Multinational Banks and Supranational Supervision

Giacomo Calzolari† Jean-Edouard Colliard‡ Győngyi Lóránth§

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Abstract

We study the supervision of multinational banks (MNBs), allowing for either national or supranational supervision. National supervision leads to insufficient monitoring of MNBs due to a coordination problem between supervisors. Supranational supervision may solve this problem and generate more monitoring. However, this increased monitoring can have unintended consequences, as it also affects the choice of foreign representation. Indeed, supranational supervision encourages MNBs to expand abroad using branches rather than subsidiaries. In some cases, it discourages foreign expansion altogether, so that financial integration paradoxically decreases.

More importantly, these changes completely neutralize the more intense monitoring that would otherwise occur with supranational supervision. Our paper provides insight into how the national boundaries of bank supervision interact with multinational banks.

Keywords: Cross-border banks, Multinational banks, Supervision, Monitoring, Regulation, Banking Union.


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†Department of Economics, University of Bologna, and CEPR, Piazza Scaravilli 2, 40126, Bologna, Italy. Phone: +39 0512098489. E-mail: giacomo.calzolari@unibo.it.

‡Department of Finance, HEC Paris, 1 rue de la Libération, 78351 Jouy-en-Josas, France. Phone: +33 1 39 67 72 90. E-mail: colliard@hec.fr.

§Faculty of Business, Economics and Statistics and CEPR, University of Vienna, Oskar-Morgenstern Platz 1, 1090, Vienna, Austria. Tel: +43 1 42 77 38 051. E-mail: gyoengyi.loranth@univie.ac.at.
Introduction

The number and importance of multinational banks (MNBs) have increased significantly over the past two decades.\(^1\) MNBs operate in complex, often uncoordinated and dissimilar supervisory regimes, involving several national supervisors which tend to act in the interest of their own countries. In such an environment, cross-border banks might be able to escape tight monitoring and supervision. A prominent example of this is the case of Dexia which, despite being supervised by the authorities of Belgium, France, Luxembourg and the Netherlands, suffered a catastrophic failure which led to a bail-out for 6 bln EUR in 2011. Failures such as this raise legitimate questions about what should be the optimal allocation of supervisory responsibilities between national supervisors. With these difficulties in mind, supervision of multinational banks has become a central part of the policy debate, and there has been a trend towards centralized supervision, albeit with different approaches, in the US, Europe and other countries.\(^2\)

Is integrated supervision, for instance in the form of a supranational supervisor (as in the Euro area), a solution to this global problem? Despite the intense policy debate on this clearly important issue, there is no theory to guide policy makers on how to organize supervision of multinational banks.\(^3\) By addressing the consequences of switching from national to supranational supervision, our paper offers the first stepping stone towards understanding the two-way feedback between the legal boundaries of MNBs and the distribution of supervision.

The aim of this paper is to study how delegation of supervision to a supranational entity can affect the way MNBs operate, the funding conditions they face, and possibly also their very decision to operate cross-border. We show that supranational supervision may indeed solve coordination problems and possibly generate more monitoring in banks. However, this increased monitoring leads to an adjustment in the choice of foreign representation by multinational banks. Supranational supervision encourages MNBs to convert their foreign subsidiaries into branches, or even to revert to a purely domestic activity altogether. Interestingly, these adjustments completely neutralize the more intense monitoring that would otherwise occur with supranational supervision. With no more monitoring actually taking place, we show that either total welfare remains unaffected, or it is reduced. At the same time, the change in the MNB’s representation form affects how potential

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\(^1\)See Claessens and Van Horen (2013).
\(^2\)We will illustrate these changes in detail in Section 1.2.
\(^3\)As for the policy debate see, for example, the Financial Stability Board 2011 document on “Global adherence to regulatory and supervisory standards on international cooperation and information exchange,” the 10 December 2012 joint paper by the Federal Deposit Insurance Corporation and the Bank of England, concerning global financial institutions, and Schoenmaker and Huttl (2015).
losses are allocated to the national and the foreign deposit insurance funds. In particular, we show that it necessarily leads to higher pressure on the deposit insurer of the country of origin.\footnote{To the extent that subsidiaries are more integrated in the local economy than branches, i.e., lend to small and medium size firms instead of large multinational companies, the change in organizational form might also have repercussions on the real economy. Cerutti, Dell’Ariccia, and Martinez Peria (2007), for example, find that foreign banks are more likely to enter via subsidiaries when they plan to penetrate host markets.}

To properly assess the impact of supranational supervision, we explicitly account for the different liability structures that a cross-border banking group may choose from. In particular, banks can operate abroad via subsidiaries or branches. Subsidiaries are foreign incorporated stand-alone entities, which are protected by limited liability. Should the subsidiary fail, depositors have no claim on the assets of the parent company. However, they do have priority over any claimholders of the parent company. Under national supervision, deposits in each country are insured by the local deposit insurance fund, and supervision is similarly split between a home and a host supervisor. Branches share liabilities and profits with the parent bank. Deposits are insured by the home country deposit insurance fund, and supervision in both units is undertaken by the home country supervisor. The Icelandic crisis showed how real the differences between branches and subsidiaries could be in crisis periods.\footnote{A famous example of these differences is the two Icelandic banks, Landsbanki and Kaupthing, operating in the UK as a branch (named Icesave) and as a subsidiary (Kaupthing UK), respectively. When Landsbanki (and with it Icesave) failed, UK depositors lost their savings because the Icelandic Deposit Guarantee Scheme could not cope with the large amount of deposit guarantees. Kaupthing depositors, however, were insured by the UK’s Deposit Insurance Scheme, and were fully compensated from it.}

These differences are summarized in Table 1.

<table>
<thead>
<tr>
<th>Subsidiary</th>
<th>Branch</th>
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<tbody>
<tr>
<td>Deposit insurer (DI) of the home unit</td>
<td>Home DI</td>
</tr>
<tr>
<td>Deposit insurer (DI) of the foreign unit</td>
<td>Foreign</td>
</tr>
<tr>
<td>Supervisor of the home unit</td>
<td>Home supervisor</td>
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<tr>
<td>Supervisor of the foreign unit</td>
<td>Foreign supervisor</td>
</tr>
<tr>
<td>Home unit responsible for foreign unit’s liabilities</td>
<td>No</td>
</tr>
<tr>
<td>Foreign unit responsible for home unit’s liabilities</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1: Branches and subsidiaries.
monitoring is expensive and the assets have a high probability of being of low quality, it is optimal for a supervisor not to monitor the bank and liquidate its assets. This decision has different consequences depending on the organizational structure of the MNB. In particular, when the supervisor of a foreign subsidiary monitors and this unit returns a positive payoff, part of it can be used to repay depositors in the home country if the home unit is unsuccessful. On the contrary, when the foreign triggers intervention, its assets cannot be used to offset losses in the home country. As a result, the foreign supervisor exerts a positive externality when he chooses to monitor. This externality is not taken into consideration by the foreign supervisor, and for this reason, the equilibrium level of monitoring can be too low.

Due to this externality, the introduction of supranational supervision unequivocally leads to more monitoring for banks with a subsidiary structure. The supranational supervisor internalizes the fact that monitoring the foreign unit is valuable for the home unit, which is a desirable outcome of supranational supervision. At the same time, centralizing supervision does not lead to different decisions for banks with a branch structure. Indeed, the home regulator of a branch represented MNB internalizes all the benefits of monitoring because it makes decisions for both units and pays for depositors in both countries.

However, there can be a second, unintended effect. Under some conditions (that we identify) the MNB chooses the subsidiary structure under national supervision precisely because it leads to low monitoring of the foreign unit. When supranational supervision is introduced and leads to more monitoring, the MNB will reconsider its organizational structure and may reorganize as a branch-MNB, or as a stand-alone bank present in one country only. When the MNB chooses to become a stand-alone bank, supranational supervision has the paradoxical impact of decreasing financial integration. More generally, the message of our paper is that, in the long-run, MNBs will strategically react to the new structure of regulation in order to skirt tightened supervision. In our model, they do it with a view to extracting more benefits from their liability structure and the deposit insurance fund.

The changes in organizational forms affect both the total expected losses and their allocation to the national deposit insurance funds. When supranational supervision induces a switch to branches, total (expected) losses to the deposit insurance funds are reduced, but will be borne entirely by the home deposit insurance fund. This reallocation of losses is particularly damaging as we also show

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6Changing the organizational structure for an MNB is not a rare event. Nordea is in the process of changing its legal structure by converting subsidiary banks in Denmark, Finland, and Norway into branches of the parent company Nordea Bank AB. Alpha Bank in March 2015 changed its branches into subsidiaries in Romania and Bulgaria.
that a branch-MNB is only profitable if the home deposit insurance fund is less well funded than the foreign one. When the MNB reverts to a stand-alone domestic bank, only the home deposit insurance fund can suffer losses, and on average these losses are larger than with a subsidiary structure. Hence, in both cases the home deposit insurance fund ends up with more liabilities. As many countries’ deposit guarantee systems are already overstretched, centralization might have the long-run effect of further reducing the credibility of some countries’ deposit insurance, leading to higher deposit rates and lower profits for multinational banks. Our model enables us to derive implications about which countries would be more likely to be affected by the introduction of a supranational regulator. In particular, we show that countries that host more headquarters will end up with a larger burden on their deposit insurance fund.

We also complete this analysis by considering the effects of moving towards common deposit insurance, as is currently debated for the European Banking Union. In particular, we show that introducing common deposit insurance does not eliminate the paradoxical result that supranational supervision may induce the MNB to close its foreign unit.

Finally, the model can also be used to deliver empirical implications about the funding conditions of MNBs, depending on their organizational structure. While funding costs for foreign subsidiaries are only affected by the credibility of the foreign deposit insurance, the home unit funding costs will be influenced by both the credibility of the home and that of the foreign deposit insurance. Branches’ funding costs, instead, are determined by the credibility of the home deposit insurance fund. Higher credibility results in lower funding rates and higher profits for banks. An MNB should thus react differently to a switch to supranational supervision, depending on the credibility of the deposit insurance fund in the home and the host countries. We show the extent to which the implications will be different for the case of an MNB incorporated in a crisis country with subsidiaries in surplus countries with credible deposit insurance, and for the symmetric case of an MNB incorporated in a surplus country that expands in crisis countries.

Our paper builds on two strands of the literature. First, several papers study frictions and conflicts of objectives between national regulators. Externalities lead independent national regulators to choose suboptimal regulatory standards, in the form of too low capital requirements (Dalen and Olsen (2003), Dell’Ariccia and Marquez (2006)), too lax intervention thresholds (Acharya (2003)), or too coarse information sharing (Holthausen and Rønde (2004)). Beck, Todorov, and Wagner (2013), Agarwal et al. (2014), and Rezende (2011) provide empirical support for the divergence of
objectives hypothesis by looking at the decisions of different supervisors while controlling for bank fundamentals.

Second, there is a literature looking at the endogenous choice of representation form of financial intermediaries based on the differences in liability structure between branches and subsidiaries (Kahn and Winton (2004), Dell’Ariccia and Marquez (2010), Luciano and Wihlborg (2013)). None of these papers consider supervision as a factor that could drive the choice between branches and subsidiaries. Harr and Rønde (2004) and Loranth and Morrison (2007) study optimal capital regulation and Calzolari and Loranth (2003) analyze optimal closure policies for branches and subsidiaries and their impact on the choice of representation form by the bank. Focarelli and Pozzolo (2005) and Cerutti, Dell’Ariccia, and Martinez Peria (2007) empirically investigate the determinants of the MNBs’ organizational choice.

We combine these two strands of the literature in a model in which regulatory treatment and frictions in supervision are key drivers of the choice of representation form by the MNB. In particular, the optimal supervisory actions depend on the bank’s representation form and, in turn, the bank’s representation form optimally responds to the anticipated supervisory actions. Taking these feedback effects into account, we show that the choice of the organizational form actually neutralizes the centralization of supervision.

As our main example of supranational supervision is the Single Supervisory Mechanism, we also contribute to a growing literature on the possible effects of this new architecture. Colliard (2014) compares supranational to national supervision, focusing on the trade-off between worse quality information and less biased incentives of supranational supervisors. Carletti, Dell’Ariccia, and Marquez (2016) argue that local supervisors will have lower incentives to collect information if decisions are taken by a central regulator. Beck and Wagner (2016) also study common supervision, but examine the problem of different regional preferences regarding financial stability.

1 Model

We first set up the model and then discuss its main assumptions.

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7 Calzolari and Loranth (2011) provide an extensive overview of the problems in multinational bank supervision.
8 A few recent papers deal with bail-in of global systematically important banks: Bolton and Oehmke (2016) and Faia and Weder di Mauro (2016). We differ from these papers as our focus is on ex ante incentives, in the form of monitoring and intervention, as opposed to ex post incentives that arise upon bank failure.
1.1 Setup

We consider a multinational bank (MNB) operating units in two countries: the home country \( h \) (where the MNB is incorporated) and the foreign country \( f \). Each unit \( i \in \{h, f\} \) invests locally in a portfolio of illiquid and risky projects that pay out \( R > 1 \) with probability \( p_i \), or return 0 with probability \( 1 - p_i \). Returns on the portfolios in the two countries are uncorrelated.\(^9\) Premature liquidation of a portfolio yields a sure payoff \( L \in [0, 1) \).

Investments are financed by one unit of insured deposits in each country. The deposit insurance (DI) fund in country \( i \) fully reimburses depositors with probability \( \alpha_i \leq 1 \), and only partially with complementary probability, as described below. Since actual reimbursement may be partial, depositors ask for a premium: instead of investing in a safe outside option returning 1, depositors are willing to lend to the bank at an endogenous interest rate \( P_i \geq 1 \).\(^{10}\)

**Liability structure.** We examine the two types of representation for the foreign unit, subsidiary and branch, that allow the bank to perform the (complete) set of activities described above.\(^{11}\)

A subsidiary shares liability for the home unit’s losses, but the reverse is not true. More precisely, after foreign depositors are paid out, the remaining assets in a solvent subsidiary are used against the home unit’s outstanding liabilities. No such transfer is legally required from a solvent home unit to an insolvent subsidiary. With a subsidiary-MNB, each national supervisor supervises its local unit and deposits are insured by the local deposit insurance fund.

A branch can be thought of as an extension of the home unit, thus forming a single entity. Insolvency occurs when the total assets of the MNB in both units fall short of total liabilities. The supervisor in the home country is in charge of supervision and insures depositors in both countries. In insolvency, the MNB’s assets are distributed to depositors pro-rata in both countries.

**Supervision.** Supervisors perform two tasks: monitoring and prudential intervention. They are assumed to be risk neutral and minimize all (expected) costs that may arise as a consequence of monitoring, intervention, or failure of the units.

National supervisors non-cooperatively elect whether to monitor and intervene in their local unit. Monitoring the local unit in country \( i \) costs \( c_i \) and results in a perfect signal on the future success or failure of the unit. In the absence of monitoring, the supervisor only knows that assets

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\(^9\)One of the drivers of MNBs’ expansion is risk diversification, which justifies uncorrelated projects. The analysis of systemic risk is beyond the scope of this paper.

\(^{10}\)With an equivalent interpretation of our model, the bank obtains funds from lenders in the wholesale funding market who expect to be bailed out with probability lower than one.

\(^{11}\)In the following, we will indicate the foreign unit simply as “the subsidiary” or “the branch” depending on the representation form.
in country $i$ pay out with probability $p_i$. To obtain sensible comparisons, we consider the case $p_h = p_f = p$.

Based on the available information, supervisor $i$ then takes a prudential decision on whether or not to intervene in the local unit. We think of intervention as conservatorship or ring-fencing activity that results in early liquidation of the portfolio of illiquid loans with the payoff $L$. Alternatively, the supervisor can decide to take no action, i.e., let the unit continue until the asset matures. Each unit can thus be in one of three states: success $s$, liquidation $l$, or failure $f$.

So as to rule out trivial cases, we make two parametric assumptions: An unmonitored unit creates economic surplus (H1) and a successful foreign unit cannot repay all depositors if the home unit is liquidated or fails (H2).

$$pR > 1 \quad (H1)$$

$$R + L < 2 \quad (H2)$$

When a central supervisor is active, it faces the same information structure and costs, $c_h$ and $c_f$, as national supervisors do. Its objective is an equally weighted sum of the expected payoffs that the national supervisors would adopt in the two countries.

**Information.** Information obtained by monitoring is truthfully shared between supervisors before any prudential decision is taken.

