# Theory Structuralism in a Rigid Framework

**Abstract.** This paper develops the first parts of a logical framework for the empirical sciences, by means of a redefinition of theory structuralism as originally developed by Joseph Sneed, Wolfgang Stegmüller, and others, in the context of a 'rigid' logic as based on a fixed (therefore rigid) ontology. The paper defends a formal conception of the empirical sciences that has an irreducible ontological basis and is unable, in general, to provide purely structural characterizations of the domain of a theory. The extreme rationalist utopia of a characterization of the real world 'up to isomorphism', therefore, is rejected.

## 1. Introduction

The following account of theory structuralism deviates from the classical presentation[1] in two respects. There are (1) some inessential formal simplifications in my account that may have some didactic advantages, but there is (2) also one essential restriction, namely, 'rigidity': we define structures and classes of structures only relative to some fixed sets of objects – the 'sorts' of a so-called 'structure frame'.

This rigid framework, firstly, has some technical merits. It allows us to develop a considerably simpler presentation of theory structuralism; it especially allows us to define a logical framework that is *comprehensive*, insofar as it enables us to develop every formal aspect of theory structuralism in a logical way, whereas in the classical framework a logical representation may hardly cover more than the basic non-modal aspects of the framework.[2]

Secondly, beside that (rather technical) aspect of a (complete) 'logification', there is another, more philosophical argument for the use of the rigid framework in the context of theory structuralism. Theory structuralism is intended *as a framework for the empirical sciences*, and, in its original form, *it is based on model theory.* However, some crucial notions of theory structuralism, e. g., the notion of the 'empirical claim' of a theory or the notion of 'reduction' of one theory to another, are defined in such a way that the whole framework becomes rigid in fact, because in all these cases a theory is not merely *structurally* compared with another theory (that represents the empirical world or the theory it is reduced to), but some concrete models of a theory are *directly linked* to other concrete models. The crucial concept is not that of a structural *isomorphism* between models (as it is commonplace in model theory), but that of a (partial) *identity* between models or that of being linked to another model by means of a fixed reduction relation. In other words, theory structuralism *in fact* makes use only of such a fragment of the model theoretic framework that is structurally more or less identical with a rigid framework, in the sense specified in the present paper. Thus, the claim is that the rigid framework is the more genuine framework for theory structuralism than the framework of classical model-theory. In particular, one of the main oddities of theory structuralism, namely, the task of ruling out ‘unintended models’, immediately vanishes if we switch to the rigid framework, because in the latter we can provide the whole framework with such a restrictive ontological layout that unintended models do not appear at all.

There are three different general aspects under which the present paper may be read. *First*, it may be considered *as a demonstration of the use of the rigid framework as a framework for the empirical sciences*. Theory structuralism rather functions as an example here, as an object of demonstration. It was chosen because it is one of the best developed formal accounts of scientific theories, and possibly the only one that deals exclusively with the empirical sciences. The purpose of the present paper, seen from the perspective of the rigid framework, is to illustrate the expressive power of this formalism as a framework for the empirical sciences and to illustrate the family resemblances with the framework of theory structuralism. *Second*, the paper may be seen *as a formal and philosophical critique of theory structuralism*, as presented in (BMS). Along that line of thought, the paper points out a number of failures of the original account and shows how we may improve them, if we transform this account into the rigid context. *Third*, this paper can be seen as the preliminary part of a research project that intends to develop *a comprehensive logical framework for the empirical sciences*. In this sense, the present work is part of a philosophical research tradition that begins with Rudolf Carnap’s seminal book *The Logical Structure of the World*. In Carnap’s terminology, the task of a ‘rational reconstruction’ of the empirical sciences is divided into two parts: ‘constitutional *theory*’, which is only concerned with the purely formal aspects of the rational reconstruction, and the development of concrete ‘constitutional *systems*’ that illustrate the meta-theory with some concrete examples and philosophical explanations.[3] The present paper focuses on the former task. Its strategy is to provide a framework that has the greatest possible expressive power, so that research does not cover unnecessary formal hindrances if it goes on to develop the more concrete parts of the program.

What we need is an abstract framework that makes as few *formal presuppositions* as possible that may force us later on to give up a particular idea because it does not fit into the framework. It is not easy to understand, however, what exactly is meant by this claim, if we adopt it for the idea of a framework for the *empirical* sciences. It is *not* meant, of course, that we must be able to express the most complex *formal structures* (in the mathematical sense), but rather that we can express the most complex *ontologies*. A formal framework that allows us to express complex *formal structures* is classical first- or second-order logic. Complex *ontologies* for empirical cases, however, in general have quite simple *formal structures*: they are often finite or countable, and they are virtually always based on the simple idea of having a number of concrete objects and ascribing a number of concrete properties to them. The *complexity*, in that case, comes from a completely different side. For example, we may have quite complex *modal notions* to express here that force us to quantify not only over basic objects, but also over structures, as built from basic objects ('possible worlds'), over formulas that express properties of basic objects, over the whole range of basic objects of any type, and over some power-sets and other set-theoretical constructs that we obtain from objects of that kind. Operations of that kind can be established in the context of a structurally powerful language like first- or second-order logic only in a quite *limited* way. However, rigid logic, i. e., a logic that is based on a fixed collection of basic objects (and thus can be seen as a *fragment* of first-order logic), does allow operations of the just-mentioned kind, without any limitation, simply because *the object level* in such a logic is simpler and more concretely construed than in the usual structurally powerful logics. Thus, we finally arrive at a situation where we have to choose definitely between two different types of *limitation*. Our formal framework can be either structurally comprehensive and ontologically limited or structurally limited and ontologically comprehensive. The former is the case in first-order and (to an even higher extent) in second-order logic. The latter situation we obtain in the context of a framework of rigid logic (that, as it turns out, is structurally weaker but ontologically stronger than first-order logic). And this is why rigid logics may turn out to provide better frameworks for the empirical sciences than the tools of classical (first- or second-order) logic. In particular, the framework of theory structuralism may appear to be much more powerful and much more manageable if we reconsider and redefine it in a rigid setting.

In section 2 the principal formal aspects of the rigid framework are developed. Section 3 contains a general discussion of theory structuralism and defines some fundamental layout principles for the present framework. On this basis, in section 4, some portions of theory structuralism are reconsidered, in the context of the rigid framework, in a more formal way.

## 2. The rigid framework

In this section we specify a simplified version of the rigid framework that, in particular, does not contain a detailed specification of a formal language. A more comprehensive specification and discussion of this framework is developed elsewhere.[4] The somewhat unusual technicalities of section 2.1 are followed by a number of informal remarks and clarifications in section 2.2.

### 2.1. Structure frames and structures

The basic objects of theory structuralism in the classical setting (BMS, I) are the structures

$$D_1, \ldots, D_k, A_1, \ldots, A_m, R_1, \ldots, R_n$$

of a particular structure type. Here $D_i$ are the principal sets, $A_i$ are the auxiliary sets (i. e., the sets of mathematical objects), and $R_i$ are relations of some specified types.

