gestern in Köln den Inhalt des Verkehrssicherheitsprogramms, mit dem der ADAC in rund 4000 Veranstaltungen über 100 000 Fünftklässler auf die Gefahren im Verkehr vorbereiten will. In anderthalbstündiger Unterrichtung wird der Zusammenhang zwischen Reaktionszeit, Brems- und Anhalteweg vor Augen geführt. So dürfen die Schüler laut ADAC auch in einem Auto mitfahren und bei 50 km/h eine Vollbremsung erleben. Foto: dpa

In the picture: **Reaction distance + braking distance = overall stopping distance**

A group of students presented the contents of the road safety programme in Cologne yesterday. With roundabout 4,000 road safety programme events, the ADAC (General German Automobile Association) wants to prepare over 100,000 fifth-graders for the dangers in road traffic. In the one and a half hour lessons, the pupils are shown quite plainly the connection between reaction time, braking distance and overall stopping distance. According to the ADAC the pupils are, for example, allowed to ride in a car and experience full brake application at 50 km/h.