**Timeline.** The following timeline summarizes the environment.

- At $t = -1$: the supervisory architecture is announced. The bank faces either supranational or national supervision.
- At $t = 0$: the MNB first chooses whether to expand abroad with a subsidiary or a branch or, alternatively, to remain a stand-alone bank in the home market.
- At $t = 1$: The bank offers payments of $P_h$ and $P_f$ to depositors in the two countries, and depositors choose whether to deposit or not.
- At $t = 2$: The supervisor in charge decides whether to monitor the unit(s) under his jurisdiction or not.
- At $t = 3$: The supervisors learn the state of units that were monitored in $t = 2$. On the basis of available information, the supervisor(s) decides whether to intervene in the unit or not.
- At $t = 4$: Payoffs realize. Liquidated assets are worth $L$, successful assets return $R$, and failed assets return 0. Depositors of a successful unit $i$ are repaid $P_i$. For an unsuccessful unit, the deposit
insurance fund in country \(i\) fully repays depositors with probability \(\alpha_i\), and partially repays them with probability \((1 - \alpha_i)\).

**Notations.** We denote with \(\sigma = S\), \(\sigma = B\) and \(\sigma = A\), the MNB’s decision of expanding abroad respectively with a subsidiary representation, with a branch, or to remain a stand-alone bank.

As will become clear later, monitoring and intervention decisions taking place on unit \(i\) at \(t = 2\) and \(t = 3\) can be summarized in a single decision \(d_i \in \{M, I, O, C\}, i = f, h\): Decision \(d_i = M\) consists of monitoring unit \(i\), keeping it open when the assets are good, intervening in it when they are bad, irrespective of the signal received about the other unit. Decisions \(d_i = O\) and \(d_i = I\) consist in not monitoring unit \(i\) and keeping it open or intervening in it, respectively, regardless of the signal received about the other unit. With a slight abuse of notation, \(d_i = C\) indicates that unit \(i\) is not monitored and is kept open when the assets of the monitored other unit are bad, and intervened when the other unit’s assets are good. The supervisory decisions for the home and foreign units will be denoted as \((d_h, d_f)\).

We denote with \(W_h(d_h, d_f)\), \(W_f(d_f)\), and \(W_b(d_h, d_f)\) the supervisors’ expected payoffs with the subsidiary represented MNB (the first two) and in the branch one (the third). \(W_h(d_h)\) is that of the home supervisor when the bank remains domestic. Similarly, \(\Pi(\sigma, d_h, d_f)\) denotes the expected profit of an MNB with the representation form \(\sigma \in \{S, B\}\) and \(\Pi(A, d_h)\) the profit of a stand-alone bank only present in country \(h\). The rates paid to depositors in countries \(h\) and \(f\) are denoted by \(P_h(S, d_h, d_f)\) and \(P_f(S, d_f)\) for the subsidiary case, \(P(B, d_h, d_f)\) for the branch case, and \(P_h(A, d_h)\) for the stand-alone case.

Figure 1 summarizes the tree of the game for periods 0 to 2 when supervision is national. Tables 3 and 4 in the Appendix summarize the payoffs for all agents and decision pairs \((d_h, d_f)\).

1.2 Discussion

**Bank supervision.** Prudential supervision comprises a range of activities intended to identify and address any practices or conditions that could jeopardize a bank’s immediate or long-term viability. This includes monitoring of unsafe or unsound practices and subsequent intervention when needed. In our simplified framework, an intervention is equivalent to liquidating a bank’s assets, but it should be interpreted more broadly as any supervisory action that leads to a less risky payoff.\(^{12}\)

Bank supervisors in our model are assumed to minimize the expected losses of the deposit

\(^{12}\)We explicitly abstract from minimum capital requirements and convertible liabilities in our model. Although these regulatory tools may affect some of the decisions of a supervisor, they are unlikely to impact on the incentives to choose one organizational form rather than another in our model.
insurance fund. A prominent example of a supervisor with such an objective function is the Federal Deposit Insurance Corporate (FDIC) in the US. Indeed, Demirguc-Kunt, Kane, and Laeven (2014) find that 57 percent of DI funds in the world have extended powers or responsibilities including a responsibility to minimize losses or risk to the fund.

More generally, Dewatripont and Tirole (1994) have argued that since dispersed depositors may be unable to monitor a bank, the supervisor should step in acting in their interest. Minimizing the expected loss of a deposit insurance fund with the control rights of debt holder guarantees that the supervisor indeed acts as a perfect representative of depositors (“representation hypothesis”).

If the supervisor adequately “represents” the bank’s depositors, its objective function is also compatible with a welfare-maximizing objective. In particular, the supervisor of a stand-alone bank in the model does maximize welfare when the deposit insurance fund pays out with probability 1. Indeed, the supervisor of a stand-alone bank suffers expected losses equal to \((1 - p)\) if he does not monitor, compared to \((1 - p)(1 - L)\) if he does. Thus, monitoring takes place if and only if \((1 - p)L > c\), which is also the welfare-maximizing decision.

The model introduces two frictions relative to this benchmark. First, under stress conditions, the deposit insurance fund might be under-funded, and the probability that the fund can repay depositors is lower than 1. This reduces the incentives to monitor the banks, as the supervisor cares less about future losses.\(^\text{13}\) Second, multinational banks can face several supervisors responsible for

\(^{13}\)This mechanism partly explains the behavior of the FSLIC during the Savings & Loan crisis, e.g., Kane (1989).
different units, which can act in a non-coordinated way. Supranational supervision solves the second friction. In Section 5.1 we will explore to what extent a possibly stronger common deposit insurance may address the first issue.

Deposit Insurance Fund. A significant feature of our model is that the DI fund in country $i$ can only pay out with probability $\alpha_i$. This can be seen as a measure of the robustness and credibility of the deposit insurance fund in country $i$. Indeed, in many countries DIs appear underfunded. In countries with high levels of government debt and with a large amount of deposits relative to GDP, the ability of the government to honor its commitment to depositors raises doubt (Demirguc-Kunt, Kane, and Laeven (2014)). It is also possible to interpret the formal deposit insurance of our model as representing informal government guarantees more generally, in which case $\alpha_i$ can be interpreted as the (exogenous) probability of a bail-out in country $i$.

Allocation of Supervisory Responsibilities. An important element of our analysis is that we consider an array of organizational forms available to the bank. The organizational form defines a liability structure and an allocation of supervisory responsibilities. Our modeling assumptions reflect real-life arrangements. Indeed, both in the EU and the US for the supervision of branches the competent authority is the one where the bank is initially licensed. However, despite the higher legal and administrative burdens, many banks still choose to establish subsidiaries with separate capital and foreign supervision (Cerutti, Dell’Ariccia, and Martinez Peria (2007)). According to a recent 2014 prudential regulation, in the United Kingdom branches of non-European foreign banks are allowed to operate upon a specific assessment of the supervision activity and approach of the home supervisor. Branches of foreign European banks are allowed to operate in the UK without authorization (before Brexit).

As a response to bank failures during the financial crisis and the lack of appropriate coordination among national regulators in handling these failures, several countries have moved towards more centralized supervision. The EU has given the European Central Bank primary responsibility for supervising the largest EU banks. Since November 2014, the banking system in the Euro area is split into two groups. The 129 most significant credit institutions are supervised directly by the European Central Bank (ECB). The less important ones are still in the hands of national authorities.

Centralization of supervision is also ongoing in the US, but is moving towards giving more power to the host country supervisors. The most notable change is that foreign banks with US presence exceeding $50$ billion are required to put their subsidiaries under an intermediate holding company

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which will be regulated and supervised as any other US bank holding company. This is in sharp
contrast with the previous light-touch regulation which mainly relied on home country supervision
of foreign subsidiaries (as well as branches).\footnote{As a recent development, the EU is now considering forcing US banks such as Goldman Sachs and JPMorgan to have additional capital and liquidity in the EU so their subsidiaries can better withstand a crisis and be separately wound up if needed by European authorities.}

With regard to the issue of centralization versus decentralized decision-making, the US dual
banking system also provides a good laboratory. While federally-chartered banks are supervised by
the FED, State-chartered commercial banks are supervised by a State supervisor (corresponding to
the “national” level) and a federal supervisor, either the Fed or the FDIC (corresponding to the
“supranational” level), in an alternating fashion. Agarwal \textit{et al.} (2014) show that State supervi-
sors in the US are systematically more lenient than Federal ones, demonstrating the differences in
objectives between state-level and federal supervisors.

\textit{Monitoring costs and information.} The cost of monitoring should be thought of as mostly related
to a bank’s complexity and opacity. Since these characteristics are specific to the single unit, we
denote the cost of monitoring unit $i$ with $c_i$. Heterogeneity can also arise from a different reliance of
economies on banks, from differences in market structures or from different legal and institutional
frameworks. We abstract instead from potential differences of expertise or cost-efficiency between
national and supranational supervisors who face the same cost, so as to focus the analysis on the
different incentives of these two levels.

Assuming that information generated by monitoring is truthfully shared is clearly a simplification
of the complex monitoring task faced by supervisors who may also be motivated by different and
conflicting interests.\footnote{See, for example, Repullo (2001) and Holthausen and Rønde (2004) on information sharing.} However, credibility is essential for bank supervisors, which drastically limits
their willingness to misrepresent ex-post verifiable information.\footnote{Even if a supervisor could conceal the information obtained with monitoring, information could still “unravel” and be perfectly inferred by the other supervisor, as shown in persuasion games (Grossman (1981) and Milgrom (1981)).}

\subsection{1.3 Benchmark: Full information}

To set up the scene and exemplify payoffs, here we briefly illustrate the special case in which
$c_h = c_f = 0$. Strategy $M$ being optimal for both units, prudential decisions are taken under full
information and are independent of the type of foreign representation.
In the case of the stand-alone bank, the deposit rate $P_h$ is implicitly defined by

$$pP_h(A, M) + (1 - p)[\alpha_h + (1 - \alpha_h)L] = 1,$$

(1)

where the square bracket is the expected repayment to depositors in case of failure, relying either on the deposit insurance (the term $\alpha_h$) or on actual liquidated assets (the term $(1 - \alpha_h)L$) if the deposit insurance is underfunded. For the subsidiary, a similar equation pins down $P_f$ as the home unit does not share liability for the subsidiary’s losses, with the difference that the rate is now determined by the deposit insurance fund’s credibility in the host country, $\alpha_f$. As for the home unit, $P_h$ satisfies:

$$pP_h(S, M, M) + (1 - p)p[\alpha_h + (1 - \alpha_h)(L + R - P_f)] + (1 - p)^2[\alpha_h + (1 - \alpha_h)L] = 1.$$  

(2)

This equation takes into account that with probability $p(1 - p)$ the home unit fails but the foreign unit is successful (third term). In such a case there are residual assets, worth $R - P_f$, in the foreign unit that are left after local depositors have been reimbursed $P_f$. This implies that $P_h(A, M) \geq P_h(S, M, M)$.

Under branch representation, each unit is liable for the losses of the other unit and the deposit rate $P$ is:

$$p^2P(B, M, M) + 2p(1 - p)[\alpha_h + (1 - \alpha_h)(R + L)/2] + (1 - p)^2[\alpha_h + (1 - \alpha_h)L] = 1.$$  

(3)

Both units need to succeed in order to pay depositors the promised rate and the home country deposit insurance covers depositors in the foreign unit, which implies that the deposit rate can in general be higher or lower in a subsidiary than in a branch, depending on $\alpha_h$ and $\alpha_f$.

At $t = 0$ the bank will choose the representation form, anticipating associated deposit rates and supervisory decisions. The stand-alone profit can be written as

$$\Pi(A, M) = p(R - P_h(A, M)),$$

(4)

while the subsidiary represented MNB yields a profit of

$$\Pi(S, M, M) = p^2[2R - P_h(S, M, M) - P_f(S, M)] + p(1 - p)[R - P_h(S, M, M)]$$

$$= p(R - P_h(S, M, M)) + p^2(R - P_f(S, M)).$$

(5)
This last equation shows that, with full information, remaining a domestic bank is sub-optimal. First, a subsidiary is a source of additional profits, while limited liability shields the home unit’s profit from the subsidiary’s losses. Second, the home deposit rate is lower ($P_h(S, M, M) \leq P_h(A, M)$) because of the additional repayments possibly available from the subsidiary. The profit in the branch case can be written instead as

$$\Pi(B, M, M) = 2p^2(R - P(B, M, M)).$$

Comparing $\Pi(B, M, M)$ and $\Pi(S, M, M)$, the next proposition summarizes the optimal organizational choice of the bank under full information.

**Proposition 1.** With full information (i.e. $c_h = c_f = 0$), the bank expands abroad. Operating with a subsidiary is more profitable than with a branch if $\alpha_f \geq \hat{\alpha}_f(\alpha_h)$ where $\hat{\alpha}_f(\alpha_h) \in [0, \alpha_h]$. Otherwise, the opposite is true.

Branch representation can only dominate subsidiary representation if the home deposit insurance fund’s credibility is sufficiently higher than the foreign one. In this case, the deposit rate is lower with a branch than with a subsidiary, which compensates for the lower probability with which the bank obtains a positive profit under the branch representation.

### 2 National supervisors and the multinational bank

Differently from Section 1.3, when there are informational frictions, i.e., $c_h$ and $c_f$ are not zero, monitoring becomes a non-trivial strategic decision for supervisors. Moreover, by choosing whether to organize as a subsidiary or a branch, the MNB effectively decides whether it faces two uncoordinated supervisors, or a single supervisor. In this section, we study the monitoring and prudential decisions of independent national supervisors both in the subsidiary and in the branch cases.

#### 2.1 National supervision in the subsidiary case

As explained in the previous section, when both units are monitored, supervisors take, for a given state, the same prudential decision. When only one or none of the units are monitored, instead, the home and the foreign supervisors may act differently. This difference arises from the asymmetric liability structure of a subsidiary-represented multinational bank. Since foreign depositors have priority over the subsidiary’s assets and the home unit has limited liability for the subsidiary’s
losses, the decision over the home unit affects neither the intervention nor the monitoring of the foreign supervisor. The situation for the home supervisor is different. If the foreign unit is kept open, the home supervisor may be able to reduce its costs by taking possession of the foreign residual assets. The availability of foreign residual assets affects both the incentives to intervene under limited information and those to monitor. The incentives to monitor will be measured by the value of information (or of monitoring). The value of monitoring the foreign unit is equal to the cost $c_f$ such that $W_f(M) = W_f(O) = 0$ and, for a given $d_f$, the value of monitoring the home unit is the cost $c_h$ such that $W_h(M, d_f) = \max_{d_h \neq M} W_h(d_h, d_f) = 0$. The next proposition gives us the full characterization of the equilibrium decisions in the subsidiary case:

**Proposition 2.** The equilibrium decisions of the supervisors of a subsidiary represented $MNB$, $(d^*_h, d^*_f)$, are qualitatively described as follows (the precise thresholds are in the Appendix).

(i) The foreign supervisor chooses $d^*_f = M$ if $c_f$ is low, otherwise $d^*_f = O$;

(ii) The home supervisor chooses $d^*_h = M$ if $c_h$ is low. Otherwise, a small $L$ induces $d^*_h = O$, and a large $L$ induces either $d^*_h = C$ if $c_f$ is low (such that the foreign supervisor monitors), or $d^*_h = I$ otherwise.

The equilibrium decisions $(d^*_h, d^*_f)$ can be seen as the outcome of three different comparisons.

- **Monitoring in the foreign unit.** In the absence of monitoring, the foreign supervisor obtains $W_f(I) = -\alpha_f(1 - L)$ with intervention, and $W_f(O) = p \times 0 - (1 - p)\alpha_f$ with no intervention. As $p > L$, with no monitoring the foreign supervisor minimizes the foreign DI’s expected losses by leaving the subsidiary open.\(^{18}\) Monitoring serves to identify and intervene in a failing unit (that would otherwise be kept open for the reasons just stated), thus securing the liquidation value $L$. Since failure occurs with probability $1 - p$ from an ex ante perspective, and the deposit insurance credibly pays only with probability $\alpha_f$, the value of monitoring is $\alpha_f(1 - p)L$, which is then contrasted with the cost of monitoring $c_f$ (part (i) of the Proposition).

- **Liquidating a non-monitored home unit.** The home supervisor’s prudential decision to liquidate $(d_h = I)$ or leave open $(d_h = O)$ a non-monitored home unit is affected by the availability of residual assets in the foreign unit. When the foreign unit is successful, the residual assets reduce the home supervisor’s costs for any decision. However, the expected value of those assets will be higher for the home supervisor upon intervention than with no intervention. Indeed, with intervention the home supervisor expects to obtain these foreign residual assets with probability $p$, while with

\(^{18}\) The condition $p > L$ follows from H1 and H2.
no intervention these assets are only useful upon failure of the home unit, hence with the lower probability $p(1 - p)$. As a consequence (despite $p > L$), intervention can take place in the home unit when the liquidation value $L$ is large enough.

