Public debt and aggregate risk ⋆

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Abstract

This paper assesses the long-run optimal level of public debt in a non Ricardian framework with aggregate fluctuations. Households are subject to aggregate and idiosyncratic risk, face borrowing constraints and the incompleteness of markets prevents them from perfectly insuring against risk. We find that aggregate fluctuations modify the cost and the motive for precautionary saving. Higher levels of public debt, by effectively reducing the cost of precautionary saving, help agents to smooth consumption when they face price and employment fluctuations. The long-run optimal level of public debt is higher in an economy with aggregate fluctuations than in an economy without.

Key words : public debt, aggregate risk, precautionary saving, credit constraints

JEL classification: E32, E62, H31

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Introduction

According to the Ricardian approach of taxes and deficits, changes in the level of public debt do not have any effect on household decisions as the timing of taxes is unimportant. This view is at odds with empirical evidence (see for instance Bernheim (1987)) and has resulted in the development of a vast literature considering deviations from the initial Ricardian setting as introduced by Barro (1974). Woodford (1990), for instance, departs from the complete market framework to consider credit constraints or as he phrases it "imperfect financial intermediation". In a simple economy embedding credit constraints, Woodford (1990) finds that changes in the level of public debt can be welfare enhancing because they can reduce the gap between the interest rate and the time preference rate. Aiyagari and McGrattan (1998) then quantitatively address the question of the optimal level of public debt in a heterogeneous agents, incomplete market model. Calibrating on the U.S. economy, they find an annual debt over GDP ratio of about 67% to be optimal. Floden (2001) uses a similar framework to look at public debt/transfers optimal combinations.

These authors underline the strong uncertainty and inequality implications behind public debt policies. An important finding is that in a non Ricardian setting with distortionary taxation, idiosyncratic risk, credit constraints and incomplete insurance markets, public debt has a key impact on the precautionary saving decision of households and subsequently on welfare. However, by considering only idiosyncratic risk, the literature above abstracts from one important macroeconomic feature: the cyclical behavior of the economy.

The introduction of uninsurable risk in the literature on the cost of business cycles has generated rich implications. In an effort to reconsider the small welfare effect of business cycles found by Lucas (1987), several authors such as Krusell and Smith (2002) or Storesletten, Telmer and Yaron (2001) have emphasized the distributional effects of aggregate fluctuations. In other words, the aggregate productivity shock is correlated with individual specific shocks to produce cyclicality in the cross-sectional variance of idiosyncratic risk. In such a framework, these authors find a greater cost of business cycles than Lucas (1987). Even more interestingly, Imrohoroglu (1989) briefly suggests that economic policies could be used to help individuals reduce the cost of business cycles. In this paper, we explore this latter suggestion and its implications for the optimal level of public debt.

This paper's main objective is to introduce a simple framework exhibiting aggregate fluctuations in a non Ricardian economy to quantify the long-run optimal level of public debt. We argue that the cyclical behavior of the economy will influence the optimal level of public debt as prices and employment fluctuation might affect the precautionary saving motive of households. Also, we examine how the level of public debt has an impact on household decisions along the phases of the cycle and across the population.

Aggregate fluctuations are modelled following the approach of Den Haan (1996) or Krusell and Smith (1998). Along the lines of Bewley-Huggett-Aiyagari type models, we build a production economy with capital market imperfections where a large number of ex-ante identical infinitely-lived agents face idiosyncratic income shocks and aggregate productivity shocks. Households' saving behavior is influenced by precautionary saving motives and borrowing constraints. Private capital and government bonds, both yielding the same interest, can be claimed to insure against future risk. Government levies proportional taxes on households and issues debt in order to finance its consumption.

Our main finding is that the long-run optimal level of public debt is 5% of annual GDP for an economy calibrated on the U.S.. The gains of being at the optimal level of debt instead of the benchmark debt to GDP ratio of 67% amounts to 0.257% of consumption. The consumption gains of being at the optimal level of debt are higher in recessions than in expansions and higher for the poorest percentile of the population. Moreover, the consumption gains we find are larger than those reported in Aiyagari and McGrattan (1998).