By contrast, the rigid framework introduces $D_i$, $A_i$, and $R_i$ by means of some fixed sets that contain either 'basic objects' or 'labels' of 'relations'. These sets we will call 'sorts' here. The elements of the sorts are considered as *rigid designators*, i. e., names that have a fixed reference. Thus, the elements of the sorts provide both *the names* and the *primitive objects* of our framework.

Note also that rigid designators are of a rather technical nature in our framework. In particular, we do *not* claim here that *the names* of a particular language have to be rigid, i. e., to have a fixed reference. Rather, the rigid layout of our framework is merely a technical device for the introduction of objects into our language, to be able to specify every kind of ontology directly, on the level of the object language. However, an extremely important special case of rigid ontologies as specified in this way will be ontologies that contain *non-rigid names.* This will be realized by an operator $\downarrow$ that interprets any object/name $c$ of our language as a 'zero place function' such that $\downarrow c$ points to the reference of $c$ (in case there is such a reference). Thus, if $c$, for example, represents the notion of *space*, the reference $\downarrow c$ may be a completely different manifold in the case of a theory that characterizes Newtonian physics and in a theory that describes general relativity. In other words, non-rigid designators are quite fundamental cases of our rigid framework, and they are not at all ruled out here. Whether a particular name $c$ is rigid or not will be one of the technical decisions that we will have to make in the context of the development of the formal specification of a particular theory.

The sorts of our framework are typified by a set of 'types' of relations between sorts. Each type is defined as a finite sequence of sorts. If $(s, s_1, \ldots, s_n)$ is a type, then the first sort $s$ always represents *the names* or 'labels' of the relations of this type. Therefore, the 'individuals' of a structure species, i. e., $D_i$ and $A_i$, are introduced here by means of sorts (thus, are restricted to fixed collections of basic objects); the 'relational objects' of a structure species, i. e., $R_i$, are introduced here by means of types.

A structure in our framework is simply a function that specifies (1) the domain sets as subsets of their respective sorts and (2) a suitable relation over the domain sets, for every specified type. Thus, the whole framework is based on a 'structure frame' that consists of a set of 'sorts' and a set of 'types'. More formally:

A *structure frame* $\mathfrak{F} = (S, T)$ consists of a non-empty set $S$ of non-empty and disjoint sets – the *sorts* of the structure frame – and a set $T$ of *types* that is defined as a subset of the set of all finite sequences of elements of $S$ such that $(s) \in T$, for every $S$ (i. e., every sort specifies a unary type).

The elements of the sorts we call the *atomic objects* of the structure frame. Thus, we obtain the set $\mathbb{S}_a$ of all atomic objects of $\mathfrak{F}$ as

$$\mathbb{S}_a := \bigcup_{s \in S} s.$$

We designate as $\tau$ the function that assigns to every atomic object $o \in \mathbb{S}_a$ its (well-defined) sort $\tau(o) \in S$.

If $t = (s, s_1, \ldots, s_n)$ is a type with $n > 0$, then we call *s the labeling sort* of $t$. (Unary types do not have labeling sorts.) To obtain a useful type structure we require the following *typification rules* to hold:

1. There are some sorts that do not function as labeling sorts of any type. These are the *basic sorts*. (These sorts correspond to first-order sorts in the classical sense.)

2. Every sort is the labeling sort of, at most, one type. (This ensures that labels identify types in a unique way.)

3. $S$ contains at least one basic sort and one labeling sort.

The set of all basic sorts we call $B$, and the set of all *labels*, i. e., objects that are contained in labeling sorts, we call $\mathbb{L}$.

A *structure* $\mathfrak{S}$ over $\mathfrak{F} = (S, T)$ is defined as a function that firstly assigns to every sort $s \in S$ a set $\mathfrak{S}(s) \subseteq s$. Secondly, the structure $\mathfrak{S}$ assigns to every sequence $(s_1, \ldots, s_n) \in T$ with $n > 1$ a set

$$\text{(S)} \quad \mathfrak{S}(\mathrm{s}_1, \ldots, \mathrm{s}_\mathrm{n}) \subseteq \mathfrak{S}(\mathrm{s}_1) \times \ldots \times \mathfrak{S}(\mathrm{s}_\mathrm{n}).$$

## 2.2. Remarks on the above definitions

(1) Ultimately, the only completely unusual feature of this notion of a structure is its *rigidity*: for every atomic sort $s$ the domain $\mathfrak{S}(s)$ of a structure $\mathfrak{S}$ is specified as a subset of the set $s$, which builds the same fixed basis, for every structure. This feature of rigidity is shared by van Fraassen's conception of *semi-interpreted* languages (see section 3.1, below), and it can also be found in the context of quantified modal logic with variable domains, where also the domain of a particular possible world must be a subset of a fixed basic domain $D$ (i. e., the union set of the domains of all possible worlds).[5]

(2) The main advantage of rigid structure frames, in the context of empirical theories, is that *the ontology* of a theory can be specified here *completely* by means of non-logical terms, i. e., objects of the structure frame. This is impossible, in the classical framework of model theory, because there we do not have rigid designators. Thus, in the classical framework we have to specify the ontology *indirectly*, on the basis of purely structural specifications (by means of Ramsey sentences and the like). This leads to the awkward situation in which every formal specification of an empirical theory is burdened with a vast amount of *unintended models*, i. e., models of a theory that have nothing to do with its intended applications. These unintended models can be ruled out only on an informal level, then, but by no means in a formal way. In sharp contrast to this unfortunate situation the rigid framework allows us to rule out unintended models from the beginning, because we only introduce the objects we really want to talk about. Thus, the *actual models* of a theory, in the rigid case, are always assumed to be also its *intended models* (cf. section 3.2, below).[6]

(3) If $(s, s_1, \ldots, s_n)$ is defined as a type, then there is *exactly one* relation over $s, s_1, \ldots, s_n$ specified in a structure (i. e., the structure specifies a subset of the Cartesian product $s \times s_1 \times \ldots \times s_n$). In non-rigid logic, by contrast, a structure ranges either over *all* relations or (in the case of Henkin-semantics) *over a subset* of all relations of a particular type $t = (t_1, \ldots, t_n)$. In rigid logic, on the other hand, each type contains a first sort $s$ of *labels* that represents the relations of a particular type $t$. Thus, the rigid type that corresponds to the just-mentioned non-rigid type $t$ has the form $(s, t)$. Take, for example, the rigid type (colors, objects), where 'colors' is a set of colors: 'red', 'blue', 'green', etc., and 'objects' is a set of objects: 'table', 'chair', 'house', etc. A rigid structure $\mathfrak{S}$, then, first specifies subsets of 'colors' and 'objects': the colors and objects that exist in that structure. Then, the structure specifies a subset of the Cartesian product $\mathfrak{S}(\text{colors}) \times \mathfrak{S}(\text{objects})$: the pairs of existing objects for which the relation of type (colors, objects)

holds. If (red, chair), for example, is an element of that relation, then we may interpret this in such a way that *the object 'chair' has the property 'red'*. In other words, $n+1$-place types of the rigid framework are always interpreted in the sense of $n$-place types of a non-rigid framework; in the rigid framework, the first place simply contains the names of the relations of the type. (If we want to be able to quantify over the whole range of all relations of a particular type $(s, s_1, \ldots, s_n)$, then we have to introduce a sufficiently large set $s$, i. e., the cardinality of $s$ has to be greater or equal to the cardinality of $s_1 \times \ldots \times s_n$.)