- **Monitoring in the home unit.** Foreign monitoring has two effects on the home supervisor’s decisions. First, it allows the home supervisor to condition his strategy on the outcome of monitoring in the foreign unit, with a conditional behavior that optimally contemplates intervention in the home unit if foreign assets are good, and leaving it open if they are not (i.e. $d_h = C$).\(^{19}\) Second, the expected value of foreign residual assets is higher when the foreign supervisor monitors, so that $W_h(d_h, M) - W_h(d_h, O) > 0$ for any $d_h$. We know that more (expected) foreign residual assets increase the payoff of intervention for the home supervisor relatively more than with other decisions, and in particular more than with monitoring the home unit. Hence, it follows that the larger foreign residual assets delivered by monitoring the foreign subsidiary reduce the value of monitoring the home unit.

**Corollary 1.** (i) **Foreign monitoring reduces the value of monitoring to the home supervisor.** (ii) *If the two countries are symmetric ($\alpha_h = \alpha_f$, $c_h = c_f$) and monitoring decisions are different, then the home supervisor monitors less than the foreign supervisor.*

Clearly, if the home supervisor has a lower monitoring cost than the foreign supervisor, $c_h \leq c_f$, or if the probability that the home deposit insurance fund ends up paying depositors is higher, $\alpha_h \geq \alpha_f$, this makes the home supervisor more likely to exert monitoring than the foreign supervisor. However, controlling for these two effects, the home supervisor actually exerts less monitoring.

Finally, since foreign residual assets are decreasing in the deposit rate $P_f$ promised to foreign depositors, and $P_f$ decreases in $\alpha_f$, we have the following:

**Corollary 2.** *A more credible foreign deposit insurance increases the availability of foreign residual assets to the home supervisor and thus reduces his incentives to monitor the home unit.*

A higher credibility of the local deposit insurance, $\alpha_h$, instead naturally increases the value of monitoring for the home supervisor, as does an increase of $\alpha_f$ for the foreign supervisor.

### 2.2 National supervision in the branch case

Under branch representation, there are three differences with the subsidiary case: (i) a single supervisor now takes the decisions $(d_h, d_f)$ for both units; (ii) the assets of the home unit can be

\(^{19}\)Indeed, bad news about the foreign unit eliminates the possibility for the home supervisor to reduce home costs with assets from the foreign unit, so that he will be less likely to intervene than when the foreign assets are good.
used to pay back depositors when the foreign unit defaults; (iii) both the domestic and foreign depositors are covered by the home deposit insurance. Note that, except for the monitoring costs $c_h$ and $c_f$, the two units are now completely symmetric. The next proposition shows the optimal decisions:

**Proposition 3.** The optimal decisions of the supervisor of a branch represented MNB, $(d^b_h, d^b_f)$, are qualitatively described as follows (the precise thresholds are in the Appendix).

(i) If monitoring costs are both low, then $d^b_h = d^b_f = M$;

(ii) If monitoring costs are both high, there is no monitoring at all. A low $L$ induces $d^b_h = d^b_f = O$, and a large $L$ induces $d^b_h = d^b_f = I$;

(iii) If $c_i$ is low and $c_j$ is high, only unit $i$ is monitored. A small $L$ induces $d^b_j = O$, and a large $L$ induces $d^b_j = C$.

Although the complete characterization is lengthy, the intuition behind Proposition 3 is simple. In the absence of monitoring, the liquidation value $L$ determines whether it is optimal to always intervene in one unit or not. If one unit is monitored, $L$ determines whether it is preferable to intervene in the other unit conditionally on success in the monitored unit or not (decision $C$). Then, when monitoring costs are low in both units, the optimum is to exert monitoring in both, when monitoring costs are both high there is no monitoring at all, and if one cost is low and the other high, only the “cheaper” unit is monitored.

As in the case of the subsidiary, a high liquidation value increases monitoring incentives in the first unit that the supervisor decides to monitor. Indeed, the supervisor can avoid a type II error if he monitors an open unit. However, the supervisor’s ability to make decisions for both units introduces an additional effect. The supervisor internalizes the fact that monitoring one unit can potentially lower the costs associated with the other unmonitored unit. No such type of internalization occurs in the case of a subsidiary, whose national supervisors take independent uncoordinated decisions.

The next corollary summarizes the main effects that shape the equilibrium decisions:

**Corollary 3.** (i) High liquidation values increase the likelihood of an intervention decision in an unmonitored unit, and the likelihood that at least one unit is monitored; (ii) Monitoring one unit reduces the value of monitoring the second unit.

With one unit monitored, expected costs decrease for the other unmonitored unit. This in turn reduces incentives to collect information on the second unit. This effect is similar to the one we discussed for the home supervisor’s in a subsidiary represented MNB. This reduction in incentives
is stronger for high liquidation values: a higher $L$ reduces the deposit rate by more in the monitored unit, and therefore increases the residual assets available for the other unmonitored unit. However, the branch liability structure magnifies the effect of the residual assets on the monitoring decisions. In particular, as assets from the two units are pulled together in case of a failure of a unit, the larger (expected) availability of residual assets further reduces monitoring incentives compared to the subsidiary. Hence, the value of monitoring the second unit is lower in a branch represented MNB compared to a subsidiary represented one.

3 Supranational supervision

We now turn to the case of a subsidiary-MNB with supranational supervision: instead of two supervisors taking monitoring and prudential decisions non-cooperatively, a single entity is responsible for both units and minimizes the total expected losses for deposit insurers in both countries. The setup is otherwise unchanged. In particular, deposit insurance is still national, potentially with unequal credibility in both countries (see Section 5.1 for a discussion about common DI) and the supranational supervisor faces the same costs of collecting information as national supervisors.

3.1 Short-run implications of supranational supervision

Our goal is to explore to what extent supranational supervision will lead to a different outcome than national supervision in the short-run, that is, without taking into account that the representation form of the MNB may react to the supervisory architecture. Formally, a supranational supervisor takes a joint decision $(d_h, d_f)$ in order to maximize the sum of the expected payoff of the home and of the foreign DI. We denote by $(d^*_h, d^*_f)$ the optimal decisions.

**Lemma 1.** National and supranational supervision with a subsidiary represented MNB may lead to a different outcome only if the decision in the foreign unit is different: If $d^*_f = d^{**}_f$, then $d^*_h = d^{**}_h$.

The intuition for this lemma is that the foreign supervisor exerts an externality on the home supervisor, while the opposite is not true. For a given decision in the foreign unit, minimizing the losses of the home deposit insurance fund is equivalent to minimizing the total losses of both funds. Hence, supranational supervision can lead to a different outcome only if it affects the supervision of the foreign unit.

The foreign supervisor does not internalize that monitoring the foreign unit is beneficial to the home DI fund and allows the home supervisor to take a decision conditional on the information
on the foreign unit. In contrast, the supranational supervisor does take this into account and thus, when decisions differ in the foreign unit, we have $d_f^* = O$ and $d_f^{**} = M$. Building on these preliminary results, the next proposition summarizes the cases in which national and supranational supervision lead to different decisions.

**Proposition 4.** If decisions under national and supranational supervision in a subsidiary represented MNB differ, then in the subsidiary we have $d_f^{**} = M$ and $d_f^* = O$, whereas in the home unit we have $d_h^{**} = C$ and $d_h^* \in \{O, I, M\}$.

Only the choice of the home supervisor in the case of national supervision is different. In particular, in all cases obtaining a different outcome with supranational supervision requires that $c_f$ is high enough, so that the foreign supervisor chooses not to monitor, but not too high, so that the monitoring is useful once the internalization effect is taken into account. Conversely, $c_h$ must be high, so that the supranational supervisor prefers to rely on monitoring the foreign unit only rather than both units.

Fig. 2 shows the supervisory decisions reached for each representation form and organization of supervision as a function of $\alpha_h$ and $\alpha_f$. Comparing the subsidiary case with the supranational case, one can see how, for intermediate values of $\alpha_h$ and $\alpha_f$, introducing a supranational supervisor shrinks the regions $(M, O)$ and $(O, O)$ and expands the region $(C, M)$. It is now useful to identify the consequences of a switch to supranational supervision on the profitability of a subsidiary represented MNB.

**Corollary 4.** Holding the representation form of the MNB constant, introducing a supranational supervisor leads to lower profit for existing subsidiary represented MNBs.

Reduced profitability is a direct consequence of more monitoring of the foreign subsidiary by the supranational supervisor and the associated decision $d_h = C$ for the home unit. Both moves involve a higher probability of intervention and thus lower profits. Corollary 4 points out the negative impact of a supervisory change on the subsidiary’s profit. Clearly, as a consequence, banks may also decide to change their foreign representation or become stand-alone domestic banks. The next section discusses the consequences of such changes.

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20 The parameters are $p = 0.8$, $R = 1.5$, $L = 0.5$, $c_h = c_f = 0.05$. 

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Figure 2: Equilibrium supervisory decisions as functions of the credibility of the national DI funds ($\alpha_h$ and $\alpha_f$) for the different types of banks and of supervision.
3.2 Long-run implications of supranational supervision

We now analyze the long-run implications of supranational supervision, investigating how the bank responds to a change in supervision. To this end, we compare the choices made by the supervisor(s) for the different organizational forms under the same parameter values. To streamline the discussion, we concentrate on the parameter region where the introduction of the supranational supervisor leads to different decisions than the ones with national supervisors.

It is easy to see that a supranational supervisor does not lead to different decisions in the case of the branch represented MNB. Indeed, the branch representation attributes all the costs to the home deposit insurance and thus the (single) home supervisor internalizes all costs and benefits from the two units. Thus, we need to compare the outcomes with a subsidiary supervised either at the national or the supranational level, with those of a branch, or a stand-alone bank.

Note that the branch represented MNB only makes (expected) profit if (with a positive probability) both units are open. The supervisor of a branch-MNB is close to the situation of the supranational supervisor of a subsidiary-MNB, in that both internalize the effect that monitoring the foreign unit has on the home unit. We know from the previous section that, when the supranational supervisor and the national supervisor take different decisions for the subsidiary, then the supranational supervisor chooses $(C,M)$. As a result, the decision in the branch case will often also be $(C,M)$, and the branch will not be viable. The only exception is that under some parameter values the supervisor of a branch may choose $(O,O)$. The following lemma summarizes the possible cases in which supranational supervision leads to different outcomes and the branch is a viable option.

**Lemma 2.** When supranational supervision induces different decisions over a subsidiary-represented MNB, i.e. $(d^*_h, d^*_f) \neq (d^{**}_h, d^{**}_f)$, and a branch-represented MNB is viable, i.e. $(d^*_h, d^*_f) = (O,O)$, then $(d^{**}_h, d^{**}_f) = (C,M)$ associated with the following (mutually exclusive) possibilities for national supervision: (a) $(d^*_h, d^*_f) = (O,O)$; (b) $(d^*_h, d^*_f) = (I,O)$.

Both cases require $\alpha_f > \alpha_h$ (the full characterization of the corresponding sets of parameters is in the Appendix).

This lemma identifies two cases of particular interest. In the first case (a) of the Lemma, in the absence of supranational supervision, the MNB can be organized as a subsidiary or as a branch, leading to the same supervisory outcomes (decisions $(O,O)$). The introduction of supranational supervision leads to more monitoring with the subsidiary structure, and does not affect the branch.
Table 2: Possible combinations of supervisory decisions, for the different organizational forms, when national and supranational outcomes differ. The symbol ∅ denotes cases in which the bank cannot profitably operate given its representation form and the supervisory decisions it induces.

<table>
<thead>
<tr>
<th>Case</th>
<th>National Subsidiary</th>
<th>Subsidiary Supranational</th>
<th>Branch</th>
<th>Stand-alone</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(O, O)</td>
<td>(C, M)</td>
<td>(O, O)</td>
<td>O</td>
</tr>
<tr>
<td>(ii)</td>
<td>(O, O)</td>
<td>(C, M)</td>
<td>∅</td>
<td>O</td>
</tr>
<tr>
<td>(iii)</td>
<td>(M, O)</td>
<td>(C, M)</td>
<td>∅</td>
<td>M</td>
</tr>
<tr>
<td>(iv)</td>
<td>∅</td>
<td>(C, M)</td>
<td>(O, O)</td>
<td>O</td>
</tr>
<tr>
<td>(v)</td>
<td>∅</td>
<td>(C, M)</td>
<td>∅</td>
<td>O</td>
</tr>
<tr>
<td>(vi)</td>
<td>∅</td>
<td>(C, M)</td>
<td>∅</td>
<td>M</td>
</tr>
</tbody>
</table>

In the second case (b), the home unit of a subsidiary-organized MNB is always closed with national supervision (decisions (I, O)), and supranational supervision actually leads to liquidating less often (decisions (C, M)). These results will be used to assess the implied changes of bank’s profitability in the ensuing analysis.

Table 2 summarizes all the cases in which supranational supervision makes a difference to the supervision of the subsidiary-MNB, and thus can encourage the MNB to change its representation form. To complete the picture, we add possible supervisory decisions for the subsidiary and the stand-alone in cases when the branch leads to either (C, M) or (I, O) and thus cannot be active.\(^{21}\)

Comparing the MNB’s profits across different representation forms, we obtain the following:

**Proposition 5.** When supranational supervision changes the optimal organizational form of the MNB, it either induces a subsidiary-MNB to become a branch-MNB (case (i) of Table 2), or it induces a subsidiary-MNB to become a stand-alone bank (cases (ii) and (iii)).

Assume that the parameters of the model are such that we are in case (i) of Table 2 (which corresponds to case (a) of the previous Lemma). Under national supervision, the MNB can choose between a subsidiary, a branch, or a stand-alone, all leading to the same outcome of any unit being left non-monitored and open. As case (i) requires $\alpha_f > \alpha_h$, the subsidiary structure allows the bank to benefit from the high-quality foreign deposit insurer and is the most profitable structure. However, when supranational supervision is introduced, the coordination problem between the two supervisors of the subsidiary is solved, which leads to tighter monitoring of the subsidiary structure and reduced profits. As a result, this structure becomes less profitable than the branch, and the MNB adopts this latter representation form instead.

\(^{21}\)This decision is easily deduced from the conditions defining the supervisory decisions in the other cases.
In cases (ii) and (iii), a branch structure is not viable, so that under national supervision the MNB chooses between a subsidiary structure and a stand-alone bank. If the supervisory decision in the home unit is the same, the subsidiary structure is more profitable than a stand-alone bank, as the subsidiary provides additional sources of profit without putting strain on the home unit’s profit. However, in these cases supranational supervision changes the supervisory decision in the home unit, thereby reducing the profits of the subsidiary structure. Hence, the bank’s best response is actually to close the foreign unit and revert to domestic banking.

Finally, in cases (iv) to (vi) of Table 2, the subsidiary is not viable with national regulators. A switch to a supranational regulator increases the profitability of subsidiaries and make them a viable alternative. However, either the branch structure with \((O,O)\) (as in case (iv) which corresponds to case (b) of the previous Lemma) or the stand-alone structure (in cases (v) and (vi)) will still dominate. Supranational supervision thus has no impact on the choice of organizational form in these cases.

Figure 3 illustrates the bank’s choice of representation as a function of \(\alpha_h\) and \(\alpha_f\). The parameters are the same as on Fig. 2, so that the choice of the MNB can be compared to the supervisory decisions associated with each structure. In particular, we see that introducing a supranational supervisor expands the region where a subsidiary faces the decision \((C,M)\), so that the MNB optimally chooses to switch to a stand-alone or a branch structure instead. Proposition 5 implies that centralization of supervision could have unintended consequences. With national supervisors, the MNB can adopt a subsidiary structure in order to face low monitoring and thus a low probability of intervention. When supervision becomes supranational, the MNB prefers a branch structure instead to avoid the increased monitoring induced by supranational supervision. In other instances, when the branch structure is not profitable, the lower profitability of the subsidiary structure (due to supranational supervision) implies that the MNB prefers to entirely forego foreign expansion, reverting to a national bank: supranational supervision has the paradoxical effect of decreasing financial integration.

Finally, whether centralizing supervision has an impact on the MNB’s representation form depends on the parameters in the following way:

**Corollary 5.** - Symmetric countries: When \(\alpha_h = \alpha_f\), centralizing supervision either has no impact on the MNB’s representation form or it induces a switch from subsidiary to stand-alone.
- Robust home deposit insurance: a more robust home deposit insurance (higher \(\alpha_h\)) makes a switch to stand-alone more likely than a switch to branch.
Figure 3: Equilibrium organization of the MNB. The area with vertical (resp., horizontal) dashed lines represents a switch from subsidiary to stand-alone (resp., branch) induced by centralized supervision.

The first point shows that when integrating similar countries, if centralized supervision has an impact then it is a switch from subsidiary to a stand-alone structure. Graphically, this means that on the 45 degree line in Fig. 3 we are necessarily in the vertically-dashed region or in a non-dashed region. The reason is that if the internalization effect makes it optimal to obtain \((C,M)\) under supranational supervision, it will lead to the same outcome with the branch, so that a branch is not profitable for the MNB in such a case. The second point follows from a high \(\alpha_h\) making the supervisor of a branch tougher, so that this representation form is less profitable for the MNB. Graphically, this means that the horizontally dashed region is necessarily on the left of the vertically dashed region in Figure 3.

