Workshop

Fractional diffusion and applications

at the Wolfgang Pauli Institute Vienna

March 14–16, 2012

Schedule

	10:00-11:00	11:00-12:00	14:00-15:00	15:00-16:00
Wed			Fedotov	Achleitner
Thu	Aigner	Kallsen	Tomovski	Sandev
Fri	Roux			

Titles and abstracts

Franz Achleitner: *On nonlinear conservation laws with a nonlocal diffusion term*
Scalar one-dimensional conservation laws with a nonlocal diffusion term corresponding to a Riesz-Feller differential operator are considered. Solvability results for the Cauchy problem with essentially bounded initial datum are adapted from the case of a fractional derivative with homogeneous symbol. The main interest of this work is the investigation of smooth shock profiles. In case of a genuinely nonlinear smooth flux function we prove the existence of such traveling waves, which are monotone and satisfy the standard entropy condition. Moreover, the dynamic nonlinear stability of the traveling waves under small perturbations is proven, similarly to the case of the standard diffusive regularization, by constructing a Lyapunov functional. We will provide an example of a single layer shallow water flow, where the pressure is governed by a nonlinear conservation law with the aforementioned nonlocal diffusion term and additional dispersion term and report on the recent progress in the analysis of smooth shock profiles.

Mario Aigner: *Finite Time Singularities in Separated Flows*
This work deals with the Navier-Stokes-Equations in the limit of high Reynolds numbers, also termed boundary layer flows, indicating the appearance of a viscous layer along and a potential flow away from some surface. The most prominent question then is whether this boundary layer is laminar or turbulent, or under which conditions the transition from an initially laminar flow to turbulence occurs. Experiments show that *separation* of the viscous layer from the surface can be a trigger event for transition. In such situations the

so-called *wall shear stress* becomes less or equal to zero. Hence, its spatio-temporal evolution is of great interest.
In the case considered here, the time dependent behavior of this stress is governed by an integro-differential equation, involving Abel integral operators and Riesz potentials. We will present novel approximation schemes by employing mapped Chebyshev polynomial expansions in spatial coordinates. Calculating the Fourier symbol of the operators shows the according Cauchy problem to mimic a (nonlinear) *backwards* fractional-diffusive heat equation. We will further investigate the general *ill-posedness* of this problem and discuss possible remedies.
Eventually, using some regularization for the discrete time stepping, we focus on finite time blow-up scenarios and their self-similar structures.

Sergei Fedotov: *Fractional subdiffusive reaction-transport equations*
I will talk about how to incorporate the nonlinear terms into non-Markovian fractional partial differential equations corresponding to subdiffusive transport. I will discuss applications of these equations in biology: chemotaxis, subdiffusion in dendrites, etc. I will show that the standard subdiffusive fractional equations with constant anomalous exponent are not structurally stable with respect to the non-homogeneous variations of exponent.

Jan Kallsen: *On uniqueness of solutions to martingale problems*
A key question in stochastic analysis concerns whether a Markov process is uniquely determined in law by its generator or its symbol, or, in a different language, whether a semimartingale is determined by its local characteristics. This issue can be rephrased as a martingale problem and it corresponds to uniqueness of solutions to ordinary diffential equations in deterministic analysis. We argue that – in spite of many well-established results –there is still a gap in the literature at least on processes with jumps. Indeed, it would be desirable to dispose of a Picard-Lindelöf result warranting uniqueness under sufficient smoothness of the coefficients. This talk discusses a new result and limitations in this regard.

Rafael Roux: *Particle systems for approximating fractional conservation laws*
I will present a particle approximation to the solution of the one-dimensional fractional conservation law using a system of interacting probabilistic particles.
Fractional conservation laws are a class of PDEs presenting a nonlinear transport term and a nonlocal diffusion term. We consider the nonlinear stochastic process naturally associated to the equation satisfied by the space derivative

of the solution to the conservation law. This process can be approximated by a system of interacting processes, called "particles". The considered particles evolve according to independent Levy processes (due to the nonlocal diffusion in the PDE) and have a drift depending on their cumulative distribution function (due to the nonlinear drift).
The cumulative distribution of the system will converge to the solution of the conservation law if the discretization parameters (time step and number of particles) converge in good proportions (depending on the respective strength of the diffusion and transport terms).

Trifce Sandev: *Anomalous and single file-type diffusion: Theoretical modeling*
Short introduction to different pathways to anomalous diffusion is presented. Particularly, generalized Langevin equation and fractional generalized Langevin equation are used to model anomalous diffusive processes. Special cases with three parameter Mittag-Leffler frictional memory kernels are investigated analytically. Mean velocity, mean particle displacement, mean square displacement and variances are derived by finding exact results for relaxation functions. The results are represented via the Mittag-Leffler type functions. Asymptotic behaviors of the particle in the short and long time limit are found and it is shown that anomalous diffusion occurs. Fractional generalized Langevin equation approach to single file-type diffusion or possible generalizations thereof is discussed.

Zivorad Tomovski: *Generalized Cauchy type problems for nonlinear fractional differential equations with composite fractional derivative operator*
This talk is devoted to proving the existence and uniqueness of solutions to Cauchy type problems for fractional differential equations with composite fractional derivative operator on a finite interval of the real axis in spaces of summable functions. An approach based on the equivalence of the nonlinear Cauchy type problem to a nonlinear Volterra integral equation of the second kind and applying a variant of the Banach's fixed point theorem to prove uniqueness and existence of the solution is presented. The Cauchy type problems for integro-differential equations of Volterra type with composite fractional derivative operator, which contain the generalized Mittag-Leffler function in the kernel, are considered. Using the method of successive approximation, and the Laplace transform method, explicit solutions of some open problems proposed by Srivastava and Tomovski (2009) are established in terms of the multinomial Mittag-Leffler function. Given the successful application of the generalized composite fractional (Hilfer) derivative for the modeling of highly non-trivial dielectric data by Hilfer, and modeling with

generalized fractional diffusion equations by Sandev et al. and with generalized space-time fractional diffusion equations by Tomovski et al., we believe that the fractional nonlinear models, extended Laplace transform formula and fractional integro-differential equations of Volterra type discussed here will be useful in many problems in science and engineering.