Unary types in the classical sense are introduced in a structure frame as binary types where the first place represents the labels of the unary type and the second place its range. On the other hand, every atomic sort of a structure frame of the rigid framework is defined as a rigid unary type. Rigid unary types of that kind function as 'zero-place' or 'empty' types in the classical sense, and we may interpret them in two different ways. First we may interpret them as 'existence predicates' that pick out all these objects that may have positive properties in the structure; rule (S) stipulates that if an object $o$ of sort $s$ does not exist in a structure $\mathfrak{S}$, i. e., $o \notin \mathfrak{S}(s)$, no sequence of objects of the form $(\ldots, o, \ldots)$ can be contained in the structure $\mathfrak{S}$. Second, we may interpret rigid unary types as 'propositions' such that any $o \in s$ is 'true', iff $o \in \mathfrak{S}(s)$.

(4) It is essential that the sorts of a structure frame are *disjoint*, because we take the atomic objects of a structure frame *as rigid designators*, i. e., as names that have a fixed reference. The notion 'rigid designator' was introduced by Saul Kripke.[7] In our framework, however, rigid designators have a rather technical nature. Unlike Kripke, we do not claim that *any name* (of a particular language) is rigid in fact (i. e., it refers causally to a particular object). Rather, rigid designators are merely the basic technical element of our framework. An important special case for these objects are the above-mentioned *non-rigid designators*, i. e., objects/names $c$ that have a varying reference $\downarrow c$ in different theories or even in different models of one and the same theory. Formally, the operator $\downarrow$ is introduced in the context of sorts $s$ that correspond to a type $(s, s')$, which is specified as a partial function from $s$ to $s'$. For each object $c \in s$, then, we define $\downarrow c$ as the definite object of $s'$ that is picked out by the partial function $(s, s')$, or by a dummy-object `NULL`, respectively, if there is no such object.

(5) In general, we assume that $S$ contains a suitably large number of mathematical standard sets like $\mathbb{R}$, $\mathbb{R}^3$, and the like. We do not always mention such sets explicitly, if we describe a particular structure frame. Additionally, we assume that, in the context of a structure frame, there is defined a suitably large number of mathematical standard functions

and relations over the mathematical standard sets. All these elements we call *auxiliary elements* of a structure frame. The remaining elements we call its *principal elements* (BMS, 10). The set of all principal atomic objects of a structure frame we call $\mathbb{S}_p$.

(6) We cannot provide a detailed specification of a *rigid logic* in the context of the present paper. We just sketch the principal layout of the simplest form of such a logic. A logic over a rigid structure frame $\mathfrak{F}$, in its simplest form, can be viewed as an instance of propositional logic with possibly uncountable propositional constants. Consider the set

$$T_\times := \bigcup_{(s_j)_{j=1}^i \in T} s_1 \times \ldots \times s_i,$$

which is well-defined for every structure frame $\mathfrak{F} = (S, T)$. Structures, then, can be redefined as subsets of $T_\times$. (Note that not *every* subset of $T_\times$ is defined as a structure, because of the unary relations that form 'existence predicates'.) Thus, a rigid logic can be defined as propositional logic over the set $T_\times$ of propositional constants. We only have to include a device $\bigwedge$ for building conjunctions of arbitrary large (and possibly uncountable) sets of formulas.[8]

This basic rigid logic is bound to a particular structure frame $\mathfrak{F}$. Thus, we call it $L_r(\mathfrak{F})$. On this basis we can introduce a first level of modal quantification over structures in the context of a *first order modal structure frame* $(\mathfrak{F}, S_m, T_m)$. Here, the elements of $S_m$ are 'modal sorts', i. e., sets that contain either accessibility relations and the like or such things as the structures, formulas, and other logical elements of the basic rigid logic; the elements of $T_m$ are 'modal types', specified by analogy with non-modal types. We obtain a set of 'modal structures' and a set of atomic formulas that allow us to quantify over structures and formulas of the basic logic, while the layout remains that of propositional logic, in principle. The result of this specification is a 'first-order modal rigid logic' $L_r(\mathfrak{F}, S_m, T_m)$. By analogy we also may introduce a second-order and a third-order modal rigid logic, etc.

In the following considerations we always assume that a suitable (modal) structure frame is given, together with a suitable formal language and a suitable definition of satisfaction of a formula in a (modal) structure.

## 3. A general discussion of theory structuralism

### 3.1. More structuralism or less structuralism?

Theory structuralism originates from the tradition of formal semantics and model theory. One of its main forerunners is Patrick Suppes, who propagated in (Suppes, 1957) a reconstruction of the empirical sciences by means of 'set theoretic predicates', with the slogan "that *philosophy of science should use mathematics, and not meta-mathematics*".[9] Another obvious source of theory structuralism is the structuralist approach of the Bourbaki group.[10] Sneed and Stegmüller adopted this approach and enriched it with some vocabulary of more recent model theory. However, the mere usage of model theoretic *vocabulary* does not imply that a theory is model theoretic in a full-fledged way. And indeed, theory structuralism is not. A full-fledged model theoretic account describes some portions of reality, *exclusively* by means of their pure structural properties. Thus, for example, the only possibility of ascribing a particular *property* to some portions of reality, is to identify the latter as *isomorphic* or at least approximately isomorphic with a particular structure or structure-characterization. This principle prohibits, in particular, our identifying two portions of reality as structurally (dis-)similar or (in-)commensurable on the mere basis of a description of their *ontology*. In other words, a *purely structuralist account*, in this sense, has to express everything that it wants to express, exclusively with appeal to structural properties (by means of Ramsey sentences), and not with appeal to *ontological properties* (by means of sentences that ascribe concrete properties to concrete objects). In this respect, theory structuralism clearly is *not* a purely structuralist account, because most of its specifications for the comparison of structures do not refer to isomorphy and other purely structural properties, but rather compare structures *directly* by means of their ontology. Theory structuralism is ultimately sort of a *mixture* between a purely structuralist account, as based on formal semantics and model theory, and an *ontological account*, as based on the idea of providing some concrete ontological descriptions of objects and their (actual or possible) relations.