### 3.3 Supranational supervision and welfare

Finally, we analyze the impact of centralized supervision on welfare when taking into account that MNBs can adjust their representation form. We first look at total welfare, which we measure as the unweighted sum of payoffs to all agents in the economy. In this model, total welfare is simply equal to the expected value of the bank’s assets in the two countries, minus monitoring costs, and thus depends only on whether the MNB opens a foreign unit or not, and on supervision decisions. We observe the following:
Corollary 6. - When supranational supervision leads to a switch from a subsidiary structure to a branch structure, there is no change in the monitoring of either unit, and total welfare is unaffected.
- When supranational supervision leads to a switch from a subsidiary structure to a stand-alone structure, the foreign unit is closed but there is no change in the monitoring of the home unit, so that total welfare decreases.
- When supranational supervision leaves the representation form unchanged, there is no impact on monitoring or on welfare.

We thus obtain the surprising result that centralized supervision is either neutral or even negative for total welfare. Indeed, the only reason why centralized supervision can increase welfare in the model is that it leads to greater monitoring of the foreign unit. However, when this happens, the MNB finds it more profitable to change its representation form to completely neutralize this effect, so that either the foreign unit is still open but not monitored, or it is simply shut down. Thus, the positive effect of centralized supervision never materializes. Of course, a more general model may lead to a less extreme outcome, but this result captures in a stark manner how a change in representation form can potentially largely undermine the benefits of centralized supervision.

Even when the impact of centralized supervision on total welfare is neutral, the associated change in representation form has redistributive consequences. Notice that depositors always break even in the model, and that supervision costs are never affected by centralized supervision. Thus, we can focus only on the payoffs received by the MNB and the two deposit insurance funds:

Corollary 7. When centralized supervision changes the MNB's representation form, total expected losses to the two deposit insurance funds decrease, and the MNB’s expected profit decreases.

This result is straightforward when the MNB becomes a stand-alone: deposit insurers become liable for one unit only, and the MNB loses all profit from the foreign unit. When the MNB switches from a subsidiary to a branch structure, we know that it could have chosen the branch structure under national supervision, so the parameters have to be such that the subsidiary form is more profitable than the branch form, hence the bank's profit decreases with centralized supervision. Since total welfare is not affected, it has to be the case that losses to the deposit insurance funds decrease. This decrease is the combination of two effects: the fact that each unit of the MNB is now liable for losses in the other unit, and the lower credibility of the home deposit insurance fund, which is less likely to pay out as the MNB’s representation form can change only when $\alpha_h \leq \alpha_f$ (Lemma 2).
While it decreases total expected losses to the two deposit insurance funds, centralized supervision also leads to a redistributive effect among them:

**Corollary 8.** *When supranational supervision changes the MNB’s optimal organizational form, it moves all potential losses to the home deposit insurance fund, which is also the less credible fund (αₜ ≤ αₗ). The losses to the home deposit insurance fund strictly increase, while the losses to the foreign deposit insurance fund become null.*

The redistributive effect is straightforward: whether the MNB’s representation form changes from subsidiary to branch or from subsidiary to stand-alone, the foreign deposit insurance fund is no longer liable, and the home deposit insurance fund either becomes liable for an extra unit (branch case), or loses access to the foreign unit’s residual assets (stand-alone case). A less obvious effect is that the deposit insurance fund now liable for all potential losses is the weaker one. Indeed, the reason why the MNB changes its representation form is because having its deposits insured by the home deposit insurer leads to less monitoring. This is the case when the home deposit insurance fund is weak and unlikely to pay out. Although in our model αₜ is exogenous, it is clear that the higher burden can further undermine the credibility of the home deposit insurance, leading to higher deposit rates and lower profits for multinational banks. As many countries’ deposit guarantee systems are already overstretched, centralization (with national deposit insurance) can have the long-run effect of further reducing the credibility of some countries’ deposit insurance.²²

Finally, Corollary 7 and Corollary 8 have a direct implication on which countries can voluntarily share supervision. Assume that there are two countries with several MNBs, some headquartered in country 1 with foreign units in country 2, and some headquartered in country 2 with foreign units in country 1. Assume a proportion φ of the MNBs are headquartered in country 1, and 1 − φ in country 2. We know that the deposit insurer in each country makes lower losses on all subsidiary units of foreign banks, and higher losses on all home units. Thus:

**Corollary 9.** *Assume that centralizing supervision changes the optimal organizational form symmetrically for domestic and foreign MNBs. If φ is sufficiently close to 1/2, then centralizing supervision reduces expected losses to the deposit insurers of both countries. Otherwise, if φ is low enough, then the deposit insurer in country 1 gains from centralizing supervision while the deposit insurer in country 2 loses out, and the opposite obtains when φ is high enough.*

²²This is a theoretical rationale for moving to complement supranational supervision with a common deposit insurance scheme, as we study in Section 5.1.
This corollary implies in particular that it will be easier for two countries to adopt a common supervision framework if MNBs are more symmetrically distributed (see Dell’Ariccia and Marquez (2006) for a similar result based on a different argument). Conversely, countries that are mostly home to MNBs will resist centralized supervision, while countries that mostly host foreign MNBs will gain from it.

4 Empirical implications

We briefly review the main testable implications of the model in this section. We separate them into two groups: (i) short-term implications, that predict changes in observables holding the representation form of the MNB constant; (ii) long-term implications, that take into account that MNBs may adapt their representation form over time.

4.1 Short-term implications

Borrowing costs. The variables $P_h$, $P_f$ and $P$ in the model measure the borrowing costs of banks. They can be deposit rates if one interprets $\alpha_h$ and $\alpha_f$ as measuring the credibility of deposit insurance in a narrow sense. More generally, these variables can measure the rates at which bank units borrow on the wholesale market, in which case the $\alpha$s measure implicit safety net guarantees. In both cases, Demirgüç-Kunt, Kane, and Laeven (2014) offer proxies that can be used to measure $\alpha_h$ and $\alpha_f$. In particular, they use the government debt-to-GDP ratio as an inverse proxy for the ability of the government to backstop the DI fund. Our analysis shows the following:

Implication 1. Holding the organizational form of the MNB constant:
- In a subsidiary-MNB, the borrowing costs of the foreign unit are decreasing in $\alpha_f$, but do not depend on $\alpha_h$. The borrowing costs of the home unit are decreasing in $\alpha_h$ and $\alpha_f$.
- In a branch-MNB, borrowing costs are decreasing in $\alpha_h$, but do not depend on $\alpha_f$.

These implications directly follow from the liability structure of the MNB and the allocation of deposit insurance responsibilities (see section 1.3). An interesting application of this is the European sovereign debt crisis, which can be interpreted as a negative shock to the $\alpha$s of some countries: the model predicts that subsidiaries of foreign banks in a crisis-hit country will see their borrowing costs rise similarly to local banks, whereas subsidiaries of crisis country banks in non-hit countries will not be as affected. Similarly, the borrowing costs of the parent bank may increase when its
foreign subsidiaries are located in countries hit by a sovereign debt crisis.

Monitoring. The amount of monitoring exerted by a supervisor is of course not a simple binary variable, and is not readily observable by outsiders. However, it is possible to find proxies and indirect measures for the decision $M$. For instance, Beck, Todorov, and Wagner (2013) propose measuring the delay with which a supervisor acts by the CDS spread of the troubled bank at the time the supervisor intervened. In the model, the supervisor intervenes in banks with bad assets earlier when they are monitored (decision $M$) than when he leaves the unit open without monitoring (decision $O$), so that the measure proposed by the authors can also be interpreted as a proxy for the monitoring intensity chosen by the supervisor. Our model implies that:

**Implication 2.** Holding the organizational form of the MNB constant:
- Monitoring of the subsidiary’s foreign unit is more likely when $\alpha_f$ is higher. Monitoring of the home unit is more likely when $\alpha_h$ is higher and $\alpha_f$ smaller (Corollaries 1 and 2).
- Monitoring of the subsidiary’s foreign unit is more likely under supranational supervision (Proposition 4).

As mentioned in footnote 13, there are historical examples of bank supervisors becoming more lenient when the pressure on the deposit insurance fund is too high. Our result suggests regressing the intensity of monitoring of a given unit more systematically (as proxied by Beck, Todorov, and Wagner (2013) for instance) on the credibility of each deposit insurance fund (using Demirguc-Kunt, Kane, and Laeven (2014)), interacted with the legal structure of the MNB. A recent illustration of the second point is given by the Greek crisis: bank supervisors of Greek banks’ subsidiaries in Romania and Bulgaria considered liquidating these subsidiaries. This would have worsened the situation of their parent banks, but this externality was not taken into account by the subsidiaries’ supervisors: from their point of view, the liquidation decision was more attractive than costly monitoring. The ECB had to extend credit lines to these subsidiaries to avoid this outcome.\(^{23}\)

### 4.2 Long-term implications

In the long-run, different organizations of supervision can give a competitive advantage to MNBs with different organizational structures. Whether an MNB chooses to expand abroad via a subsidiary or a branch can be observed empirically. Moreover, the model delivers predictions on the

\(^{23}\)See “ECB puts in place secret credit lines with Bulgaria and Romania”, Financial Times Online, July 16, 2015.
choice of whether to expand abroad at all. The literature on cross-border bank acquisitions typically considers the choice between a stand-alone structure and a subsidiary-organized MNB (e.g., Karolyi and Taboada (2015)).

**Implication 3.** (i) All else equal, a lower $\alpha_h$ and a higher $\alpha_f$ make the subsidiary representation form more profitable than the branch form (Proposition 1).

(ii) Supranational supervision makes the branch form more profitable compared to the subsidiary form, and can discourage cross-border expansion altogether (Proposition 5).

(iii) Supranational supervision increases the interest rate offered to depositors in both the home and the foreign unit if the MNB switches from subsidiary to branch.

Although there may be many reasons that induce a bank to expand abroad and to do so with different types of foreign representation (for example tax incentives), the first point seems consistent with a recent Greek case. The Greek Central Bank indicated that, as of March 2015, all foreign units of Greek banks were subsidiaries, with the unique exception of Alpha Bank, organized with branch representation in Romania and Bulgaria. Facing the deterioration of the credibility of the Greek national deposit insurance, the foreign branches of Alpha Bank faced the largest withdrawal of deposits of all foreign units of Greek banks (all the others being subsidiaries). Even more interestingly, these foreign branches of Alpha Bank were shortly after acquired (July 2015) by foreign subsidiaries of other Greek banks which then managed them as subsidiaries backed-up by the more solid Romanian and Bulgarian national deposit insurance.\textsuperscript{24}

The second point implies that centralizing supervision, as the European Single Supervisory Mechanism does, should lead to a different organization of MNBs, with more MNBs choosing a branch form and, potentially, fewer cross-border banks.

The third point follows from the observation that $P_h(S, O, O) < \min(P_h(B, O, O), P_h(A, O))$ (see the proof of Lemma 2). For given supervisory decisions, depositors in the home unit of a subsidiary are more protected because they can recover part of the profits of the foreign unit. As centralized supervision can result in a move away from the subsidiary structure, this benefit disappears and the interest rate offered to depositors has to increase.

\textsuperscript{24}See, for example, “Greek Eurobank Takes Over Alpha Bank’s Branch Network in Bulgaria,” July 18, 2015, at www.novinite.com.
5 Extensions

5.1 Common deposit insurance

The analysis of common supervision developed so far assumed that the deposit guarantee scheme remains national. However, a common deposit insurance (CDI) may seem another natural step to go, and is currently in the Banking Union agenda of the EU and already established in the US. Here we address this possibility by assuming that the supranational supervisor relies on a CDI fund with a credibility parameter $\alpha_c$, which conceivably depends on the credibility of national deposit insurance funds, $\alpha_h$ and $\alpha_f$.

Although one could conceive of different institutional arrangements, here we consider on purpose the rather “optimistic” case in which the more reliable national deposit insurance scheme transfers its credibility to the less reliable one. In particular, to fix ideas, we assume the home national DI insurance scheme is more reliable, so that $\alpha_c = \alpha_h \geq \alpha_f$ and the effects of the CDI can be determined by comparative statics on $\alpha_f$.

Considering an MNB that is already supervised at the supranational level, the following proposition shows the impact of introducing a common deposit insurance:

**Proposition 6.** When supervision is supranational, adding a common deposit insurance scheme with $\alpha_c = \alpha_h \geq \alpha_f$:
- Does not affect the supervision of a branch-MNB;
- Increases monitoring incentives in the foreign unit of a subsidiary-MNB;
- Decreases monitoring incentives in the home unit of a subsidiary-MNB.

With branch representation, a CDI has no effect at all since the home DI is already in charge under national deposit insurance. With a subsidiary-MNB, deposits in the foreign unit are more likely to be covered by the common deposit insurance fund than they were to be covered by the foreign fund, so that potential losses increase. The value of monitoring the foreign unit is thus higher. Both the higher credibility of the foreign unit and the increase in monitoring imply that deposits in the foreign unit are safer, so that $P_f$ decreases. As a result, the foreign unit has larger residual assets, which reduces the value of monitoring in the home unit (Corollary 1). Thus, even if one optimistically assumes that the CDI inherits the credibility of the more reliable deposit scheme,

25Alternatively, one could assume $\alpha_c$ is between $\min\{\alpha_h, \alpha_f\}$ and $\max\{\alpha_h, \alpha_f\}$, which requires us to study the effects of an increase in DI credibility for one unit and a decrease for the other unit.
the introduction of common deposit insurance can have the unintended consequence of decreasing monitoring in one unit. This also implies the following:

**Corollary 10.** *Introducing supranational supervision together with common deposit insurance can still induce a subsidiary-MNB to close its foreign unit and become a stand-alone bank.*

Indeed, Proposition 6 shows that if the supranational supervisor chooses the strategy \((C, M)\), introducing a common deposit insurance only reinforces this choice (i.e., the foreign unit is monitored but the home unit is not). Moreover, the result in Corollary 4 that supranational supervision makes the subsidiary-MNB less profitable holds true for any value of \(\alpha_h\) and \(\alpha_f\), and thus irrespective of whether the deposit insurance is common or not. Hence, more monitoring on the subsidiary with supranational supervision and CDI can still make stand-alone banking more profitable.

### 5.2 Representation form-sensitive insurance premia

As is well known in the theoretical literature, risk-based insurance premia, while not commonly used, can go a long way towards alleviating moral hazard in banking (see, e.g., Rochet (1992)). In this model, one needs to go even further to align incentives with the social optimum. First, insurance premia should depend on the credibility of the deposit insurance fund: deposits insured by a less credible fund should be charged a lower premium. Second, the premia should also depend on the representation form of the bank. Indeed, for a given premium, a subsidiary-MNB enjoys an implicit subsidy, when compared to a branch or a stand-alone: with probability \(\alpha_f \times p(1 - p)\), its foreign creditors are repaid by the foreign deposit insurance fund, even though the home unit redistributes profits to shareholders. This is the reason why centralized supervision can lead the MNB to close its foreign unit: in some cases, it might be profitable for the MNB to expand abroad only if it enjoys this implicit subsidy. If centralized supervision suppresses it, then the best reaction of the MNB is to revert to domestic banking.

To see how a premium based on the representation form can align the incentives of the bank with the social optimum, assume that the MNB needs to pay fees \(F_h(k)\) and \(F_f(k)\) upfront to the deposit insurers of countries \(h\) and \(f\), respectively, where \(k \in \{S, B, A\}\) stands for the representation form chosen by the MNB. A fairly priced deposit insurance would imply that the bank pays exactly
the expected costs to each deposit insurer. This is equivalent to having:

\[ F_h(S) = -W_h(d_h^*, d_f^*), \quad F_f(S) = -W_f(d_f^*) \]

\[ F_h(B) = -W_b(d_h^b, d_f^b), \quad F_f(B) = 0 \]

\[ F_h(A) = -W_h(d_h^a), \quad F_f(A) = 0. \]

The same formulas hold under supranational supervision, replacing \((d_h^*, d_f^*)\) with \((d_h^{**}, d_f^{**})\). To see why with such premia the bank’s organizational form is now socially optimal, observe that both deposit insurers are now indifferent regarding the choice of the bank (in particular, centralized supervision no longer creates “winners” and “losers”). The bank will, for instance, choose \(S\) over \(B\) if and only if \(\Pi(S, d_h^*, d_f^*) - F_h(S) - F_f(S) \geq \Pi(B, d_h^b, d_f^b) - F_h(B)\), which is simply equivalent to \(\Pi(S, d_h^*, d_f^*) + W_h(S, d_h^*, d_f^*) + W_f(d_f^*) \geq \Pi(B, d_h^b, d_f^b) + W_b(d_h^b, d_f^b)\): the bank is in effect maximizing aggregate welfare.