The intuition behind our results is the following. Credit constraints and uncertainty lead agents to engage in precautionary saving. The result of precautionary saving is a higher level of capital that in turn lowers the interest rate away from the time preference rate. Aggregate fluctuations, as it is correlated with the labor market process, exacerbate the level of risk faced by agents. An *employment fluctuation effect* increases both the risk of losing one's job and the time one could spend in unemployment in recessions. This strengthens households' precautionary saving motive. A *price fluctuation effect* changes the level of prices between recessions and booms making it more costly to accumulate precautionary saving in recessions. As a result of these effects, the capital stock rises and the interest rates move further away from the time preference rate. A higher level of public debt has opposing effects on welfare. A *crowding out effect* crowds out private capital and reduces welfare. A *cost of precautionary saving effect* increases the interest rate, reduces the cost of precautionary saving and enhances welfare. The optimal level of public debt we find balances the welfare decreasing and increasing effects.

We also compare our benchmark economy to a model without aggregate fluctuations. The latter model yield a lower optimal level of public debt of 2.5% of annual GDP. Because the *employment fluctuation effect* and the *price fluctuation effect* change the precautionary saving motive and cost, the *cost of precautionary saving effect* is stronger in an aggregate fluctuations setting than in an idiosyncratic risk setting. Thus the optimal level of public debt is higher in a setting with aggregate fluctuations.

The rest of the paper is organized as follows. Next section describes the benchmark economy. Section 2 details our main results. The last section concludes.

1 The Model

Our benchmark economy is a Bewley-Huggett-Aiyagari type dynamic stochastic general equilibrium model augmented to allow aggregate fluctuations à la Den Haan (1996) or Krusell and Smith (1998) and public debt. Insurance markets are incomplete. Agents face idiosyncratic and aggregate risks and are borrowing constrained. These three assumptions lead agents into precautionary saving (Aiyagari (1994)).

Our benchmark model can be related to the model in Aiyagari and McGrattan (1998) but presents three deviations of importance apart from its aggregate fluctuations feature. First, the productivity process here is simpler as agents can only be employed or unemployed. Second, leisure is not valued. Those two deviations greatly simplify the model with aggregate fluctuations. Third, there is no exogenous growth in our benchmark economy. Aiyagari and McGrattan (1998) specify an exogenous annual growth rate of 1.85%. This assumption exogenously reduces the cost of public debt for individuals since public debt interest repayments are diminished by this exogenous growth factor. It is noteworthy that making this assumption lead to a higher optimal level of public debt.

1.1 Firms

We assume that there is a continuum of firms which have a neoclassical production technology and behave competitively in product and factor markets. The output is given by:

$$Y_t = z_t F(K_t, N_t)$$

where K is aggregate capital and N aggregate labor used in production. The function F exhibits constant returns to scale with respect to K and N, has positive and strictly diminishing marginal products, and satisfies the Inada conditions. Capital depreciates at a constant rate δ.

The economy is subject to an exogenous aggregate shock noted z. There are two possible aggregate states: a good state where $z = z_g$ and a bad state where $z = z_b$. The aggregate shock follows a first-order Markov process with transition probability $\eta_{z|z'} = \Pr(z_{t+1} = z'/z_t = z)$. Thus $\eta_{z|z'}$ is the probability that the aggregate state tomorrow is z' given that it is z today. We note η the matrix that describes the transition from one aggregate state to another such that:

$$\eta = \begin{pmatrix} \eta_{gg} & \eta_{gb} \\ \eta_{bg} & \eta_{bb} \end{pmatrix}$$

Finally, our setting assumes that inputs market are competitive. The wage w and the interest rate r verify:

$$r_t + \delta = z_t F_K(K_t, N_t) \tag{1}$$
$$w_t = z_t F_N(K_t, N_t) \tag{2}$$

1.2 The government

The government issues public debt and levies taxes to finance public expenses. Both the revenue of capital and labor are taxed proportionally at an identical rate τ. The government's budget constraint verifies:

$$G_t + r_t B_t + TR_t = B_{t+1} - B_t + T_t$$

with

$$T_t = \tau(w_t N_t + r_t A_t)$$

G_t is the level of public expenses, B_t the level of public debt, T_t tax revenues and TR_t a lump sum transfer to households that amounts to zero at the equilibrium [1] . A_t accounts for total average wealth in the economy. It is the sum of average physical capital K and public debt B such that $A_t = K_t + B_t$.