For example, the 'constraint' of a theory (BMS, II.2) generally specifies a particular class or set of models that (1) share their ontology (i. e., their domains) and (2) share some properties. The former is an ontological property, and only the latter a structural one. On the other hand, the 'reduction' of a theory to another one (BMS, 277, DVI-5) is defined (at least at a first glance) in structuralist terms, because there has to be a reduction-relation $\rho$ that maps the models of one theory to the models of the other in a particular way. The problem that appears here is that the notion of reduction of an empirical theory to another, in purely structural terms, tends to trivialize the whole idea of reduction.[11] The question that we have to ask is the following: Is what we want to say when we describe constraints or reduction relations *in fact* something purely structural, is it *in fact* a purely ontological thing or a mixture of both? In a way, the latter must be evidently true, because every empirical theory clearly *has* both ontological and structural aspects: it ascribes some properties to some objects, therefore *identifies* an ontology with a structure. However, this sort of combination of structural and ontological aspects clearly must be a feature of every kind of a 'pure' ontological framework, and it is a feature of the rigid framework, in particular. Whereas it is true, in this sense, that there is no purely ontological framework at all (unless we may be willing to call a mere *collection* of objects, without any structural commitment, a 'framework'), it is also quite clear that theory structuralism mixes the ontological and the structural aspects in a different way. The problem here seems to be that theory structuralism (at least in some sense) *wants* to provide a purely structuralist framework, but fails to do so.

Let us take this for granted and ask how we may improve this failure of theory structuralism. There seem to be two possible strategies. First, one may try to improve theory structuralism by making it even more structuralist. Second, one may try to improve it by simply giving up the structuralist intention and reconsidering the whole account in a different framework that has a more ontological nature. The first strategy obviously would lead to a more or less complete counteraction of the original task of theory structuralism, namely, to provide a formal framework for the *empirical* sciences, simply because it is the very foundation of this task to take the empirical sciences as different from pure mathematics in having quite essential *ontological* aspects. In other words, it cannot be a vital aim of theory structuralism to provide a purely structural account of the empirical sciences, insofar as this would be just question-begging: the empirical sciences *are not* purely structural – that's it. In the empirical sciences, we (per definition) do not talk about pure structures, but rather about some objects in the real world having these and those properties. All this ontological talk, of course, also *has a structural aspect*, which may have led theory structuralists to adopt the model theoretical framework for their purposes. The very aim of theory structuralism, however, never was the complete structuralization of empirical knowledge in such a way that the world is characterized up to isomorphism, but a much more modest one, namely, to "develop a 'representation scheme' for scientific knowledge" (BMS, xvii). This representational attitude implies, of course, that the whole structuralist framework must be designed in such a way that it always enables us to 'encapsulate' some aspects of the empirical world by simply *referring to it* (because there is no complete structuralization that allows us to replace direct reference with a structure).

Given these considerations there can be no doubt that the second strategy is best suited for an improvement of the structuralist framework. Following this strategy, the present paper aims to reconceptualize some portions of the original framework of theory structuralism in a rigid framework. In this framework, constraints, to come back to the first of our examples, simply describe the same structural properties of concrete ontologies, as in the original conception. We may say that the notion of a constraint, as presented in (BMS), like many other notions, *is* a rigid notion, in fact. On the other hand, such things like reduction of a theory to another one clearly must obtain in the present framework a much more ontological nature. There must be a concrete relation $\rho$ that *reduces* the ontology of one theory to the ontology of the other. Only on the basis of this concrete relation $\rho$ (that must be seen as the direct expression of some *empirical facts*) are we able to define the notion of being a reduction of a theory (by means of having corresponding models, in a rather obvious way).[12]

A historical remark. Theory structuralism is by far not the only example that allows us to support the claim that a framework for the empirical sciences has to be *less structural* than a framework for pure mathematics, and therefore is rigid in fact. The logics that were considered by Frege, Russell, Carnap, and other classical philosophers of logic, until the end of the 1920s, where rigid.[13] Rudolf Carnap, who developed in his seminal (Carnap, 1928) an account of theories of the empirical sciences, which was firstly introduced as a purely structural one (§ 16), came to the conclusion, at the end of his book, that a purely structural characterization fails and has to be replaced by an account that is based on so-called *founded relations* (§ 154), and therefore is rigid in fact. Moreover, the whole philosophical discussion in the context of the so-called 'new theory of reference' (Kripke, Putnam) is nothing else but a broad establishment of the claim that a purely structural identification of empirical objects is generally impossible.[14] Probably the most explicit example of a logical framework for the empirical sciences that is rigidly construed in fact is Bas van Fraassen's use of *semi-interpreted languages* (as inspired by Evert Willem Beth). Semi-interpreted languages are formal languages that are semantically interpreted only in the context of a fixed set $F$ of objects. Thus, semi-interpreted languages and rigid logics have the same conceptual basis.[15]

### 3.2. The problem of unintended models

Theory structuralism, like the rigid framework, is intended as a framework for the empirical sciences, but not for theories of pure mathematics.[16] Nevertheless, theory structuralism is based on the classical framework of model theory, so that, given a particular structure type of a theory, every structure

$$D_1, \ldots, D_k, A_1, \ldots, A_m, R_1, \ldots, R_n$$

of that type must be considered. Theory structuralism, then, provides some restrictions to structures in an obvious way. Firstly, it defines some *characterizations* $\mathcal{C}$, i. e., some rules that stipulate such things, like that a particular relation $R_i$ is defined as a bijective function or that a particular set $D_j$ has to be finite. Secondly, and in particular, it describes the respective *laws* $\mathcal{L}$ of a theory $\mathbf{T}$ and defines *the class* $\mathbf{M}$ *of all models of* $\mathbf{T}$, essentially, as all models of the sentences $\mathcal{C} \cup \mathcal{L}$.

A main problem of theory structuralism is that the class $\mathbf{M}$ inevitably contains a vast number of *unintended models*, i. e., models that characterize 'realities' that have nothing but their formal structure in common with the reality that the theory intends to describe. In other words, what clearly would be a virtue in the context of a theory of pure mathematics, namely, to provide a purely structural and not an ontological characterization of a theory, turns out to be a serious shortcoming in the context of a theory of the empirical sciences.

Note also that this problem of unintended models becomes more serious the simpler a particular theory is. On the one hand, we may construe the utopian and extremely rationalist picture of a holistic, all-encompassing theory of the empirical world that 'converges' with a mathematical theory, because it has just one model: the real world as it actually is. In that case, a purely structuralist account would be the right choice, of course. On the extreme opposite side, if we take an extremely simple example of an empirical theory, e. g., a theory that only ascribes one property to one object, in the context of the claim

'Snow is white'

then we may represent this theory structurally, by means of the structures of type

$$D, R,$$

where $D$ is a set with one element and $R$ is a unary function over $D$. The *law* of the theory, of course, is the claim

$$\forall x \in D : R(x).$$

The 'intended model' **I** of the theory is the set $D$ that contains the object 'snow' and the relation $R$ that represents the property 'white'. However, besides this intended model **I** the models of the theory obviously contain a vast amount of models, namely, all structures of the just-specified type, where $R$ is true for the only element of $D$.