Note that, while theoretically natural, this solution is difficult to implement. First, it requires pricing in the credibility of the deposit insurance fund. This is particularly problematic, as it requires public acknowledgment that a deposit insurance fund may not be adequately funded. Second, it requires to have a good understanding of the often complicated structure of the entire MNB, as this structure has a first-order impact on the distribution of losses in case the MNB defaults. Third, the insurance premium should correctly anticipate and price in the optimal supervisory decisions. To our knowledge, none of these elements are priced in existing deposit insurance schemes, which is in line with our assumptions.

Finally, note that this approach does not make centralized supervision redundant or useless, quite the opposite: centralized supervision still has the impact of changing \((d_h^*, d_f^*) = (O, O)\) into \((d_h^{**}, d_f^{**}) = (C, M)\). What representation form-sensitive deposit insurance achieves is to charge the MNB for switching from subsidiary to branch when this is not socially optimal. Indeed, a conclusion from our model is that making insurance premia depend on the MNB’s structure, while complicated, is necessary to control the MNB’s incentives to change its representation form so as to extract more implicit subsidies from deposit insurance funds.
6 Conclusion

We have shown how the allocation of supervisory authorities between a national and a supranational supervisor affects the way MNBs operate, their profit and funding conditions. In particular, we propose a framework for understanding the interaction between the structure of bank supervision and the organizational form of MNBs. We show that national and supranational bank supervisors take different monitoring and prudential decisions in MNBs depending on whether these adopt a branch or a subsidiary structure. Conversely, the differences in supervisory actions affect the MNB’s choice of whether to expand abroad using a branch, a subsidiary, or not at all.

This interaction has important implications for regulatory reforms in banking. In particular, we show that the centralization of bank supervision at a supranational level, as recently implemented with the European banking union, can have unintended consequences on the organization of MNBs. Our results indicate that supranational supervision can, in some instances, reduce the willingness of banks to expand abroad, which clearly runs against the objective of any banking union. Another possibility is that supranational supervision gives a competitive advantage to branches over subsidiaries. As both types of foreign units may differ in their lending technologies, this effect can also imply undesirable consequences of supranational supervision, “un-leveling” the playing field. Unexpectedly, with any of these moves, the MNB is able to neutralize the more intense monitoring that it would otherwise expect with supranational supervision. With no more monitoring taking place, either welfare remains unaffected or it is reduced by supranational supervision.

Finally, our approach can also be used to compare the situation of national deposit insurance funds with that of common deposit insurance, envisaged as a next step of the banking union in Europe. Actually, the discrepancy in Europe between the level of supervision and the level of deposit insurance is a unique phenomenon. In the United States, for instance, access to the Federal deposit insurance automatically implies supervision by a Federal authority. We have shown that centralization may also bring about unexpected consequences also in this case. In particular, introducing supranational supervision can still have the same paradoxical effect of decreasing financial integration when common deposit insurance is in place.
A Appendix

A.1 Summary of payoffs

We summarize the payoffs obtained by all agents in all states of the world and for each organizational form of the bank. First, Table 3 gives the probabilities to be in different states \((s_h, s_f) \in \{s, f, l\}^2\) (home unit and foreign unit) depending on the decisions \((d_h, d_f)\) taken by the supervisor(s). Note that we neglect cases that can easily be obtained by symmetry. For instance, since \((d_h, d_f) = (O, M)\) leads to \((s, l)\) with probability \(p(1 - p)\), \((d_h, d_f) = (M, O)\) leads to \((l, s)\) with the same probability.

Table 3: Probabilities of the different states, for given supervisory decisions.

<table>
<thead>
<tr>
<th>((M, M))</th>
<th>((O, M))</th>
<th>((I, M))</th>
<th>((C, M))</th>
<th>((O, O))</th>
<th>((I, O))</th>
<th>((I, I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((s, s))</td>
<td>((s, s))</td>
<td>((l, s))</td>
<td>((s, s))</td>
<td>((s, s))</td>
<td>((l, s))</td>
<td>((l, s))</td>
</tr>
<tr>
<td>((s, l))</td>
<td>((s, l))</td>
<td>((l, l))</td>
<td>((s, l))</td>
<td>((s, f))</td>
<td>((l, f))</td>
<td>((l, l))</td>
</tr>
<tr>
<td>((l, s))</td>
<td>((f, s))</td>
<td>((l, s))</td>
<td>((f, s))</td>
<td>((f, s))</td>
<td>((l, f))</td>
<td>((l, f))</td>
</tr>
<tr>
<td>((l, l))</td>
<td>((f, s))</td>
<td>((l, l))</td>
<td>((l, l))</td>
<td>((l, f))</td>
<td>((l, f))</td>
<td>((l, l))</td>
</tr>
</tbody>
</table>

Reading: when \((d_h, d_f) = (M, M)\), we obtain \((s_h, s_f) = (s, s)\) with probability \(p^2\), \((s_h, s_f) = (s, l)\) with probability \(p(1 - p)\), \((s_h, s_f) = (l, s)\) with probability \((1 - p)p\), and \((s_h, s_f) = (l, l)\) with probability \((1 - p)^2\).

Second, Table 4 below gives the payoff to each type of agent in each state \((s_h, s_f)\) and for each organizational form. Combining Tables 3 and 4 thus gives the expected payoffs of all agents in each organizational form and for all supervisory decisions. We use some additional notation. For each state \((i, j) \in \{s, l, f\}^2\), in the subsidiary case we denote by \(u_h(i, j)\) and by \(u_f(j)\) the payoffs to depositors in countries \(h\) and \(f\), by \(w_h(i, j)\) and by \(w_f(i, j)\) the payoffs to the deposit insurers, and by \(\pi(i, j)\) the bank’s payoff. In the branch case, we can aggregate all agents and we similarly use the notations \(u_b(i, j), w_b(i, j)\) and \(\pi_b(i, j)\). In the stand-alone case, we will use the following notation: \(u_h(i), w_h(i)\) and \(\pi(i)\).
Table 4: Payoffs for depositors, supervisors, and the MNB, for the different organization forms.

<table>
<thead>
<tr>
<th>State ((i, j))</th>
<th>(w_h(i, j))</th>
<th>(u_h(i, j))</th>
<th>(\pi(i, j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((s, s))</td>
<td>0</td>
<td>(P_h)</td>
<td>(2R - 2P_h - P_f)</td>
</tr>
<tr>
<td>((f, s))</td>
<td>(-\alpha_h(1 - R + P_f))</td>
<td>(\alpha_h + (1 - \alpha_h)(R - P_f))</td>
<td>0</td>
</tr>
<tr>
<td>((l, s))</td>
<td>(-\alpha_h(1 - L - R + P_f))</td>
<td>(\alpha_h + (1 - \alpha_h)(L + R - P_f))</td>
<td>0</td>
</tr>
<tr>
<td>((s, f)) and ((s, l))</td>
<td>(-\alpha_h)</td>
<td>(\alpha_h)</td>
<td>(R - P_h)</td>
</tr>
<tr>
<td>((f, f)) and ((f, l))</td>
<td>(-\alpha_h)</td>
<td>(\alpha_h)</td>
<td>0</td>
</tr>
<tr>
<td>((l, f)) and ((l, l))</td>
<td>(-\alpha_h(1 - L))</td>
<td>(\alpha_h + (1 - \alpha_h)L)</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Subsidiary representation.

<table>
<thead>
<tr>
<th>State (i)</th>
<th>(w_h(i))</th>
<th>(u_h(i))</th>
<th>(\pi(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s)</td>
<td>0</td>
<td>(P_h)</td>
<td>(R - P_h)</td>
</tr>
<tr>
<td>(l)</td>
<td>(-\alpha_h(1 - L))</td>
<td>(\alpha_h + (1 - \alpha_h)L)</td>
<td>0</td>
</tr>
<tr>
<td>(f)</td>
<td>(-\alpha_h)</td>
<td>(\alpha_h)</td>
<td>0</td>
</tr>
</tbody>
</table>

(b) Stand-alone bank. Note that \(w_f(i)\) and \(u_f(i)\) for the foreign unit of the subsidiary-MNB are equal to \(w_h(i)\) and \(u_h(i)\) in the stand-alone case, replacing \(\alpha_h\) with \(\alpha_f\).

<table>
<thead>
<tr>
<th>State ((i, j))</th>
<th>(w_b(i, j))</th>
<th>(u_b(i, j))</th>
<th>(\pi_b(i, j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((s, s))</td>
<td>0</td>
<td>(P)</td>
<td>(2(R - P))</td>
</tr>
<tr>
<td>((f, f))</td>
<td>(-2\alpha_h)</td>
<td>(\alpha_h)</td>
<td>0</td>
</tr>
<tr>
<td>((l, l))</td>
<td>(-2\alpha_h(1 - L))</td>
<td>(\alpha_h + (1/2)(1 - \alpha_h)(L + R))</td>
<td>0</td>
</tr>
<tr>
<td>((s, l)) and ((l, s))</td>
<td>(-\alpha_h(2 - R - L))</td>
<td>(\alpha_h + (1/2)(1 - \alpha_h)(R + L))</td>
<td>0</td>
</tr>
<tr>
<td>((s, f)) and ((f, s))</td>
<td>(-\alpha_h(2 - R))</td>
<td>(\alpha_h + (1/2)(1 - \alpha_h)R)</td>
<td>0</td>
</tr>
<tr>
<td>((l, f)) and ((f, l))</td>
<td>(-\alpha_h(2 - L))</td>
<td>(\alpha_h + (1/2)(1 - \alpha_h)L)</td>
<td>0</td>
</tr>
</tbody>
</table>

(c) Branch representation.
A.2 Proof of Proposition 1

Using equations (5) and (6), we write the difference in profits between the subsidiary and the branch representation forms as:

\[ \Pi(S, M, M) - \Pi(B, M, M) = p(R - P_h(S, M, M)) + p^2(R - P_f(S, M, M)) - 2p^2(R - P(B, M, M)) \]

\[ = p^2(P(B, M, M) - P_f(S, M, M)) + p(P(B, M, M) - P_h(S, M, M)) \]

\[ + p(1 - p)(R - P(B, M, M)). \quad (A.1) \]

Notice in particular that this expression is positive when \( P(B, M, M) = P_h(S, M, M) = P_f(S, M, M) \): for the same interest rates, the subsidiary structure is more profitable than the branch.

As \( P_f(S, M, M) \) enters positively in \( P_h(S, M, M) \), it is clear that \( \Pi(S, M, M) - \Pi(B, M, M) \) decreases in \( \alpha_f \): all else equal, a more credible foreign deposit insurance makes the subsidiary structure more profitable.

Using equations (2) and (3), we can replace \( P_h(S, M, M) \) and \( P(B, M, M) \) by their actual values, and after simplification we obtain:

\[ \Pi(S, M, M) - \Pi(B, M, M) = (1 - p)[\alpha_f(1 - L)(1 - \alpha_h(1 - p)) - \alpha_h p(2 - R - L)]. \quad (A.2) \]

When \( \alpha_f = 0 \), we have \( \Pi(S, M, M) - \Pi(B, M, M) = -\alpha_h p(2 - R - L) < 0 \), using Assumption H2.

When \( \alpha_f = 1 \), we have \( \Pi(S, M, M) - \Pi(S, M, M) = (1 - L)(1 - \alpha_h) + \alpha_h p(R - 1) > 0 \). Since \( \Pi(S, M, M) - \Pi(S, M, M) \) is increasing in \( \alpha_f \), there exists a unique value \( \hat{\alpha}_f \in (0, 1) \) such that the difference is positive for \( \alpha_f \geq \hat{\alpha}_f \) and negative otherwise.

A.3 Proof of Proposition 2

We will show that the equilibrium decisions of both supervisors are as follows:

- If foreign monitoring costs are high, i.e., \( c_f \geq \alpha_f(1 - p)L \), then \( d_f^* = O \) and:
  
  - If the liquidation value is small, \( L \leq \lambda_1 \): then \( d_h^* = M \) if \( c_h \leq \kappa_1 \), and \( d_h^* = O \) otherwise.
  
  - If the liquidation value is large, \( L > \lambda_1 \): then \( d_h^* = M \) if \( c_h \leq \kappa_1 - \kappa_2 \), and \( d_h^* = I \) otherwise.

- If foreign monitoring costs are low, i.e., \( c_f < \alpha_f(1 - p)L \): then \( d_f^* = M \) and:
- If the liquidation value is small, $L \leq \lambda_2$: then $d_h^* = M$ if $c_h \leq \kappa_1$, and $d_h^* = O$ otherwise.

- If the liquidation value is large, $L > \lambda_2$: then $d_h^* = M$ if $c_h \leq \kappa_1 - \kappa_3$, and $d_h^* = C$ otherwise.

where the values of $\lambda_1, \lambda_2, \kappa_1, \kappa_2,$ and $\kappa_3$ are as follows:

\[
\lambda_1 = p^2(2 - R) + p(1 - p)(2 - \alpha_f) \quad (A.3)
\]
\[
\lambda_2 = \frac{p(2 - R) + (1 - p)(1 - \alpha_f)}{1 + (1 - p)(1 - \alpha_f)} \quad (A.4)
\]
\[
\kappa_1 = \alpha_h(1 - p)L \quad (A.5)
\]
\[
\kappa_2 = \alpha_h(L - \lambda_1) \quad (A.6)
\]
\[
\kappa_3 = p\alpha_h[1 + (1 - p)(1 - \alpha_f)](L - \lambda_2) \quad (A.7)
\]

**Proof.** As shown in the text, $d_f^* = O$ or $M$ depending on whether $c_f$ is higher than $\alpha_f(1 - p)L$. It remains to compute the best response of the home supervisor in each case.

- If $d_f^* = O$. The home supervisor can choose between $M, I,$ and $O$. We have:

\[
W_h(M, O) - W_h(O, O) = p(1 - p)[w_h(l, s) - w_h(f, s)] + (1 - p)^2[w_h(l, f) - w_h(f, f)] - c_h
\]
\[
= \alpha_h(1 - p)L - c_h \quad (A.8)
\]
\[
W_h(O, O) - W_h(I, O) = p[pw_h(s, s) + (1 - p)w_h(f, s) - w_h(l, s)]
\]
\[
+ (1 - p)[pw_h(s, f) + (1 - p)w_h(f, f) - w_h(l, f)]
\]
\[
= \alpha_h[p - L - p^2(R - P_f(S, O))] \quad (A.9)
\]
\[
W_h(M, O) - W_h(I, O) = p^2[w_h(s, s) - w_h(l, s)] + p(1 - p)[w_h(s, f) - w_h(l, f)] - c_h
\]
\[
= p\alpha_h[1 - L - p(R - P_f(S, O))] - c_h \quad (A.10)
\]

Using that $pP_f(S, O) = 1 - (1 - p)\alpha_f$, these equations yield the first part of the proposition.

- If $d_f^* = M$. The home supervisor can choose between $M, I, O,$ and $C$. However, $(I, M)$ is always dominated by $(C, M)$: it is straightforward to compute that $W_h(C, M) > W_h(I, M)$ is equivalent to $p > L$, which is an implication of assumptions $H1$ and $H2$. Thus, we do not have to consider
strategy \( I \). For the remaining ones, we have:

\[
W_h(M, M) - W_h(O, M) = p(1 - p)[w_h(l, s) - w_h(f, s)] + (1 - p)^2[w_h(l, l) - w_h(f, l)] - c_h
\]

\[
= \alpha_h(1 - p)L - c_h
\]\n
\[
(A.11)
\]

\[
W_h(O, M) - W_h(C, M) = p[pw_h(s, s) + (1 - p)w_h(f, s) - w_h(l, s)]
\]

\[
= \alpha_h[p - L - p(R - P_f(S, M))]
\]\n
\[
(A.12)
\]

\[
W_h(M, M) - W_h(C, M) = p^2[w_h(s, s) - w_h(l, s)] + (1 - p)^2[w_h(l, l) - w_h(f, l)] - c_h
\]

\[
= \alpha_h[(1 - 2p)L + p^2(1 + P_f(S, M) - R)] - c_h
\]\n
\[
(A.13)
\]

Using that \( pP_f(S, M) = 1 - (1 - p)[\alpha_f + (1 - \alpha_f)L] \), these equations yield the second part of the proposition, which concludes the proof.

\[\blacksquare\]

**A.4 Proof of Corollary 1**

For the reasons stated in the text, more expected foreign residual assets increase the payoff of intervention for the home supervisor relatively more than with other decisions, so that

\[
\max\{W_h(I, M), W_h(O, M)\} - \max\{W_h(I, O), W_h(O, O)\} \geq W_h(M, M) - W_h(M, O)
\]\n
\[
(A.14)
\]

The left hand side is the increase in the home supervisor’s payoff due to foreign monitoring when he either intervenes or keeps the home unit open. The right hand side is the necessarily lower increase in payoff when the home supervisor monitors. The condition can be rewritten as

\[
W_h(M, O) - \max\{W_h(I, O), W_h(O, O)\} \geq W_h(M, M) - \max\{W_h(I, M), W_h(O, M)\}
\]

\[
(A.15)
\]

In particular, this equation implies that the following, weaker conditions holds:

\[
W_h(M, O) - \max\{W_h(I, O), W_h(O, O)\} \geq W_h(M, M) - \max\{W_h(C, M), W_h(I, M), W_h(O, M)\}
\]

\[
(A.16)
\]

This condition means that foreign monitoring reduces the value of monitoring for the home supervisor, and proves result (i).