Also note that to keep the model as simple as possible, no asset pricing component is introduced here. Thus the interest on government bond is the same as the return on private capital. It is arguable that the existence of a safe asset might provide households with an additional means of insurance but given the long-run structure of this particular model, the expected effects of such an extension are small.

1.3 Households

The economy is populated by a continuum of *ex ante* identical infinitely lived households of unit mass. Their preferences are summarized by the func-

[1] Because of the aggregate fluctuations property of the model, we have to be cautious about how we close the model with respect to the government budget constraint. More details can be found in Appendix A.

tion V:

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j u(c_t) \right) \right\} \tag{3}$$

where β is the discount factor. We assume that this discount factor is random. Thus the discount factor β can differ across agents and can vary over time. We specify that the latter follows a three-states first-order Markov process. This assumption on the discount factor helps to reproduce the wealth distribution as shown in Krusell and Smith (1998). The discount factor verifies:

$$\begin{cases} \beta_0 = 1 \\ \beta_{j\geq 1} \in]0;1[\end{cases}$$

c_t is the household level consumption. The utility function we use has a standard CRRA specification and writes:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} \text{ if } \sigma \neq 1 \\ \log(c) \text{ if } \sigma = 1 \end{cases}$$

Agents are subject to idiosyncratic unemployment shocks. Let s be the household's labor market status. A household can either be unemployed ($s = u$) or employed ($s = e$). Households are also subject to shocks at the aggregate level. Aggregate shocks exacerbate idiosyncratic unemployment risks. The unemployment rate and the unemployment duration are higher in recessions than in booms. Therefore, transitions on the labor market are correlated to the aggregate state. We note $\Pi_{zz'|ss'}$ the joint transition probability to a state (s', z') conditional on a state (s, z). The matrix that jointly describes the transition from a state (s, z) to a state (s', z') is the following:

$$\Pi = \begin{pmatrix} \Pi_{bbuu} & \Pi_{bbue} & \Pi_{bguu} & \Pi_{bgue} \\ \Pi_{bbeu} & \Pi_{bbee} & \Pi_{bgeu} & \Pi_{bgee} \\ \Pi_{gbuu} & \Pi_{gbue} & \Pi_{gguu} & \Pi_{ggue} \\ \Pi_{gbeu} & \Pi_{gbee} & \Pi_{ggeu} & \Pi_{ggee} \end{pmatrix}$$

where $\Pi_{ggee} = \Pr(z_{t+1} = z_g, s_{t+1} = e | z_t = z_g, s_t = e)$.

When agents are in an employed state, they receive the wage w. However when agents are unemployed their income corresponds to their home production that we note θ. Insurance markets are incomplete so that agents can only partially self-insure against idiosyncratic risk. Following Aiyagari and McGrattan (1998) no borrowing is allowed. The only way for households to self-insure against idiosyncratic risk is to accumulate physical capital and government bonds both yielding the same return r. Their overall holding in the later assets is noted a. Therefore a typical household solves the following problem:

$$\max_{c_t,\, a_{t+1}} \mathbb{E}_0\{\sum_{t=0}^{\infty}(\prod_{j=0}^{t}\beta_j)u(c_t)\},$$

subject to:

$$a_{t+1} + c_t = (1 + (1-\tau)r_t)a_t + \chi(s_t)w_t + TR_t$$
$$c_t \geq 0$$
$$a_{t+1} \geq 0$$

with

$$\chi(s_t) = \begin{cases} \theta \text{ if } s_t = u, \\ (1-\tau) \text{ if } s_t = e \end{cases}$$

