Workshop

Transport equations in the life sciences

at the Wolfgang Pauli Institute Vienna

November 21-23, 2011

Schedule

	10:30-11:15	11:15-12:00	14:15-15:00	15:00-15:45	16:15-17:00
Mon			Fellner	Stelzer	Di Francesco
Tue	Lachowicz	Frouvelle	Canizo	Hittmeir	
Wed	Desvillettes	Haskovec	Blanchet	15:30, room C 209 Dolbeault	

Titles and abstracts

Adrien Blanchet: *City equilibria*

Jose A. Canizo: *Asymptotic behavior for the aggregation equation with diffusion* (joint work with Jose A. Carrillo and Maria Schonbek)
We consider the equation $\partial_t \rho = \nabla \cdot ((\nabla W * \rho)\rho) + \Delta\rho$, a diffusive equation with a nonlinear and nonlocal term given by a self-interaction through a potential W. We will give well-posedness results and study its asymptotic behavior. If W satisfies some suitable bounds, one can prove that the behavior is dominated by diffusion, this is, solutions behave for large times essentially like those of the heat equation. Precise estimates on rates of convergence to the fundamental solution to the heat equation can be given by using entropy methods.

Laurent Desvillettes: *Selection/Competition/Mutations equations: an asymptotic study*
This work is based on a collaboration with A. Calsina (Univ. Autonoma of Barcelona), S. Cuadrado (Univ. Autonoma of Barcelona), and G. Raoul (Univ. Cambridge).
It is possible to model populations which are structured with respect to a quantitative trait by integrodifferential equations which take into account the effects of selection, competition,and mutations. The large time behavior of those equations (and the asymptotic behavior when the mutation rate tends to 0) is complex and strongly depends on the assumptions satisfied by the coefficients in the integrodifferential equations. In the simplest case (when

the asymptotic state [when both time tends to infinity and mutation rate tends to 0] is one Dirac mass), we provide an expansion with respect to both small parameters for the solution of the integrodifferential equations.

Marco DiFrancesco: *Stationary states of quadratic diffusion equations with nonlocal attraction*

Jean Dolbeault: *Free energies, nonlinear flows and functional inequalities*
This lecture will be devoted to a review of results based on *entropy methods* in nonlinear diffusion equations. The basic example is the fast diffusion equation in the euclidean space and the study of the asymptotic behaviour of the solutions in self-similar variables. Recent results (in collaboration with G. Toscani) provide interesting refinements for the study of the asymptotic behaviour of the solutions, based on best matching asymptotic profiles rather than on self-similar rescalings. As a result, we obtain for instance improved Sobolev inequalities which solve an old open question raised by H. Brezis and E. Lieb. Nonlocal improvements of standard functional inequalities will also be introduced, based on duality and nonlinear flows approaches. They suggest deep links connecting mean field models like the Keller-Segel system with purely local nonlinear diffusion.

Klemens Fellner: *Aggregation patterns in non-local equations: discrete stochastic and continuum modelling*
(joint work with E. Hackett-Jones, K. Landman, University of Melboune)
Non-local evolution equations featuring interaction of individuals due to a repulsive-aggregative potential are observed to produce a rich dynamical behaviour, which leads to a multitude of stationary pattern. For interaction potential with suitable attractive singularities convergence to measure solutions is deduced from a gradient flow structure in Wasserstein metric. Alternatively propagate singular repulsive interaction potential regular solutions. However, the case of interaction potential with both singular and repulsive singularities remains an open problem, for which we present an interesting comparison of numerical results with a stochastic lattice model.

Amic Frouvelle: *Macroscopic limits of a system of self-propelled particles with phase transition* (joint work with Pierre Degond and Jian-Guo Liu)
The Vicsek model, describing alignment and self-organisation in large systems of self-propelled particles, such as fish schools or flocks of birds, has attracted a lot of attention with respect to its simplicity and its ability to reproduce complex phenomena. We consider here a time-continuous version of this model, in the spirit of the one proposed by P. Degond and S. Motsch, but

where the rate of alignment is proportional to the mean speed of the neighboring particles. In the hydrodynamic limit, this model undergoes a phase transition phenomenon between a disordered and an ordered phase, when the local density crosses a threshold value. We present the two different macroscopic limits we can obtain under and over this threshold, namely a nonlinear diffusion equation for the density, and a first-order non-conservative hydrodynamic system of evolution equations for the local density and orientation.