As for result (ii), it suffices to show that if \( \alpha_h = \alpha_f \) and \( c_h = c_f \), then if \( d^*_h = M \) we necessarily have \( d^*_f = M \). The proof of this is direct from the proof of Proposition 2.

\[\blacksquare\]
A.5 Proof of Corollary 2

According to the proof of Proposition 2, we have \( d^*_h = M \) if and only if \( c_h \leq \min(\kappa_1, \kappa_1 - \kappa_2) \) when \( c_f \geq \alpha_f(1 - p)L \), and if and only if \( c_h \leq \min(\kappa_1, \kappa_1 - \kappa_3) \) when \( c_f < \alpha_f(1 - p)L \). Using equations (A.3), (A.4), (A.6), and (A.7), it is clear that both \( \min(\kappa_1, \kappa_1 - \kappa_2) \) and \( \min(\kappa_1, \kappa_1 - \kappa_3) \) are decreasing in \( \alpha_f \).

A.6 Proof of Proposition 3

If \( c_f \leq c_h \), denote \( c_{low} = c_f \), \( d^b_{low} = d^b_f \), \( c_{high} = c_h \), \( d^b_{high} = d^b_h \), and symmetrically if \( c_h < c_f \). We will prove that the branch supervisor’s optimal decision is:

- If the liquidation value is small, \( L < \lambda_3 \), \( (d^b_{low}, d^b_{high}) \) is equal to \( (M, M) \) if \( c_{high} \leq \kappa_1 \), \( (O, M) \) if \( c_{low} \leq \kappa_1 \), and \( (O, O) \) if \( c_{low} > \kappa_1 \);
- If the liquidation value is intermediate, \( L \in [\lambda_3, \lambda_4] \), \( (d^b_{low}, d^b_{high}) \) is equal to \( (M, M) \) if \( c_{high} > \kappa_1 - \kappa_4 \) and \( c_{low} \leq \kappa_4 + \kappa_5 \), and \( (O, O) \) if \( c_{low} > \kappa_4 + \kappa_5 \);
- If the liquidation value is high, \( L > \lambda_4 \), \( (d^b_{low}, d^b_{high}) \) is equal to \( (M, M) \) if \( c_{high} \leq \kappa_1 - \kappa_4 \), \( (C, M) \) if \( c_{high} > \kappa_1 - \kappa_4 \) and \( c_{low} \leq \kappa_4 + \kappa_5 \), and \( (I, O) \) if \( c_{low} > \kappa_4 + \kappa_5 \);

where the values of \( \lambda_3, \lambda_4, \kappa_4, \) and \( \kappa_5 \) are as follows:

\[
\begin{align*}
\lambda_3 &= p(2 - R) \quad \text{(A.17)} \\
\lambda_4 &= pR(1 - p) + p^2(2 - R) \quad \text{(A.18)} \\
\kappa_4 &= p\alpha_h(L - \lambda_3) \quad \text{(A.19)} \\
\kappa_5 &= \alpha_h(\lambda_4 - pL). \quad \text{(A.20)}
\end{align*}
\]

Proof. We consider the case \( c_f \leq c_h \), the other case being symmetric. Observe that strategies \( (I, I) \) and \( (I, M) \) are dominated by \( (I, O) \) and \( (C, M) \), respectively: leaving one unit open brings \( pR \), while closing it yields \( L \). Since \( pR > L \), the result follows.

When no unit is monitored, we need to compare \( (I, O) \) and \( (O, O) \), which gives:

\[
W_b(O, O) - W_b(I, O) = p[pw_b(s, s) + (1 - p)w_b(s, f) - w_b(l, s)] + (1 - p)[pw_b(s, f) + (1 - p)w_b(f, f) - w_b(l, f)] = \alpha_h[pR(1 - p) + p^2(2 - R) - L]. \quad \text{(A.21)}
\]
When only one unit is monitored, since \( c_f \leq c_h \), it is unit \( f \). Thus, we need to compare \((O, M)\) and \((C, M)\):

\[
W_b(O, M) - W_b(C, M) = p[pw_b(s, s) + (1-p)w_b(f, s) - w_b(l, s)] = \alpha_h p[p(2 - R) - L]
\]

We have thus shown that \((O, M)\) strictly dominates \((C, M)\) if and only if \( L < \lambda_3 \), and \((O, O)\) strictly dominates \((I, O)\) if and only if \( L < \lambda_4 \). Since \( \lambda_4 > \lambda_3 \), we have three cases to consider:

- \( L \leq \lambda_3 \), so that \((C, M)\) and \((I, O)\) are dominated, and \((d^h_h, d^f_f) \in \{(O, O), (O, M), (M, M)\}\).

We have the following comparisons:

\[
W_b(M, M) - W_b(O, M) = p(1-p)[w_b(l, s) - w_b(f, s)] + (1-p)^2[w_b(l, l) - w_b(f, f)] - c_h
\]

\[
= \kappa_1 - c_h \tag{A.23}
\]

\[
W_b(O, M) - W_b(O, O) = p(1-p)[w_b(s, l) - w_b(s, f)] + (1-p)^2[w_b(f, l) - w_b(f, f)] - c_f
\]

\[
= \kappa_1 - c_f \tag{A.24}
\]

\[
W_b(M, M) - W_b(O, O) = p(1-p)[2w_b(s, l) - w_b(s, f)] + (1-p)^2[w_b(l, l) - w_b(f, f)] - c_f - c_h
\]

\[
= 2\kappa_1 - c_h - c_f \tag{A.25}
\]

These comparisons give us the first part of the proposition: since \( c_h \geq c_f \), \( c_h \leq \kappa_1 \Rightarrow c_h + c_f \leq 2\kappa_1 \), and \( c_f \geq \kappa_1 \Rightarrow c_h + c_f \geq 2\kappa_1 \).

- \( L \in [\lambda_3, \lambda_4] \), so that \((O, M)\) and \((I, O)\) are dominated, and \((d^h_h, d^f_f) \in \{(O, O), (C, M), (M, M)\}\).

We have to introduce two new comparisons:

\[
W_b(M, M) - W_b(C, M) = p^2[w_b(s, s) - w_b(l, s)] + (1-p)^2[w_b(l, l) - w_b(f, l)] - c_h
\]

\[
= \kappa_1 - \kappa_4 - c_h \tag{A.26}
\]

\[
W_b(C, M) - W_b(O, O) = p[w_b(l, s) - pw_b(s, s) - (1-p)w_b(f, s)] + p(1-p)[w_b(s, l) - w_b(s, f)]
\]

\[
+ (1-p)^2[w_b(f, l) - w_b(f, f)] - c_f = \kappa_1 + \kappa_4 - c_f. \tag{A.27}
\]

\((M, M)\) is optimal when \( c_h \leq \kappa_1 - \kappa_4 \) and \( c_h + c_f \leq 2\kappa_1 \) (equation \((A.25)\)), but clearly the former condition implies the latter since \( c_f \leq c_h \). Conversely, if \( c_f \geq \kappa_1 + \kappa_4 \) then we also have \( c_f + c_h \geq 2\kappa_1 \),
so that \( c_f \geq \kappa_1 + \kappa_4 \) is a necessary and sufficient condition to have \((C, M)\). This proves the second part of the proposition.

- \( L > \lambda_4 \), so that \((O, O)\) and \((O, M)\) are dominated, and \((d^b_h, d^b_f) \in \{(I, O), (C, M), (M, M)\}\).

We introduce two additional comparisons:

\[
W_b(C, M) - W_b(I, O) = (1 - p)[pw_b(s, l) + (1 - p)w_b(f, l) - w_b(l, f)] - c_f
= \kappa_4 + \kappa_5 - c_f
\]

\[
W_b(M, M) - W_b(I, O) = p^2[w_b(s, s) - w_b(l, s)] + (1 - p)[pw_b(s, l) + (1 - p)w_b(l, l) - w_b(l, f)] - c_h - c_f
= \kappa_1 + \kappa_5 - c_h - c_f
\]

For \((M, M)\) to be optimal we need both \( c_h \leq \kappa_1 - \kappa_4 \) and \( c_h + c_f \leq \kappa_1 + \kappa_5 \). Direct computation shows that \( 2(\kappa_1 - \kappa_4) \leq \kappa_1 + \kappa_5 \) is equivalent to \((1 - p)L \leq p(L - \lambda_3) + pR(1 - p)\), which is true as \( pR \geq L \). Hence \( c_h \leq \kappa_1 - \kappa_4 \) and \( c_f \leq c_h \) imply \( c_h + c_f \leq \kappa_1 + \kappa_5 \). Conversely, in order to have \((I, O)\) we need \( c_h + c_f \geq \kappa_1 + \kappa_5 \) and \( c_f \geq \kappa_4 + \kappa_5 \). Since \( c_h \geq c_f \), \( c_f \geq \kappa_4 + \kappa_5 \) implies that \( c_h + c_f \geq 2(\kappa_4 + \kappa_5) \), which is higher than \( \kappa_1 + \kappa_5 \) as the previous comparison has shown. This proves the third part of the proposition.

\section*{A.7 Proof of Corollary 3}

Point (i) is obvious from Proposition 3. For point (ii), we consider the three different cases in the proof of the proposition. When \( L < \lambda_3 \), the value of monitoring the first unit is \( W_b(O, M) - W(O, O) + c_f = \kappa_1 \), and the value of monitoring the second unit is \( W_b(M, M) - W_b(O, M) + c_h = \kappa_1 \), so that both values are equal. When \( L \in [\lambda_3, \lambda_4] \), the value of monitoring the first unit is \( W_b(C, M) - W_b(O, O) + c_f = \kappa_1 + \kappa_4 \), and the value of monitoring the second unit is \( W_b(M, M) - W_b(C, M) + c_h = \kappa_1 - \kappa_4 \). As \( \kappa_4 > 0 \), we obtain the desired result. Finally, when \( L > \lambda_4 \), the value of monitoring the first unit is \( W_b(C, M) - W_b(I, O) + c_f = \kappa_4 + \kappa_5 \), against \( W_b(M, M) - W_b(C, M) + c_h = \kappa_1 - \kappa_4 \) for the second unit. We showed in the previous proof that \( 2(\kappa_1 - \kappa_4) \leq \kappa_1 + \kappa_5 \), which is equivalent to \( \kappa_1 - \kappa_4 \leq \kappa_4 + \kappa_5 \), giving the desired result.

\section*{A.8 Proof of Lemma 1}

By contradiction, assume this is not the case and we have \( d^*_h = d^*_f \) with \( d^*_h \neq d^*_f \). The supranational supervisor chooses a pair of decisions. In particular, since the pair \((d^*_h, d^*_f)\) is optimal, we must
have
\[ W_h(d_h^*, d_f^*) + W_f(d_f^*) \geq W_h(d_h^*, d_f^{**}) + W_f(d_f^{**}), \tag{A.30} \]
but since \( d_h^* \) is optimal for the home supervisor in the national case, it must be a best response to \( d_f^* = d_f^{**} \), and in particular we must have
\[ W_h(d_h^*, d_f^*) \geq W_h(d_h^*, d_f^{**}). \tag{A.31} \]
Both inequalities cannot hold unless \( d_h^* = d_h^{**} \), a contradiction.\(^{26}\)

### A.9 Proof of Proposition 4

Lemma 1 allows us to focus on identifying cases in which supranational supervision leads to a different decision in the foreign unit. As \( p > L \), the decision in the foreign unit is either \( M \) or \( O \), so that we have either \( d_f^* = O \) with \( d_f^{**} = M \) or \( d_f^* = M \) with \( d_f^{**} = O \). We first show that only the former is possible:

**Lemma 3.** Supranational supervision with a subsidiary represented MNB leads to more monitoring in the foreign unit: if \( d_f^* \neq d_f^{**} \), then necessarily \( d_f^* = O, d_f^{**} = M \).

**Proof:** By contradiction, assume the only other possible case, which is \( d_f^* = M, d_f^{**} = O \). Note that the interest rate \( P_f \) now depends on the decision on supervision, and is either \( P_f(S, M) \) if the foreign unit is monitored, or \( P_f(S, O) \) when it is left open.\(^{27}\) Denote \( W_h(d_h, d_f, P_f) \) the payoff to the home deposit insurer (\( W_f \) does not depend on \( P_f \)). Since \( d_f^* = M \), we must have \( W_f(M) \geq W_f(O) \).

Regarding the supranational supervisor, a necessary condition for \( O \) to be optimal is to have:
\[ W_h(d_h^{**}, O, P_f(S, O)) + W_f(O) \geq W_h(d_h^{**}, M, P_f(S, O)) + W_f(M). \tag{A.32} \]
Simplifying and rearranging, we thus have:
\[ 0 \leq W_f(M) - W_f(O) \leq W_h(d_h^{**}, O, P_f(S, O)) - W_h(d_h^{**}, M, P_f(S, O)) \tag{A.33} \]

\(^{26}\)Implicitly, the proof assumes that interest rates are the same under both scenarios, which may not be the case. Note that \( P_f \) only depends on \( d_f \), so that \( P_f \) is indeed equal under both types of supervision when \( d_f^{**} = d_f^* \). \( P_h \) might be different, but it can easily be checked that this quantity plays no role in \( W_h \) and \( W_f \).

\(^{27}\)Notice that deposit rates are determined before supervisors take their decisions. This implies that for a pair of decisions \((d_h^{**}, d_f^{**})\) to be optimal for the supranational supervisor, it must be that the other decisions are dominated considering, for these decisions, the same deposit rate that would emerge with \((d_h^*, d_f^*)\).
Neglecting the borderline case in which \( c_f = \alpha_f (1 - p) L \), the term on the right hand side must be strictly positive. However, since \( d_i^* = O \) we cannot have \( d_i^{**} = C \), and for any other \( d_i^{**} \) the term in the right hand side is always null, a contradiction.

We can now prove the proposition. More specifically, we will prove that the three cases in which decisions under national and supranational supervision differ are as follows:

- \((d_h^v, d_f^v) = (O, O)\) and \((d_h^{**}, d_f^{**}) = (C, M)\). This case obtains for \( L \in [\lambda_2, \lambda_1] \), \( c_h \geq \kappa_1 \), and \( c_f \in [\alpha_f (1 - p) L, \alpha_f (1 - p) L + \kappa_3] \).

- \((d_h^v, d_f^v) = (I, O)\) and \((d_h^{**}, d_f^{**}) = (C, M)\). This case obtains for \( L > \lambda_1 \), \( c_h \geq \kappa_1 - \kappa_2 \), and \( c_f \in [\alpha_f (1 - p) L, \alpha_f (1 - p) L + (1 - p) \alpha_h (p - L)] \).

- \((d_h^v, d_f^v) = (M, O)\) and \((d_h^{**}, d_f^{**}) = (C, M)\). This case obtains for \( L > \lambda_2 \), \( c_h \leq \min(\kappa_1 - \kappa_2, \kappa_1) \), \( c_f \geq \alpha_f (1 - p) L \), and \( c_f - c_h \leq \alpha_f (1 - p) L + (\kappa_3 - \kappa_1) \).

Assume that \((d_h^{**, v}, d_f^{**, v}) \neq (d_h^v, d_f^v)\). We deduce from Lemmas 1 and 3 that \( d_f^* = O \) and \( d_i^{**} = M \). Let us show that \( d_h^{**} = C \). By contradiction, assume this is not the case. As \((d_h^{**}, M)\) is optimal for the supranational supervisor, it must in particular be better than \((d_h^{**}, O)\), which writes as:

\[
W_h(d_h^{**}, M) + W_f(M) \geq W_h(d_h^{**}, O) + W_f(O)
\]

As already shown in the proof of Lemma 3, for \( d \neq C \) we have \( W_h(d, M) = W_h(d, O) \), so that the inequality cannot hold.