This is not to say that it is *impossible* to describe an empirical theory in such a framework. Clearly, however, as long as we do not *have* a holistic and all-encompassing theory in the above-mentioned sense, the structuralist setting always provides us with this enormous bunch of unintended models that are completely irrelevant for the empirical claim of a theory, and are therefore useless.

The rigid framework provides a solution to that problem of unintended models, which is obviously valid for all possible examples of empirical theories *below* the threshold of the above-mentioned extreme rationalist case. In our example we would take a structure frame $\mathfrak{F} = (S, T)$ where $S$ consists of only two sorts $s$ and $s'$: $s$ contains the single object 'snow' and $s'$ the single object 'white'. $T$ contains the single type $(s', s)$, and the law of the theory is simply that the claim $(\text{white}, \text{snow})$ is true. Thus, we obtain exactly one model here that, of course, is also the only intended model of the theory.

We then may enrich our framework for more and more complex cases in the context of more and more complex structure frames and laws. Nonetheless, however complex the setting for an empirical theory may be, it seems to be always possible here to find a characterization of a theory that completely rules out unintended models, because it divides the structures of a structure frame into two subsets: (1) the structures in which the laws of a theory are *false*, and (2) the intended models of a theory. We assume this remark with the following *first layout principle* for a redefinition of theory structuralism in the rigid framework:

> In the rigid framework, in general, the models and the intended models of a theory are identical.

This affects the whole layout of our presentation of theory structuralism in section 4 below, because the distinction between theory-cores and intended applications falls away here: the 'actual models' and the '(theoretical) content' of a theory, in the rigid setting, are exactly everything the theory tells us about the world.

In the rigid framework the ontology of a theory can be specified in such a *restrictive* way that the models of a theory are *exactly* its intended models. The (intended) models of a theory are *the picture that a theory draws from the world*. This picture, then, may be true or false, empirically adequate or not.

Note also that this convergence between models and intended models clearly does *not* imply that it is impossible here *to distinguish* between different intended applications of a theory. The theory of classical particle mechanics, for example, may be refined in such a way that we can distinguish formally between the applications to the solar system (and its subsystems), the pendulum, and the harmonic oscillator. We must be able, of course, to distinguish between all these things even in the rigid framework (by means of some constraints or some restrictions of the ontology of a theory). The only thing that we claim here is that the set of all models of a theory, in the rigid context, ultimately shall contain all and exactly all its intended applications (see also the remarks at the end of section 4.2, below).

### 3.3. The problem of internal model restriction

We now turn to the problem of model restriction, not on the *external* level of comparison of the models of a theory with the intended (empirical) world, but on the *internal* level of the formal establishment of the models of a theory. Internal model restriction, in this sense, is realized, in the structuralist framework, on (at least) five different levels:

1. The level of specification of the 'ontology' of a theory
2. The level of 'characterization' of 'potential models' of a theory
3. The level of the specification of the 'laws' of a theory
4. The level of the specification of the 'constraints' of a theory
5. The level of the specification of the 'intertheoretical links' of a theory

The main philosophical and formal problem that we may identify here is situated already on the first level. The structuralist framework, as specified in a classical model theoretic setting, implies that we can specify *only parts of the ontology of a theory* on the elementary level of formal theory specification. If we take the structures

$$D_1, \ldots, D_k, A_1, \ldots, A_m, R_1, \ldots, R_n$$

of a particular type, then we stipulate as rigid elements of a theory (a) the auxiliary sets (i. e., sets like $\mathbb{R}, \mathbb{R}^3, \mathbb{N}$, etc., that are present in every structure of a theory) and (b) the relations $R_1, \ldots, R_n$ that have to be specified in the context of every structure. These objects are fixed ontological parts of a theory; every structure and every model of a theory must contain them and (in the case of relations) must interpret them. So far the structuralist framework *is* completely rigid, from scratch. In sharp contrast to this, we have the principal sets

$D_1, \ldots, D_k$ of the theory, which are completely unrestricted sets that may contain anything whatsoever. (We at most may establish some *formal* restrictions of the cardinality of a particular $D_i$ and the like.) Thus, as already explicated in the previous section, the ontology of a theory is simply ill-founded in the context of a specification of theory structuralism, as established in a classical model theoretic setting.

Thus, we henceforth take for granted that, to *compare* the ontologies of different theories and *restrict* ourselves to the intended models of a theory, it is inevitable to establish a *completely restricted ontology* for our theory. In the rigid setting we realized this by introducing even the sets $D_1, \ldots, D_k$ on the basis of a predefined set of *sorts* so that every $D_i$ is defined as a subset of a particular sort $s$. Ultimately, one may well say that *this* is the only real difference between our presentation and the classical one (although this difference, of course, has a number of serious consequences).

Levels (2) to (5) of internal model restriction, are realized in the context of four different levels of specification of *formal rules and axioms*:

(2) The 'characterizations' of the 'potential models' of a theory are nothing more than further restrictions and specifications that we provide for some particular elements of the ontology of a theory. A particular relation of the theory, for example, may be specified as a bijective function, another as a reflexive relation; a particular set of a theory may be specified as a finite or countable set, and so on. The only restriction that takes place here is that the respective rules and axioms must not contain references to more than one principal element (either a set or a relation) of the structure species of a theory (BMS, 14, DI-4).

(3) The 'laws' of a theory, then, are also rules and axioms that specify properties of the elements of the ontology of a theory. The only difference between a law and characterizations is that a law always has to contain references to *at least two* different principal elements of the ontology (BMS, 16). For example, $E = mc^2$ may be seen as a law, $c \approx 3 \cdot 10^5 m/s$ as a characterization.

(4) According to theory structuralism, however, the models of a theory generally are not sufficiently specified, on the basis of its characterizations and laws. In most cases of empirical theories we need (at least) two further specifications that restrict the models of a theory. The first one consists of 'constraints': rules that describe the formal relations between the models of a theory. A typical example of a constraint is the equality of mass constraint in classical particle mechanics (BMS, II.2.1), which stipulates that if two models of that theory both contain a particle $p$, then $p$ must have the same mass in both models. Thus, the

constraints specify 'consistent subclasses of models', and they restrict the class **M** of a theory (that we obtained from the characterizations and laws) to a class of consistent subclasses of **M**. (Note also that, formally, the constraints lift the specification of models of a theory, from the level of classes of structures to the level of classes of classes of structures).

(5) The second important group of further specifications that restrict the models of a theory are 'intertheoretical links' that define connections between a theory and other theories. These connections, for example, may reduce the non-theoretical parts of a theory to another theory that allows us to calculate or to visualize these non-theoretical elements. Formally, intertheoretical links are nothing more than relations that are specified between the (potential) models of different theories. Thus, the models of a theory are restricted here to only these structures that are actually linked to other theories.

