Remark to Humenberger, H. (2018)

Balanced areas in quadrilaterals—on the way to Anne's Theorem

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In this article two possibilities were mentioned to prove that a point not lying on the axis of symmetry of a kite (which is not a rhombus) cannot have the 'balanced area property'. Here we present another even shorter one (idea by R. Bischof, student teacher at the University of Vienna).

Proof

If *I* does not lie on the axis of symmetry *e* one can draw parallels to the sides through *I*. At least two of these parallels intersect *e* in the interior of the kite (*E*, parallel line to *AB*; the parallels to the shorter sides intersect *e* always in the interior, the parallels to the longer sides may intersect *e* outside).

The point *E* has the balanced area property because it lies on the axis of symmetry. Now looking at the areas we have: $|\Delta BEA| = |\Delta BIA|$ (same base and same altitude), and $|\Delta CDE| \neq |\Delta CDI|$ because *IE* is not parallel to *CD* (otherwise the kite would be a rhombus). Therefore, the point *I* cannot have the balanced area property.