The existence of aggregate risk leads us to distinguish between individual state variables and aggregate state variables. The individual state variables are given by the vector (a, s, β). The aggregate state variables are summarized by the vector (z, Γ) where $\Gamma(a, s, \beta)$ is a distribution of agents over asset holdings, employment status and preferences. To determine the wage and the interest rate, households need to forecast the aggregate stock of physical capital. Therefore, they need to know the wealth distribution. That is why wage and interest rate depend on that wealth distribution. In the computational appendix, we explain how we avoid manipulating the wealth distribution by approximating it with some of its moments using the methodology developed in Den Haan (1996) and Krusell and Smith (1998). We detail the computational strategy we used to solve the model in appendix A.

1.4 Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s, \beta; z, \Gamma), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital $K(z, \Gamma)$, factor prices $\{r(z, \Gamma), w(z, \Gamma)\}$, tax rate τ and a law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$ which satisfy these conditions:

(i) Given the aggregate states, $\{z, \Gamma\}$, prices $\{r(z, \Gamma), w(z, \Gamma)\}$ and the law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$, the decision rules $\{c(a, s, \beta; z, \Gamma), a'(a, s, \beta; z, \Gamma)\}$ solve the following dynamic programming problem:

$$v(a, s, \beta; z, \Gamma) = \max_{c,\ a'} \{u(c) + \beta E\left[v(a', s', \beta'; z', \Gamma')|(s, \beta; z, \Gamma)\right]\}$$

subject to:

$$c + a' = (1 + r(z, \Gamma)(1 - \tau))a + w(z, \Gamma)\chi(s) + TR$$
$$c \geq 0$$
$$a' \geq 0$$

and

$$\Gamma' = H(\Gamma, z, z')$$

(ii) Market price arrangements are:

$$r(z, \Gamma) = zF_K(K, N) - \delta$$
$$w(z, \Gamma) = zF_N(K, N)$$

(iii) Government budget constraint holds.

(iv) Capital market verifies:

$$K + B = \int a'(a, s, \beta; \Gamma, z) d\Gamma$$

(v) Consistency: agents' optimization problem is satisfied given the law of motion H and the law of motion is consistent with individual behavior.

1.5 Calibration

For the sake of comparison, the model economy is calibrated to match certain observations in the U.S. data. We let one period in the model be one quarter in the data. To avoid confusion we have converted all values to their annual equivalents throughout the paper. Quarterly values will only be discussed in this calibration section. To remain simple and allow comparisons, we closely follow Krusell and Smith (1998) when calibrating the characteristics of the labor market and the aggregate risk.

1.5.1 Technology

We choose the production function to be Cobb-Douglas:

$$Y_t = z_t F(K_t, N_t) = z_t K_t^{\alpha} N_t^{1-\alpha} \quad 0 < \alpha < 1$$

Technology parameters are standard. The capital share of output α is set to 0.36 and the capital depreciation rate δ is 0.025. As Krusell and Smith (1998) we assume that the value of the aggregate shock z is equal to 0.99 in recessions (z_b) and 1.01 in booms (z_g).

1.5.2 Preferences and discount factor

In the benchmark economy we assume a logarithmic utility function. We now detail the calibration steps to generate a realistic wealth distribution, the observed U.S. wealth Gini index and the capital-output ratio. Here, our calibration differs from Krusell and Smith (1998). In their economy agents can borrow whereas here, for the sake of comparison and simplicity, we follow Aiyagari and McGrattan (1998) and no borrowing is allowed.