In contrast to our criticism of level (1), none of these four aspects of the axiomatic specification of the models of a theory will be questioned here, in principle. We take for granted that the identification of these four aspects of model restriction is one of the great merits of theory structuralism as a framework for the empirical sciences, because they provide these specifications with refinements that we miss in other frameworks. What I want to point out here is only the rather *technical* question of the establishment of these four aspects, in the context of a particular formal framework. If this framework provides a logic that is *expressive enough*, then all these axioms and rules may be established *simply as formulas* (axioms), namely:

2. Formulas that refer to only one principal element of the theory

3. Formulas that refer to at least two principal elements of the theory

4. Formulas that refer to sets of models of a theory

5. Formulas that refer to the models of this and other theories

Thus, the following *second layout principle* for the below-specified framework can be formulated:

> The framework should be at least that powerful so that axioms of type (2) to (5) can be introduced as formulas of the framework.

This layout principle implies, in particular, that our framework must allow us to quantify (without any serious restriction) over the structures of a basic language, in the context of the object language of a suitable (first order) modal rigid logic, as specified over it.

## 4. Theory structuralism in a rigid framework

In this final section we develop a presentation of theory structuralism, which is based on rigid logic and the above specified layout principles and provides counterparts for some of the definitions in (BMS, ch. I and II). Because of questions of space we do not discuss the most important notions of theory-nets, reduction, and evolution of theories. These aspects are/will be discussed elsewhere.[17]

### 4.1. Theory-elements

A crucial aspect of theory structuralism is that it is not restricted to the specification of one single 'theory-element', but considers arbitrarily large collections of theories and theory-elements, and relations between them. As a consequence of this we decide to construe our logical framework in such a way that it is based on a structure frame which is *sufficiently large* so that it is possible to specify every theory-element that is relevant, in the context of our enquiry. This implies, in particular, that *the ontology* of a theory-element must be specified independently of a structure frame (because its structure frame, in general, will have a much larger ontology than this one single theory-element). Formally, we specify the ontology of a theory-element on the basis of a pair

$$(B_p, \mathbb{L}_p),$$

where $B_p$ is a set of subsets of basic sorts of the structure frame and $\mathbb{L}_p$ is a set of principal labels of the structure frame. These two objects clearly form the direct counterparts to the sets $D_1, \ldots, D_k$ and the relations $R_1, \ldots, R_n$ of a structure (species), in the classical presentation of theory structuralism. (The auxiliary sets and relations are the same, for every theory-element that we specify in the context of a structure frame, thus we do not mention them explicitly here.)

$B_p$ and $\mathbb{L}_p$ specify *the whole ontology* of a theory (with the exception of its auxiliary objects). However, because we are concerned with *empirical theories*, the ontological specification has to be supplemented by a third fundamental set of objects: the *theoretical terms* of a theory. This set $\mathbb{T}$ shall be specified simply as a set of *atomic objects* of the basic structure frame, i. e., it may contain any label or any element of a basic sort. Thus, theoretical objects may be either 'properties' or 'things', without any general restriction.

In addition to these three sets that specify the ontology of a theory-element we will have only one more set $\mathcal{A}$ of *axioms* that contains all the necessary specifications of characterizations, laws, constraints and links, in the sense of the types of rules and axioms (2) to (5) that we mentioned in section 3.3, above. More precisely, a *theory-element* $\mathbf{T}$ (as based on a modal structure frame $\mathfrak{F}_m = (\mathfrak{F}, S_m, T_m)$ and a modal structure $\mathfrak{M}$) is specified in the following form:

$$\mathbf{T} = (B_p, \mathbb{L}_p, \mathbb{T}, \mathcal{A}),$$

and we have:

1. for every $s \in B_p$ there is a basic sort $s'$ with $s \subseteq s'$. This sort shall be picked out by the function $\tau(s)$.

2. $\mathbb{L}_p \subseteq \mathbb{L}$ is a set of labels of principal types.

3. $\mathbb{T} \subseteq \mathbb{S}_a$ is a set of atomic objects of the basic structure frame.

4. $\mathcal{A}$ is a set of $L_r(\mathfrak{F}, S_m, T_m)$-formulas.

Given these specifications we define *the ontology* $\mathbb{O}$ of a theory as

$$\mathbb{O} := \left( \bigcup_{s \in B_p} s \right) \cup \mathbb{L}_p.$$

We sometimes may also consider the abbreviated description of a theory-element, in the form

$$\mathbf{T} = (\mathbb{O}, \mathbb{T}, \mathcal{A}).$$

For obvious reasons, we usually may also require that $\mathbb{T} \subseteq \mathbb{O}$.

As explicated above, the set $\mathcal{A}$ of axioms of the theory-element contains characterizations, laws, constraints, and links, in the sense of the classical framework. We distinguish between these four types of axioms, in the following way:

1. $\phi \in \mathcal{A}$ is a *characterization* iff there is either an object $x \in \mathbb{L}_p$ or a set of objects $x \in B_p$ such that every non-logical and non-auxiliary term in $\phi$ refers to $x$.

2. $\phi \in \mathcal{A}$ is a *law* iff it is not a characterization and every non-logical and non-auxiliary term in $\phi$ refers only to objects in $\mathbb{O}$.

3. $\phi \in \mathcal{A}$ specifies a *constraint* iff it specifies a unary property of sets of $\mathfrak{F}$-structures.

4. $\phi \in \mathcal{A}$ specifies a *link* iff it specifies a binary relation between $\mathfrak{F}$-structures.

We define the set $\mathcal{C}$ of characterizations, the set $\mathcal{L}$ of laws, the set $\mathcal{E}$ of constraint specifications ('$\mathcal{E}$' for 'Einschränkungen') and the set $\mathcal{V}$ of link specifications ('$\mathcal{V}$' for 'Verbindungen') as the respective subsets of $\mathcal{A}$. (Note that if there are formulas in $\mathcal{A}$ that are not specified as elements of one of these sets, then they have no effect for the specification of a theory.) – Given these specifications we sometimes may expand the specification of a theory-element to:

$$\mathbf{T} = (B_p, \mathbb{L}_p, \mathbb{T}, \mathcal{C}, \mathcal{L}, \mathcal{E}, \mathcal{V}).$$

Here, the different sets of axioms $\mathcal{C}, \mathcal{L}, \mathcal{E}, \mathcal{V}$ are introduced explicitly. (If a theory-element does not specify axioms of one of these four types we may leave out the respective set.)

The definition of a theory-element in (BMS, 89, DII-17) does not mention any of the elements that are contained in our definition. However, beside of the 'intended applications' that fall away in our account, we are able to introduce all the remaining elements of a 'theory-core'

$$(\mathbf{M_p}, \mathbf{M}, \mathbf{M_{pp}}, \mathbf{GC}, \mathbf{GL})$$

by means of some more or less obvious explicit definitions.