Jan Haskovec: *On two models of biological diffusive aggregation*
We introduce two models of biological aggregation, based on randomly moving particles with individual diffusivities depending on the perceived average population density in their neighbourhood. In the first-order model the location of each individual is subject to a density-dependent random walk, while in the second-order model the density-dependent random walk acts on the velocity variable, together with a density-dependent damping term. The main novelty of our models is that we do not assume any explicit aggregative force acting on the individuals; instead, aggregation is obtained exclusively by reducing the diffusivity in response to higher perceived density. We formally derive the corresponding mean-field limits, leading to nonlocal degenerate diffusions. Then, we carry out the mathematical analysis of the first-order model, in particular, we prove the existence of weak solutions and show that it allows for measure-valued steady states. We also perform linear stability analysis and identify conditions for pattern formation. Moreover, we discuss the role of the nonlocality for well-posedness of the first-order model. Finally, we present results of numerical simulations for both the first- and second-order model on the individual-based and continuum levels of description. This is a joint work with Martin Burger and Marie-Therese Wolfram.

Sabine Hittmeir: *Nonlinear diffusion and additional cross-diffusion in the Keller-Segel model*
The main feature of the two-dimensional Keller-Segel model is the blow-up behaviour of solutions for supercritical masses. We introduce a regularisation of the fully parabolic system by adding a cross-diffusion term to the equation for the chemical substance. This regularisation provides another helpful entropy dissipation term allowing to prove global existence of solutions for any initial mass. In the parabolic-elliptic case this model can be reformulated to the Keller-Segel model with nonlinear cell diffusion. Therefore solutions are known to be globally bounded. In the second part of the talk we return to the fully parabolic model and replace the cell diffusion and the additional

cross-diffusion by nonlinear versions. This generalisation allows for global existence results in up to three space dimensions. Numerical simulations will be presented.

Miroslav Lachowicz: *Individually-based Markov processes modeling nonlinear systems in mathematical biology*
The general approach that allows to construct the Markov processes describing various processes in mathematical biology (or in other applied sciences) is presented. The Markov processes are of a jump type and the starting point is the related linear equations. They describe at the micro–scale level the behavior of a large number N of interacting individuals (entities). The large individual limit ("$N \to \infty$") is studied and the intermediate level (the meso–scale level) is given in terms of nonlinear kinetic–type equations. Finally the corresponding systems of nonlinear ODEs (or PDEs) at the macroscopic level (in terms of densities of the interacting subpopulations) are obtained. Mathematical relationships between these three possible descriptions are presented and explicit error estimates are given. The general framework is applied to propose the microscopic and mesoscopic models that correspond to well known systems of nonlinear equations in biomathematics.
Reference: M. Lachowicz, Individually-based Markov processes modeling nonlinear systems in mathematical biology, Nonlinear Analysis Real World Appl., 2011, doi:10.1016/j.nonrwa.2011.02.014

Ines Stelzer: *Entropy Structure of a Cross-Diffusion Tumour-Growth Model*
We consider the mechanical tumour-growth model of Jackson and Byrne describing tumour encapsulation influenced by a cell-induced pressure coefficient. It consists of nonlinear parabolic cross-diffusion equations in one space dimension for the volume fractions of the tumour cells and the extracellular matrix. Exploiting the existence of entropy variables and of an entropy functional, yielding gradient estimates, we can prove the global-in-time existence of non-negative and bounded weak solutions to the initial-boundary-value problem when the cell-induced pressure coefficient is smaller than a certain explicit critical value. Moreover, when the production rates vanish, the volume fractions converge exponentially fast to the homogeneous steady state. Finally, we will present some numerical results.