To reproduce the shape of the U.S. wealth distribution we first assume that unemployed agents receive income too and fix the home production income θ to be 0.10[2]. This assumption produces a large group of poor agents. Next we use the preference heterogeneity setting discussed in Krusell and Smith (1998) to generate a long thick right tail[3]. We impose that the discount factor β takes on three values $\{\beta_l, \beta_m, \beta_h\}$ where $\beta_l < \beta_m < \beta_h$:

$$\begin{pmatrix} \beta_l \\ \beta_m \\ \beta_h \end{pmatrix} = \begin{pmatrix} 0.9750 \\ 0.9880 \\ 0.9985 \end{pmatrix}$$

Thus an agent with a discount factor β_m is more patient than an agent with a discount factor β_l. To calibrate the transition matrix, we impose that the invariant distribution for discount factors has 10% of the population at the lowest discount rate β_l, 70% at the medium discount factor β_m and 20% at the highest discount factor β_h. As Krussell and Smith (1998) we assume that there is no immediate transition between extreme values of the discount factors. Finally, we set the average duration of the lowest discount factor and

[2] This corresponds to about 12% of the average wage at the equilibrium.

[3] There are several ways to reproduce a thick right tail. One would be to give rich agents higher propensity to save or higher returns on saving for instance by introducing entrepreneurs (e.g., Quadrini (2000)). For the sake of simplicity we explore here the preference heterogeneity setting introduced by Krusell and Smith (1998).

the highest discount factor to be 50 years (200 quarters) to roughly match the length of a generation. These assumptions yield the following transition matrix [4] :

$$\Upsilon = \begin{pmatrix} 0.9950 & 0.005 & 0.0000 \\ 0.0007 & 0.9979 & 0.0014 \\ 0.0000 & 0.0050 & 0.9950 \end{pmatrix}$$

The results of this calibration are shown in Table 1. The preference heterogeneity setting helps to reproduce the shape and the skewness of the U.S. wealth distribution [5] and yields a Gini index of 0.82. The quarterly capital-output ratio we target here is 10.6 [6] .

1.5.3 Labor market processes

Our calibration of the aggregate shock and the labor market process follows Krusell and Smith (1998). The process for z is set so that the average duration of good and bad times is 8 quarters. Therefore, the transition matrix η for aggregate state changes is defined by:

$$\eta = \begin{pmatrix} 0.8750 & 0.1250 \\ 0.1250 & 0.8750 \end{pmatrix}$$

The average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times. We also set the unemployment rate accordingly: in good periods it is 4% and in bad periods it is 10%. These assumptions enable us to define the transition matrixes for labor market status for each aggregate state change: Π^{gg} for a transition from a good period to a good period, Π^{bb} for a transition from a bad period to a bad period, Π^{gb} for a transition from a good period to a bad period and Π^{bg} for a transition from a bad period to a good period [7] :

[4] For further details on the calibration of this matrix, see appendix B.
[5] The data we report on the U.S. distribution comes from Krusell and Smith (1998) and Budria-Rodriguez, Diaz-Gimenez, Quadrini and Rios-Rull (2002).
[6] The value of the capital-output ratio can change with the definition of capital. Here we adopt the definition in Quadrini (2000). Thus aggregate capital results from the aggregation of plant and equipment, inventories, land at market value, and residential structures. This definition is close to the findings of Prescott (1986) and is also used for instance in Floden and Linde (2001). This yields a capital-output ratio of 2.65 on an annual basis that we convert to its quarterly equivalent of 10.6.
[7] Further details on this calibration can be found in Appendix B.

$$\Pi^{bb} = \begin{pmatrix} 0.6000 & 0.4000 \\ 0.0445 & 0.9555 \end{pmatrix} \quad \Pi^{bg} = \begin{pmatrix} 0.2500 & 0.7500 \\ 0.0167 & 0.9833 \end{pmatrix}$$

$$\Pi^{gb} = \begin{pmatrix} 0.7500 & 0.2500 \\ 0.0729 & 0.9271 \end{pmatrix} \quad \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

Finally the joint transition matrix Π for labor market statuses and aggregate states can be defined as:

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix} = \begin{pmatrix} 0.5250 & 0.3500 & 0.0313 & 0.0938 \\ 0.0388 & 0.8361 & 0.0021 & 0.1229 \\ 0.0938 & 0.0313 & 0.2916 & 0.5833 \\ 0.0911 & 0.1158 & 0.0243 & 0.8507 \end{pmatrix}$$

1.5.4 Government

We fix the ratio of government purchases to GDP to 0.217. The debt over GDP ratio, noted b is set to the quarterly value of 267% which is equivalent to an annual value of 67%. Those values are the observed ratios in the U.S. as reported by Aiyagari and McGrattan (1998).