We start with the *potential models* $\mathbf{M_p}(\mathbf{T})$ (BMS, 15, DI-7) of a theory. Given the function $\mu$ that assigns to every $L_r(\mathfrak{F}, S_m, T_m)$-formula its models, the set of all potential models must be based on the set

$$\mu(\mathcal{C})$$

of all models of the characterization-axioms. However, we need some more restrictions here, because the potential models also have to restrict the models of a theory to exactly all these structures that specify positive properties, *only* for the principal objects of $\mathbb{O}(\mathbf{T})$. In other words: $\mathbf{M_p}(\mathbf{T})$ must not specify any positive property of any atomic object of a structure frame that is not contained in $\mathbb{O}(\mathbf{T})$. Thus, we define $\mathbf{M_p}(\mathbf{T})$ as the set of all models of $\mathcal{C}$ such that it additionally holds, for every $\mathfrak{S} \in \mathbf{M_p}$:

$$\forall o \in \mathbb{S}_p : o \notin \mathbb{O}(\mathbf{T}) \rightarrow o \notin \mathfrak{S}(\tau(o)),$$

i. e., every principal atomic object that does not belong to the ontology of $\mathbf{T}$ also does not exist in a structure of $\mathbf{M_p}$ (and this implies that it cannot have any positive property in that structure). Alternatively, the definition of $\mathbf{M_p}(\mathbf{T})$ can also be given by means of the following one-liner:

$$\mathbf{M_p}(\mathbf{T}) := \{\mathfrak{S} \mid \mathfrak{S} \in \mu(\mathcal{C}) \wedge [\forall o \in \mathbb{S}_p : o \notin \mathbb{O}(\mathbf{T}) \rightarrow o \notin \mathfrak{S}(\tau(o))]\}.$$

The *actual models* $\mathbf{M}(\mathbf{T})$ (BMS, 20) of a theory $\mathbf{T}$, then, are defined as:

$$\mathbf{M}(\mathbf{T}) := \mathbf{M_p}(\mathbf{T}) \cap \mu(\mathcal{L}).$$

For the definition of the 'partial potential models' of a theory we need the following auxiliary device: If $O \subseteq \mathbb{S}_a$ is a set of objects (i. e., of elements of any atomic sort) of a structure frame then $\mathfrak{S} \setminus O$ is the (well-defined) structure $\mathfrak{S}'$ such that:

1. $\mathfrak{S}'(s) = \mathfrak{S}(s) \setminus O$, for every $s \in S$
2. for every sequence of objects $c_1, \ldots, c_n$ with $c_i \in \mathfrak{S}'(\tau(c_i))$ it holds

$$(c_1, \ldots, c_n) \in \mathfrak{S}'(\tau(c_1), \ldots, \tau(c_n)) \leftrightarrow \\ (c_1, \ldots, c_n) \in \mathfrak{S}(\tau(c_1), \ldots, \tau(c_n))$$

Intuitively, $\mathfrak{S} \setminus O$ is the structure that results, if we 'subtract' the vocabulary out of $O$ from it. (Note that $O$ may contain both labels of relations and elements of basic sorts.) On this basis, the *partial potential models* $\mathbf{M_{pp}}(\mathbf{T})$ (BMS, 57, DII-3) of a theory $\mathbf{T}$ are defined as:

$$\mathbf{M_{pp}}(\mathbf{T}) := \{\mathfrak{S} \mid \exists \mathfrak{S}' : \mathfrak{S}' \in \mathbf{M_p}(\mathbf{T}) \wedge \mathfrak{S} = \mathfrak{S}' \setminus \mathbb{T}\}$$

The partial potential models are all these models of a theory-element that does not ascribe positive properties to any theoretical object of the theory-element.

The constraints and links are integrated into this picture in two steps. First, we define, for every $\phi \in \mathcal{E}$ and every $\phi' \in \mathcal{V}$, *the constraint* $\mathbf{C}(\mathbf{T}, \phi)$ (BMS, 47, DII-2) and *the abstract link* $\mathbf{L}(\mathbf{T}, \phi')$ (BMS, 61, DII-4):

$$\mathbf{C}(\mathbf{T}, \phi) := \{X \subseteq \mathbf{S} \mid \phi(X)\}$$
$$\mathbf{L}(\mathbf{T}, \phi') := \{\mathfrak{S} \in \mathbf{S} \mid \phi'(\mathfrak{S})\}$$

Here, $\mathbf{S}$ is the set of all $\mathfrak{F}$-structures. – On this basis we define the *global constraints* $\mathbf{GC}(\mathbf{T})$ (BMS, 78, DII-10) and the *global links* $\mathbf{GL}(\mathbf{T})$ (BMS, 78f, DII-11) of the theory:

$$\mathbf{GC}(\mathbf{T}) := \bigcap_{\phi \in \mathcal{E}(\mathbf{T})} \mathbf{C}(\mathbf{T}, \phi)$$
$$\mathbf{GL}(\mathbf{T}) := \bigcap_{\phi \in \mathcal{V}(\mathbf{T})} \mathbf{L}(\mathbf{T}, \phi)$$

And we obtain obvious definitions for the *theoretical content* $\mathbf{Cn_{th}}(\mathbf{T})$ (BMS, 82, DII-13), and the *content* $\mathbf{Cn}(\mathbf{T})$ (BMS, 85, DII-15) of a theory:

$$\begin{aligned}
\mathbf{Cn_{th}}(\mathbf{T}) &:= \wp(\mathbf{M}(\mathbf{T})) \cap \mathbf{GC}(\mathbf{T}) \cap \wp(\mathbf{GL}(\mathbf{T})) \\
\mathbf{Cn}(\mathbf{T}) &:= \{X \mid \exists X' \in \mathbf{Cn_{th}}(\mathbf{T})\ \exists\theta : \\
&\quad\ \theta \text{ is a bijective function from } X \text{ to } X' \\
&\quad\ \wedge \forall\, \mathfrak{S}, \mathfrak{S}' : \mathfrak{S} = \theta(\mathfrak{S}') \rightarrow \mathfrak{S} = \mathfrak{S}' \setminus \mathbb{T}(\mathbf{T})\}
\end{aligned}$$

The theoretical content simply combines all these restrictions for the models of a theory that are contained in the axioms: characterizations, laws, links, and constraints. The content, then, restricts the theoretical content to these sub-structures that we obtain if we subtract the theoretical vocabulary $\mathbb{T}$. Thus, the really significant aspects of a theory-element, concerning questions of truth and empirical adequacy, are $\mathbf{Cn_{th}}(\mathbf{T})$ and $\mathbf{Cn}(\mathbf{T})$.