Finally, we need to determine \( d_h^v \), which can be \( M, O \), or \( I \). All three cases are possible, and we derive a full characterization of each case:

- \((d_h^v, d_f^v) = (O, O)\) and \((d_h^{**}, d_f^{**}) = (C, M)\): By Proposition 2 we know that in order to obtain \((d_h^v, d_f^v) = (O, O)\) we need \( L \leq \lambda_1 \), \( c_h \geq \kappa_1 \) and \( c_f \geq \alpha_f (1 - p) L \). By Lemma 1, these same conditions imply that \((O, O)\) dominates all other alternatives with \( d_f = O \) for the supranational supervisor. Moreover, we have just shown that \((C, M)\) dominates all alternatives with \( d_f = M \). Thus, in order to ensure that \((d_h^{**}, d_f^{**}) = (C, M)\), we simply need to check that the supranational supervisor prefers \((C, M)\) to \((O, O)\), which writes as:

\[
W_h(C, M) + W_f(M) - W_h(O, O) - W_f(O) - c_f \geq 0
\]

\[
\Leftrightarrow \alpha_f (1 - p) L + \kappa_3 \geq c_f
\]  

(A.35)
Note that in order to have \( W_h(C, M) + W_f(M) - W_h(O, O) - W_f(O) \geq 0 \) and \( c_f \geq \alpha_f(1-p)L \), we need \( \kappa_3 \geq 0 \), which is equivalent to \( L \geq \lambda_2 \). Computations show that \( \lambda_2 \leq \lambda_1 \) is equivalent to \( \alpha_f^2 p(1-p) + (1 - \alpha_f) + pR(1-p(1-\alpha_f)) + 2(1 - \alpha_fp(1-p)) \geq 0 \), which is true (sum of positive terms). This shows the first part of the proposition.

- \((d_h^*, d_f^*) = (I, O)\) and \((d_h^*, d_f^*) = (C, M)\): The reasoning is similar. In order to have \((d_h^*, d_f^*) = (I, O)\), we need \( L > \lambda_1 \), \( c_h \geq \kappa_1 - \kappa_2 \) and \( c_f \geq \alpha_f(1-p)L \). We just need to compare the supranational supervisor’s payoff with \((C, M)\) and \((I, O)\):

\[
W_h(C, M) + W_f(M) - W_h(I, O) - W_f(O) - c_f \geq 0
\]

\[
\Leftrightarrow (1-p)[(1-p)w_h(f, l) - w_h(l, f)] + W_f(M) - W_f(O) - c_f \geq 0
\]

\[
\Leftrightarrow \alpha_f(1-p)L + \alpha_h(1-p)(p-L) \geq c_f,
\]

(A.36)

from which we deduce the second part of the proposition.

- \((d_h^*, d_f^*) = (M, O)\) and \((d_h^*, d_f^*) = (C, M)\): \((d_h^*, d_f^*) = (M, O)\) is obtained for \( c_f \geq \alpha_f(1-p)L \) and \( c_h \leq \min(\kappa_1 - \kappa_2, \kappa_1) \). \((C, M)\) is preferred to \((M, O)\) by the supranational supervisor depending on the sign of:

\[
W_h(C, M) + W_f(M) - W_h(M, O) - W_f(O) + c_h - c_f
\]

\[
= p^2 w_h(l, s) + (1-p)^2[w_h(f, l) - w_h(l, f)] + W_f(M) - W_f(O) + c_h - c_f
\]

\[
= \alpha_f(1-p)L + \kappa_3 - \kappa_1 + c_h - c_f
\]

(A.37)

Notice in particular that we must have \( c_f \in [\alpha_f(1-p)L, \alpha_f(1-p)L + \kappa_3] \) so that \( \kappa_3 \) needs to be positive, hence \( L > \lambda_2 \).

**A.10 Proof of Corollary 4**

Using Proposition 4, if a subsidiary is active under national supervision (that is, we do not have \((d_h^*, d_f^*) = (I, O)\)), then either supranational supervision does not change the subsidiary’s profit, or we have a change from \( \Pi(S, O, O) \) or \( \Pi(S, M, O) \) to \( \Pi(S, C, M) \). We first prove that \( \Pi(S, C, M) \) is lower than the other two expressions. For \((d_h, d_f)\) equal to either \((O, O)\) or \((M, O)\), we have:

\[
\Pi(S, d_h, d_f) = p^2(2R - P_h(S, d_h, d_f) - P_f(S, d_f)) + p(1-p)(R - P_h(S, d_h, d_f))
\]

(A.38)
and for \((C, M)\),\(^{28}\)

\[
\Pi(S, C, M) = p(1 - p)(R - P_h(S, C, M)). \tag{A.39}
\]

It is easy to prove that \(P_h(S, M, O) \leq P_h(S, O, O)\), so that \(\Pi(S, O, O) \leq \Pi(S, M, O)\). Hence, it is enough to show that \(\Pi(S, C, M) \leq \Pi(S, O, O)\). We first compute the deposit rates \(P_h(S, O, O)\) and \(P_h(S, C, M)\), which are given by:

\[
pP_h(S, O, O) + (1 - p)[\alpha_h + (1 - \alpha_h)p[R - P_f(S, O)]] = 1. \tag{A.40}
\]

\[
p[\alpha_h + (1 - \alpha_h)[R + L - P_f(S, M)] + (1 - p)pP_h(S, C, M) + (1 - p)p_h] = 1. \tag{A.41}
\]

Replacing \(P_f(S, O)\) and \(P_f(S, M)\) by their expressions, we obtain:

\[
P_h(S, O, O) = \frac{1 - (1 - p)[\alpha_h + (1 - \alpha_h)[pR - 1 + (1 - p)p_f]]}{p} \tag{A.42}
\]

\[
P_h(S, C, M) = \frac{(1 - \alpha_h)(2 - pR - L) - \alpha_f(1 - \alpha_h)(1 - L)(1 - p) + \alpha_h(1 - p)}{(1 - p)p} \tag{A.43}
\]

Finally, we substitute these quantities into (A.38) and (A.39) to obtain:

\[
\Pi(S, O, O) - \Pi(S, C, M) = (1 - \alpha_h)L[(1-p)\alpha_f - 1] + p[(1 + \alpha_h)pR - \alpha_h(1 + p - (1 - p)\alpha_f)]. \tag{A.44}
\]

This quantity is increasing in \(\alpha_f\) and \(R\), and decreasing in \(L\). Hence, using assumptions \(H1\) and \(H2\), it is higher than the quantity we obtain for \(\alpha_f = 0, R = 1/p, L = 2 - R:\)

\[
\Pi(S, O, O) - \Pi(S, C, M) \geq p[1 + \alpha_h(1 - p)] - [(1 - \alpha_h)(2 - (1/p)) + \alpha_h p]. \tag{A.45}
\]

This last quantity can be reexpressed as \(\alpha_h \frac{1 - p}{p}[(1 - \alpha_h)(1 - p) + \alpha_hp^2]\), so that \(\Pi(S, O, O) - \Pi(S, C, M) \geq 0\). This concludes the proof that a subsidiary-MNB’s profits are always negatively impacted by supranational supervision.

\section*{A.11 Proof of Lemma 2}

The proof can be found in the Online Appendix.\(^{28}\)

\(^{28}\)Notice that in this case the home unit will be kept open only if the foreign unit is discovered to be failing.
A.12 Proof of Proposition 5

We will make use of the following quantities, which complete the values of $P_h, P_f$ given in the previous proofs:

\[ P_h(A, O) = \frac{1 - (1 - p)\alpha_h}{p}, \quad (A.46) \]
\[ P_h(B, O, O) = \frac{1 - 2p(1 - p)[\alpha_h + (1 - \alpha_h)R/2] - (1 - p)^2\alpha_h}{p^2}, \quad (A.47) \]

Case (i). We need to prove that the MNB finds it optimal to adopt a subsidiary structure under national supervision, $\Pi(S, O, O) \geq \max(\Pi(B, O, O), \Pi(A, O))$, and a branch structure under supranational supervision, $\Pi(B, O, O) \geq \max(\Pi(S, C, M), \Pi(A, O))$.

It is immediate that $\Pi(S, O, O) \geq \Pi(A, O)$: the subsidiary structure is composed of the same home unit, plus a foreign unit that can only bring additional profit. In order to compare $\Pi(S, O, O)$ and $\Pi(B, O, O)$, we can simply follow the proof of Proposition 1 and replace $L$ by 0. Applying this method to equation (A.2) gives us:

\[ \Pi(S, O, O) - \Pi(B, O, O) = (1 - p)[\alpha_f(1 - \alpha_h(1 - p)) - \alpha_h(p(2 - R))] \quad (A.48) \]

Using that case (i) requires $\alpha_f > \alpha_h$ (Lemma 2), we have:

\[ \Pi(S, O, O) - \Pi(B, O, O) > (1 - p)\alpha_h[1 - \alpha_h(1 - p) - \alpha_hp(2 - R)]. \quad (A.49) \]

Since $2 - R < 1$ by Assumption H2, the term in brackets is larger than $1 - \alpha_h(1 - p) - \alpha_hp = 0$, which proves that $\Pi(S, O, O) > \Pi(B, O, O)$ and thus that the MNB chooses a subsidiary structure under national supervision.

Direct computations give:

\[ \Pi(B, O, O) - \Pi(A, O) > 0 \iff (1 - \alpha_h)(1 - p)(pR - 1) > 0, \quad (A.50) \]

which is true by Assumption H1.

To conclude, we can show that $\Pi(A, O) \geq \Pi(S, C, M)$. Direct computation shows that:

\[ \Pi(A, O) - \Pi(S, C, M) = (1 - L)(1 - \alpha_f(1 - p)(1 - \alpha_h)) - \alpha_h(1 - L + p^2) - pR(1 - p - \alpha_h). \quad (A.51) \]
This quantity is decreasing in $\alpha_f$ and in $L$. Hence, it is higher than what we obtain with $\alpha_f = 1, L = 2 - R$:

$$\Pi(A, O) - \Pi(S, C, M) \geq p^2(R - 1) - p(1 - p)(1 - \alpha_h). \quad \text{(A.52)}$$

This last quantity is increasing in $\alpha_h$ and thus higher than $p[pR - 1]$, which we obtain with $\alpha_h = 0$.

Using assumption H1, this shows that $\Pi(A, O) - \Pi(S, C, M) \geq 0$.

Case (ii). As the branch is not viable, we need to prove that the MNB prefers the subsidiary structure under national supervision and the stand-alone structure under supranational supervision. This is equivalent to $\Pi(S, O, O) \geq \Pi(A, O) \geq \Pi(S, C, M)$, and both inequalities have already been shown (importantly, the proof of $\Pi(A, O) \geq \Pi(S, C, M)$ did not use $\alpha_f > \alpha_h$).

Case (iii). This is very similar to case (ii), in that we need to prove $\Pi(S, M, O) \geq \Pi(A, M) \geq \Pi(S, C, M)$. Proving the first inequality is immediate. It is also easy to show that $\Pi(A, M) \geq \Pi(A, O)$. Since $\Pi(A, O) \geq \Pi(S, C, M)$, this implies that $\Pi(A, M) \geq \Pi(S, C, M)$.

### A.13 Proof of Corollary 5

**Symmetric countries:** This is a direct consequence of the last statement in Lemma 2.

Robust home deposit insurance: We use the different cases (i)-(iii) defined in Table 2. We need to prove that if we take two values $\alpha_{h1}$ and $\alpha_{h2}$, with $\alpha_{h1} < \alpha_{h2}$, then we can have case (i) with $\alpha_h = \alpha_{h1}$ and case (ii) with $\alpha_h = \alpha_{h2}$, but not the opposite. Both cases (i) and (ii) are characterized by $(d_h^*, d_f^*) = (O, O)$ and $(d_h^{**}, d_f^{**}) = (C, M)$, and the only difference between these two cases comes from the decisions with the branch-MNB. If parameters are such that case (ii) obtains, we need to have either $(C, M)$ or $(I, O)$ in the branch-MNB. Using the proof of Proposition 3, there are two cases to consider for these decisions. If $L > \lambda_4$ (notice that $\lambda_4$ does not depend on $\alpha_h$), then case (i) can never obtain: a decrease in $\alpha_h$ will eventually lead to centralized supervision having no impact, but not to case (i). If $L \in [\lambda_3, \lambda_4]$, then case (ii) obtains for $c_{\text{high}} > \kappa_1 - \kappa_4$ and $c_{\text{low}} \leq \kappa_1 + \kappa_4$, while case (i) obtains for $c_{\text{low}} > \kappa_1 + \kappa_4$. Since $\kappa_1 + \kappa_4$ is increasing in $\alpha_h$, it is impossible to find $\alpha_{h1}$ and $\alpha_{h2}$ with $\alpha_{h1} < \alpha_{h2}$ such that case (ii) obtains with $\alpha_{h1}$ and case (i) obtains with $\alpha_{h2}$. The same reasoning also applies to the comparison of case (i) and case (iii). Notice that since the decision in the branch case does not depend on $\alpha_f$, this parameter has no impact on whether we are in case (i) or in case (ii).
A.14 Proof of Corollaries 6 to 9

These corollaries are proven directly in the main text.

A.15 Proof of Proposition 6

The first point of the proposition is straightforward.

As for the second point, denote by \((d_h^n, d_f^n)\) the optimal decision with supranational supervision and national deposit insurance, and by \((d_h^c, d_f^c)\) the optimal decision with supranational supervision and common deposit insurance. Both with and without common deposit insurance, we know that the optimal decision belongs to the set \{\((O, O), (I, O), (M, O), (O, M), (M, M), (C, M)\)\}. We want to show that it is impossible to have \(d_f^n = M\) and \(d_f^c \neq M\), that is, \(d_f^c = O\). By contradiction, assume that this is the case. Then, there are two cases to consider:

- \((d_{hn}^n, d_{fn}^n) \neq (C, M)\): since both \((d_{hn}^n, d_{fn}^n)\) and \((d_{hc}^c, d_{fc}^c)\) are different from \((C, M)\), we know that the same decisions would have been reached under national supervision (Proposition 4). In order for \(d_f^c = O\) to be optimal for the foreign supervisor under national supervision, we need to have \(\alpha_c (1 - p) \leq c_f\). However, in order for \(d_f^n\) to be optimal under national supervision, we need \(\alpha_f (1 - p) \geq c_f\). Since \(\alpha_c > \alpha_f\), this is a contradiction.

- \((d_{hn}^n, d_{fn}^n) = (C, M)\): it needs to be the case that \((C, M)\) is preferred to \((d_{hc}^c, O)\) under national deposit insurance, whereas the opposite is true under common deposit insurance. The proof of Proposition 4 contains all the pairwise comparisons between \((C, M)\) and \((I, O), (O, O),\) and \((M, O)\) from the perspective of a supranational supervisor. It can be checked that in each case an increase in \(\alpha_f\) improves the payoff of \((C, M)\) over the alternative, which shows that adding common deposit insurance cannot lead the supranational supervisor to prefer any \((d_{hc}^c, O)\) over \((C, M)\).

The reasoning for the third point is similar. By contradiction, assume that we have \(d_{hc}^c = M\) and \(d_{hn}^n \in \{C, I, O\}\). Again, there are two cases:

- \(d_{hn}^n = d_{fn}^n\): it must be the case that \(W_h(d_{hn}^n, d_{fn}^n) = W_h(d_{hc}^c, d_{fc}^c) > W_h(M, d_{fn}^n)\) under national deposit insurance. Conversely, we must have \(W_h(M, d_{fn}^n) > W_h(d_{hc}^c, d_{fc}^c)\) under common deposit insurance. Since \(d_{fn}^n = d_{fn}^n\), the only difference between these two cases is that residual assets in the foreign unit are higher under common deposit insurance. We know from Corollary 1 that this reduces monitoring incentives in the home unit, so that we cannot have \(d_{hc}^c = M\) and \(d_{hn}^n \neq M\).

- \(d_{hn}^n \neq d_{fn}^n\): given the second point, it must be that \(d_{hn}^n = O, d_{fn}^n = M\). So there are only two possibilities: (i) \((d_{hn}^n, d_{fn}^n) = (I, O), (d_{hc}^c, d_{fc}^c) = (M, M)\), or (ii) \((d_{hn}^n, d_{fn}^n) = (O, O), (d_{hc}^c, d_{fc}^c) = (M, M)\). In case (i), we need in particular to have \(W_h(I, O) \geq W_h(M, O)\) with national deposit insurance,
and $W_h(I, M) \leq W_h(M, M)$ with common deposit insurance. These two conditions are equivalent to:

\[
\begin{align*}
p\alpha_h[1 - L - p(R - P_f(S, O))] & \leq c_h \quad \text{(A.53)} \\
p\alpha_h[1 - L - p(R - P_f(S, M))] & \geq c_h. \quad \text{(A.54)}
\end{align*}
\]

Both conditions are compatible if and only if $P_f(S, M) \geq P_f(S, O)$, but this is not the case: for a given $\alpha_f$ we have $P_f(S, O) - P_f(S, M) \geq 0$, and taking into account that $\alpha_f$ is larger for $P_f(S, M)$ due to common deposit insurance makes the difference even more positive. Hence, case (i) is impossible. Similarly, for case (ii) we need in particular to have $W_h(O, O) \geq W_h(M, O)$ under national deposit insurance, and $W_h(M, M) \geq W_h(O, M)$ under common deposit insurance. The first condition is equivalent to $c_h \geq \alpha_h(1 - p)L$, whereas the second one is equivalent to $c_h \leq \alpha_h(1 - p)L$. Hence, we cannot have both, and case (ii) is not possible either. Thus, there is no case in which we can have $d^n_h = M$ and $d^n_h \in \{C, I, O\}$.

\[\blacksquare\]

### A.16 Proof of Corollary 10

This corollary is proven directly in the main text. \[\blacksquare\]
References


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Kane, E. J. (1989): “The High Cost of Incompletely Funding the FSLIC Shortage of Explicit Capital,” *Journal of Economic Perspectives*, 3(4), 31–47. 9


B Supplementary Appendix for “Multinational Banks and Supra-national Supervision”

For online publication only.