### 4.2. An example: classical particle mechanics

We finally describe one typical example of a theory-element, in order to show how to transform examples from the classical presentation (BMS) to the rigid framework. The example is classical particle mechanics, as presented in (BMS, III.3). – The theory-element of classical particle mechanics **CPM** is defined as:

$$\mathbf{CPM} := (B_p, \mathbb{L}_p, \mathbb{T}, \mathcal{C}, \mathcal{L}, \mathcal{E}).$$

Here are the specifications of the respective components:

$B_p$ := $\{P, T, S\}$.
$\mathbb{L}_p$ := $\{c_1, c_2, s, m, f\}$.
$\mathbb{T}$ := $\{m, f\}$
$\mathcal{C}$ := (1) $P$ is a finite, non-empty set.
(2) $c_1 : T \mapsto \mathbb{R}$ and $c_2 : S \mapsto \mathbb{R}^3$ are bijective.
(3) $s : P \times T \mapsto S$ and $c_2 \circ s_p \circ \check{c}_1$ is smooth for all $p \in P$.
(4) $m : P \mapsto \mathbb{R}^+$.
(5) $f : P \times T \times \mathbb{N} \mapsto \mathbb{R}^3$
$\mathcal{L}$ := $\forall p, \alpha : m(p) D^2 c_2 \circ s_p \circ \check{c}_1(\alpha) = \sum_{i \in \mathbb{N}} f(p, \check{c}_1(\alpha), i)$
$\mathcal{E}$ := (1) $\forall \mathfrak{S}, \mathfrak{S}' \in X\ \forall p : p \in \mathfrak{S}(P) \cap \mathfrak{S}'(P) \rightarrow \mathfrak{S}(m(p)) = \mathfrak{S}'(m(p))$.
(2) $\exists \circ : \pi \times \pi \mapsto \pi$ where $\pi := \bigcup\{\mathfrak{S}(P) \mid \mathfrak{S} \in \mathbf{M_p}(\mathbf{CPM})\}$ and: $\forall \mathfrak{S} \in X\ \forall p, p' \in \mathfrak{S}(P) : (p \circ p' \in \mathfrak{S}(P)) \rightarrow \mathfrak{S}(m(p \circ p')) = \mathfrak{S}(m(p)) + \mathfrak{S}(m(p'))$
(3) $\forall \mathfrak{S}, \mathfrak{S}' \in X\ \forall p \in \mathfrak{S}(P) \cap \mathfrak{S}'(P)\ \forall t \in \mathfrak{S}(T) \cap \mathfrak{S}'(T)\ \forall i : \mathfrak{S}(f(p, t, i)) = \mathfrak{S}'(f(p, t, i))$

These specifications are more or less literal reproductions from (BMS, III.3). For our present purpose, the only important aspect of this specification is that it is in full accordance with the above specified formal rules. Note in particular that the three constraint-axioms all contain the free variable $X$. This variable is of type $\wp(\mathbf{S})$, i. e. it ranges over the power set of the set of all structures of the structure frame. As pointed out above (p. 15, remark (4)) a constraint is a property of sets of models of a theory, thus it formally has to be specified as a $\wp(\mathbf{S})$-property. (Following (BMS), we do not introduce links, in the context of this example.)

Characterization rule (1) refers to the set $\mathfrak{S}(P)$ that is picked out by a particular structure $\mathfrak{S}$. Thus, in spite of the limited size of $\mathfrak{S}(P)$, the respective sort $P$ may be an arbitrarily large set. In general, the sort $T$ may contain every possible temporal state of any system relevant to **CPM**, the sort $P$ every possible spatial object of any system relevant to **CPM**. The result should be a theory-element whose content $\mathbf{Cn}(\mathbf{T})$ converges with the intended applications of **CPM**, as characterized in (BMS, 107).

Note also that the choice of the respective sets $P$, $T$, and $S$ is the main pragmatic aspect in our specification. The stipulation of $P$, $T$ and $S$ restricts the amount of possible intended applications, but still leaves open enough space for specifying different instances of a theory with *different* sets of intended applications. In other words, those unintended models in the classical framework that are *completely* unintended (because their ontology does not match the ontology of the theory in question) are ruled out here, whereas we do obtain a particular amount of structures as *possible candidates* for different sets of intended applications. The 'larger' our structure frame is the larger the scope for the specification of different sets of intended applications will be (and vice versa).

## Notes

[1] The main source for this paper is (Balzer et al., 1987). Henceforth I quote this book always in the form (BMS, page), (BMS, section), (BMS, page, definition), etc. Other works on theory structuralism that I used here are especially (Sneed, 1971; Stegmüller, 1973; Stegmüller, 1979; Stegmüller, 1986; Balzer and Moulines, 1996; Balzer et al., 2000; Moulines, 2002).

[2] For a discussion of the problem of a logification of theory structuralism, in the context of classical logic and set theory, see (Rantala, 1980). Cf. also (Balzer and Moulines, 1996, ch. 12 and 13), where rigorous set theoretical and category theoretical foundations of theory structuralism are discussed. In contrast to these accounts, the present proposal may be seen as a rigorous logical foundation for theory structuralism.

[3] See (Carnap, 1928, §§ 100 and 179). See also (Leitgeb, 2011), where the 'Aufbau' is reconsidered, on the basis of an up-to-date formal framework.

[4] See (Author, 201xb).

[5] The classical presentation of this type of quantified modal logic is (Kripke, 1963).

[6] Note also that in the rigid framework purely structural descriptions are possible as well. Because a typical rigid logic is higher order the formulation of Ramsey sentences is not at all a problem here. Thus, one may decide to characterize the whole amount of theoretical and empirical objects exclusively by means of rigid designators (my proposal), but one also may decide to have a more traditional layout in which either the theoretical or the empirical objects or both are characterized in a purely structural way, by means of Ramsey sentences.

[7] See (Kripke, 1980).

[8] The technique of reducing a logic to propositional logic is developed in (Damböck, 2009).

[9] I found this slogan only in its indirect quotation in (van Fraassen, 1980, 65).

[10] See (Bourbaki, 1968, ch. IV) and (Stegmüller, 1979).

[11] Cf. (Niebergall, 2000) where it is demonstrated how to 'reduce' a particular empirical theory to a purely structural and empirically meaningless thing like 'propositional logic'. Another critical discussion of the structuralist account of reduction is provided in (Hoering, 1984).

[12] The discussion of reduction is one task that has to be postponed in the present paper, even though it seems to be one of the crucial aspects of the framework of theory structuralism. In the present context we only shall discuss the specification of single theory-elements, in a rigid framework. See also note 17, below.

[13] For an account of the development of the modern semantic picture of logic, from that rigid starting point, see (Goldfarb, 1979). The present account, indeed, is a plea to go back to the rigid roots of pure logic, wherever we are concerned with theories of the empirical sciences and not with theories of pure mathematics.

[14] See (Kripke, 1980). One should also note here that quantified modal logic is a version of predicate logic that shows a number of parallels with rigid logic. Cf. the first remark in section 2.2, below.

[15] See (van Fraassen, 1967; van Fraassen, 1969; van Fraassen, 1970) and (Beth, 1960).

[16] See (BMS, XVf).

[17] Some aspects of the dynamic of scientific theories are discussed in (Author, 201xa). At any rate, a more detailed account of 'theory nets' and reduction requires a detailed formalization of rigid logic, which is provided in (Author, 201xb). An adoption of rigid logic to more complex problems of theory structuralism is intended for future work of the author. This shall include, in particular, the problem of reduction, theory nets, and invariance principles as considered in (BMS, ch. IV-VI) and (Sneed, 1979).

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