B.1 Proof of Lemma 2

So as to reduce the number of cases to consider, we first prove the following intermediate result:

Lemma 4. If \((d^*_h, d^*_f) \neq (d^{**}_h, d^{**}_f)\) and \(\alpha_h \geq \alpha_f\), then \((d^b_h, d^b_f) = (C, M)\).

Proof: According to Proposition 4, in order to have \((d^*_h, d^*_f) \neq (d^{**}_h, d^{**}_f)\) we need \(L \geq \lambda_2\). This implies that \(L \geq \lambda_3\) and hence only \((C, M), (M, M), (O, O)\), and \((I, O)\) can be optimal in the branch case. Define \(\Delta_{CM - OO} = [W_b(C, M) - W_b(O, O)] - [W_h(C, M) + W_f(M) - W_h(O, O) - W_f(O)]\): this represents the payoff of \((C, M)\) relative to \((O, O)\) under branch, minus the same difference with a supranational supervisor. Using (A.27) and (A.35), we have

\[
\Delta_{CM - OO} = \left[\kappa_1 + \kappa_4 - c_f\right] - \left[\alpha_f(1-p)L + \kappa_3 - c_f\right] = (\alpha_h - \alpha_f)(1-p)L + p(1-p)\alpha_h(1 - \alpha_f)(1 - L).
\]

If \(\alpha_h \geq \alpha_f\) we have \(\Delta_{CM - OO} > 0\), which means that if \((C, M)\) dominates \((O, O)\) under supranational supervision, it is also the case with a branch.

We repeat the analysis for the comparison between \((C, M)\) and \((I, O)\) and between \((C, M)\) and \((M, M)\). Using (A.28) and (A.36), and then (A.13) and (A.26), we have:

\[
\begin{align*}
\Delta_{CM - IO} &= \left[\kappa_4 + \kappa_5 - c_f\right] - \left[\alpha_f(1-p)L + \alpha_h(1-p)(p - L) - c_f\right] \\
&= (\alpha_h - \alpha_f)(1-p)L + p(1-p)\alpha_h(R - 1). \quad (B.1)
\end{align*}
\]

\[
\begin{align*}
\Delta_{CM - MM} &= \left[\kappa_4 - \kappa_1 + c_h\right] - \left[c_h + \kappa_3 - \kappa_1\right] \\
&= p(1-p)\alpha_h(1 - \alpha_f)(1 - L). \quad (B.2)
\end{align*}
\]

Again, if \(\alpha_h \geq \alpha_f\) we surely have \(\Delta_{CM - IO} \geq 0\) and \(\Delta_{CM - MM} \geq 0\). We conclude that if \((C, M)\) dominates \((I, O), (O, O),\) and \((M, M)\) in the supranational case and \(\alpha_h \geq \alpha_f\), then \((C, M)\) is also optimal in the branch case. \(\blacksquare\)
We now prove Lemma 2. We start by excluding all the other cases than those mentioned in the Lemma.

According to Proposition 4, we need \( c_f \geq \alpha_f(1 - p)L \). Using Lemma 4, we have \( \alpha_f \geq \alpha_h \) so that \( c_f \geq \kappa_1 \). It is thus impossible to have \((d^h_k, d^h_k) = (M, M)\). Still using Proposition 4, we need \( L \geq \lambda_2 \). As \( \lambda_2 > \lambda_3 \), we have \( L \geq \lambda_3 \) and hence \((d^h_k, d^h_k) = (M, O)\) or \((d^b_k, d^b_f) = (O, M)\) are impossible. Hence only \((O, O)\) is feasible in the branch case.

We cannot have \((d^* h, d^* f) = (M, O)\): According to Proposition 4, we need \( c_h \leq \kappa_1 \) in such a case, but according to Proposition 3 we need \( c_h \geq \kappa_1 + \kappa_4 > \kappa_1 \) to have \((d^b_k, d^b_f) = (O, O)\).

We now consider the first case, \((d^*_h, d^*_f) = (O, O), (d^{**}_h, d^{**}_f) = (C, M)\) and \((d^b_k, d^b_f) = (O, O)\). Collecting all the conditions for this case to obtain, we have:

\[
L \in [\lambda_2, \lambda_1] \quad \text{(B.3)}
\]
\[
L \leq \lambda_4 \quad \text{(B.4)}
\]
\[
c_h \geq \kappa_1 \quad \text{(B.5)}
\]
\[
c_h \geq \kappa_1 + \kappa_4 \quad \text{(B.6)}
\]
\[
c_f \geq \alpha_f(1 - p)L \quad \text{(B.7)}
\]
\[
c_f \geq \kappa_1 \quad \text{(B.8)}
\]
\[
c_f \geq \kappa_1 + \kappa_4 \quad \text{(B.9)}
\]
\[
c_f \leq \alpha_f(1 - p)L + \kappa_3 \quad \text{(B.10)}
\]

We first notice that \( \lambda_3 \leq \lambda_2 \), so that \( \kappa_4 \geq 0 \). Hence, we can neglect (B.5) and (B.8). We need to pick \( L \in [\lambda_2, \min(\lambda_4, \lambda_1)] \). It is possible to find such an \( L \): we have already shown that \( \lambda_1 > \lambda_2 \), while \( \lambda_4 > \lambda_2 \) is equivalent to:

\[
2p(R - 1)[1 - (1 - \alpha_f)p] + (1 - \alpha_f)[pR - 1] \geq 0, \quad \text{(B.11)}
\]

which is true under assumption H1.

To satisfy the remaining inequalities, we can pick an arbitrarily high \( c_h \), but \( c_f \) needs to simultaneously satisfy (B.7), (B.9) and (B.10). This requires at least that these inequalities are compatible. This is clearly the case for (B.7) and (B.10) because \( \kappa_3 \geq 0 \) when \( L \geq \lambda_2 \). For (B.9) and (B.10),
we need:

\[ \kappa_1 + \kappa_4 \leq \alpha_f (1 - p) L + \kappa_3 \]

\[ \Leftrightarrow L \geq \frac{\alpha_h p (1 - \alpha_f)}{\alpha_f - \alpha_h + \alpha p \alpha_h (1 - \alpha_f)} = \bar{L}_1. \]  \hfill (B.12)

To summarize, we can find \( c_h, c_f \) satisfying all inequalities if and only if \( L \geq \bar{L}_1 \), \( L \) satisfies (B.3), (B.4), and \( L \leq 2 - R \) (our assumption H2). These five inequalities need to be compatible. We know that \( \lambda_2 < 2 - R \), hence it remains to show that \( \bar{L}_1 \leq 2 - R \), \( \bar{L}_1 \leq \lambda_1 \), and \( \bar{L}_1 \leq \lambda_4 \). We have:

\[ \bar{L}_1 \leq 2 - R \quad \Leftrightarrow \quad R \leq 1 + \frac{\alpha_f - \alpha_h}{\alpha_h (1 - \alpha_f) + (\alpha_f - \alpha_h)} = \bar{R}_1 \]  \hfill (B.13)

\[ \bar{L}_1 \leq \lambda_1 \quad \Leftrightarrow \quad R \leq \frac{2 - \alpha_f (1 - p)}{p} - \frac{\alpha_h (1 - \alpha_f)}{p \alpha_f - \alpha_h + \alpha p \alpha_h (1 - \alpha_f)} = \bar{R}_2 \]  \hfill (B.14)

\[ \bar{L}_1 \leq \lambda_4 \quad \Leftrightarrow \quad R \leq \frac{2p}{2p - 1} - \frac{\alpha_h (1 - \alpha_f)}{(2p - 1) [\alpha_f - \alpha_h + \alpha p \alpha_h (1 - \alpha_f)]} = \bar{R}_3. \]  \hfill (B.15)

\( \bar{R}_1, \bar{R}_2, \text{and} \bar{R}_3 \) can all be lower than 2, hence these conditions are not automatically satisfied. Remember that we cannot make \( R \) arbitrarily small: due to Assumption H1, we need at least \( R \geq 1/p \). We thus need to check that \( \bar{R}_1, \bar{R}_2, \text{and} \bar{R}_3 \) are larger than \( 1/p \):

\[ \bar{R}_1 \geq 1/p \quad \Leftrightarrow \quad \alpha_h \leq \frac{\alpha_f (2p - 1)}{2p - 1 + p (1 - p) (1 - \alpha_f)} = \bar{\alpha}_1 \]  \hfill (B.16)

\[ \bar{R}_2 \geq 1/p \quad \Leftrightarrow \quad \alpha_h \leq \frac{\alpha_f (1 - \alpha_f (1 - p))}{[1 - \alpha_f (1 - p)] [1 - p (1 - \alpha_f)] + (1 - \alpha_f)} = \bar{\alpha}_2 \]  \hfill (B.17)

\[ \bar{R}_3 \geq 1/p \quad \Leftrightarrow \quad \alpha_h \leq \frac{\alpha_f (1 - 2p (1 - p))}{1 - 2p (1 - p) (1 - (1 - \alpha_f) p)} = \bar{\alpha}_3. \]  \hfill (B.18)

To summarize, if we pick \( \alpha_h \) lower than \( \bar{\alpha}_1, \bar{\alpha}_2, \text{and} \bar{\alpha}_3 \) (which are all strictly positive quantities), then we can find \( R \in [1/p, \min(\bar{R}_1, \bar{R}_2, \bar{R}_3)] \), so that we can find an \( L \) satisfying all the conditions we need, which guarantees that there are \( c_f \) and \( c_h \) as we require. The full characterization of the

\[ ^{29} \text{In the last case we use the fact that assumptions H1 and H2 imply } \frac{1}{2}. \]
parameters satisfying all conditions is as follows:

\[
p \geq \frac{1}{2}, \ \alpha_f \in [0, 1] \tag{B.19}
\]

\[
\alpha_h \leq \min(\bar{\alpha}_1, \bar{\alpha}_2, \bar{\alpha}_3) \tag{B.20}
\]

\[
R \in [1/p, \min(\bar{R}_1, \bar{R}_2, \bar{R}_3, 2)] \tag{B.21}
\]

\[
L \in [\lambda_1, \min(2 - R, \bar{L}_1, \lambda_2, \lambda_4)] \tag{B.22}
\]

\[
c_h \geq \kappa_1 + \kappa_4 \tag{B.23}
\]

\[
c_f \in [\max(\kappa_1 + \kappa_4, \alpha_f(1 - p)L), \alpha_f(1 - p)L + \kappa_3]. \tag{B.24}
\]

In particular, because of the way we constructed these intervals, they define a non-empty set of parameters.

We now turn to the second case, \((d_{h}^{*}, d_{f}^{*}) = (I, O), (d_{h}^{**}, d_{f}^{**}) = (C, M)\) and \((d_{h}^{b}, d_{f}^{b}) = (O, O)\). Collecting all the conditions for this case to obtain, we have:

\[
L > \lambda_1 \tag{B.25}
\]

\[
L \leq \lambda_4 \tag{B.26}
\]

\[
c_h \geq \kappa_1 - \kappa_2 \tag{B.27}
\]

\[
c_h \geq \kappa_1 + \kappa_4 \tag{B.28}
\]

\[
c_h \geq \kappa_1 \tag{B.29}
\]

\[
c_f \geq \alpha_f(1 - p)L \tag{B.30}
\]

\[
c_f \geq \kappa_1 + \kappa_4 \tag{B.31}
\]

\[
c_f \geq \kappa_1 \tag{B.32}
\]

\[
c_f \leq \alpha_f(1 - p)L + (1 - p)\alpha_h(p - L) \tag{B.33}
\]

We have \(L \geq \lambda_1 \geq \lambda_3\), so that \(\kappa_2 \geq 0\) and \(\kappa_4 \geq 0\). Hence, (B.27), (B.32), and (B.29) are implied by (B.28) and (B.31). It is clear that we can always find a sufficiently high \(c_h\) to satisfy (B.28). In order to find a \(c_f\) satisfying (B.30), (B.31), and (B.33), these three inequalities must be compatible, which is clear for (B.30) and (B.33). For (B.31) and (B.33) to be compatible, we need:

\[
\kappa_1 + \kappa_4 \leq \alpha_f(1 - p)L + (1 - p)\alpha_h(p - L)
\]

\[
\Leftrightarrow L[\alpha_h - (\alpha_f - \alpha_h)(1 - p)] \leq \alpha_h[p(1 + p - pR)]. \tag{B.34}
\]
While the right-hand side is always positive (consider the extreme case $R = 2$), the left-hand side can be negative. There are two cases to consider: if $\alpha_h \leq \frac{\alpha_f(1-p)}{2-p} = \tilde{\alpha}_4$, then (B.34) is automatically satisfied and (B.31) and (B.33) are compatible, otherwise we need:

$$L \leq \frac{\alpha_h p[1 + p - pR]}{\alpha_h - (\alpha_f - \alpha_h)(1 - p)} = \bar{L}_2$$ \hspace{1cm} (B.35)

We thus have three or four conditions on $L$: $L \geq \lambda_1$, $L \leq \lambda_4$, $L \leq 2 - R$ and, if $\alpha_h \geq \tilde{\alpha}_4$, $L \leq \bar{L}_2$.

We already know that $\lambda_4 \geq \lambda_1$. $2 - R \geq \lambda_1$ if and only if:

$$R \leq \frac{2 + p\alpha_f}{1 + p} = \bar{R}_4$$ \hspace{1cm} (B.36)

Finally, when $\alpha_h \geq \tilde{\alpha}_4$, $\lambda_1 \leq \bar{L}_2$ is equivalent to:

$$R \leq \frac{2 - \alpha_f(1 - p)}{p} - \frac{\alpha_h (1 - \alpha_f)}{p(\alpha_f - \alpha_h)} = \bar{R}_5.$$ \hspace{1cm} (B.37)

In particular, both $\bar{R}_4$ and $\bar{R}_5$ are necessarily lower than 2. The last thing to check is that we can have $R \geq 1/p$, $R \leq \bar{R}_4$, and $R \leq \bar{R}_5$. We obtain

$$\bar{R}_4 \geq 1/p \iff \alpha_f \geq \frac{1 - p}{p^2} = \bar{\alpha}_5$$ \hspace{1cm} (B.38)

$$\bar{R}_5 \geq 1/p \iff \alpha_h \leq \frac{\alpha_f(1 - \alpha_f(1 - p))}{2(1 - \alpha_f) + \alpha_f p} = \bar{\alpha}_6.$$ \hspace{1cm} (B.39)

In particular, it can be checked that $\tilde{\alpha}_4 \geq \bar{\alpha}_6$. This implies that when $\alpha_h \geq \tilde{\alpha}_4$, we cannot find $R$ that is simultaneously lower than $\bar{R}_5$ and higher than $1/p$. Finally, $\alpha_f \geq \bar{\alpha}_5$ is possible only if $\bar{\alpha}_5 \leq 1$, which requires $p > \frac{\sqrt{5} - 1}{2}$. To summarize, the full characterization of the parameters satisfying equations (B.25) to (B.33) is:

$$p \geq \frac{\sqrt{5} - 1}{2}$$ \hspace{1cm} (B.40)

$$\alpha_f \geq \bar{\alpha}_5$$ \hspace{1cm} (B.41)

$$\alpha_h \leq \tilde{\alpha}_4$$ \hspace{1cm} (B.42)

$$R \in [1/p, \bar{R}_4]$$ \hspace{1cm} (B.43)

$$L \in [\lambda_1, \min(2 - R, \lambda_4)]$$ \hspace{1cm} (B.44)

$$c_h \geq \kappa_1 + \kappa_4$$ \hspace{1cm} (B.45)

$$c_f \in [\max(\kappa_1 + \kappa_4, \alpha_f(1 - p)L), \alpha_f(1 - p)L + \kappa_3], (B.46)$$
which defines a non-empty set of parameters.