2 Results

We now present the results obtained with our benchmark economy. A first section reports the aggregate behavior of the model. A second examines the long-run welfare effects of public debt in an aggregate fluctuations setting. In a third, we move on to the business cycle and distributional effects of public debt. In the next to last section we compare the benchmark economy to simpler idiosyncratic risk only models. Finally, we run a robustness test with an alternative calibration that changes the labor market process.

2.1 Public debt in an aggregate fluctuations setting

We start by discussing the aggregate behavior of our benchmark model when our policy experiment is carried out. This experiment consist in changing the long-run debt to GDP ratio in the economy. Our computations are reported in Figure 1. Increasing the level of public debt increases the supply of assets in the economy. Consequently, the before tax interest rate increases. Because the repayment of debt interests is higher, the income tax rate increases. Nevertheless, the after tax interest rate unambiguously increases. In turn, public debt has a crowding out effect on private capital: higher levels of debt decrease the aggregate amount of private capital in the economy. The crowding out of capital induces the observed decline in output[8].

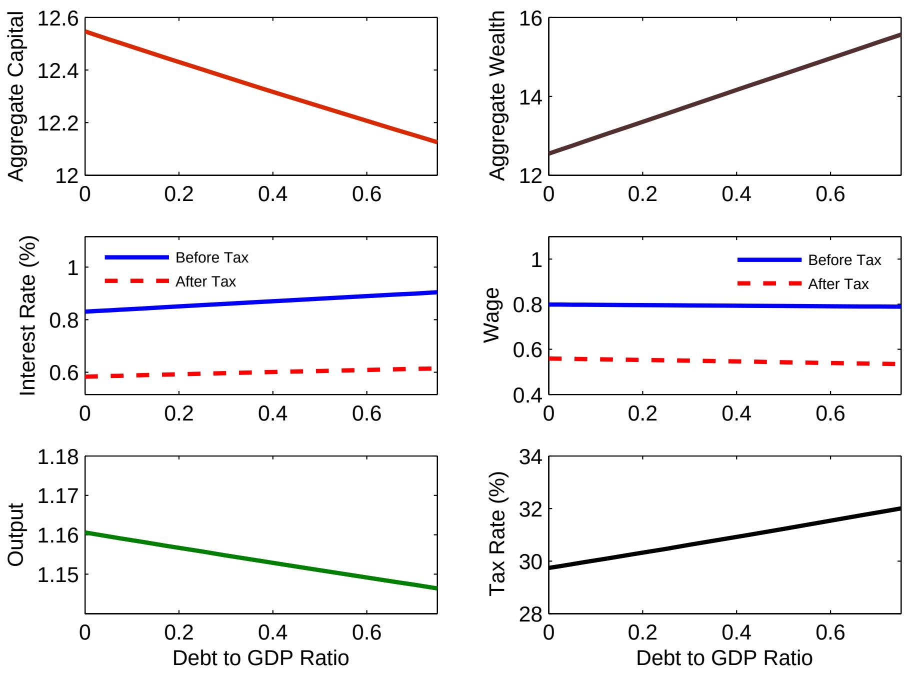

Fig. 1. Aggregate behavior of the benchmark model

However, the decline in physical capital is smaller than the increase in public debt. The increase in the after-tax interest rate reduces the gap between the after-tax interest rate and the rate of time preference. The cost of postponing consumption to build up a buffer stock of saving is then reduced. Households choose to hold more assets at the steady-state equilibrium. That is why the overall private wealth level, which is the combination of private capital and public debt, is higher.

[8] In the absence of a crowding out effect of physical capital, the increase in wealth would have been the same as the increase in public debt. The steady state consumption would have been higher.

2.2 *Welfare analysis and optimal level of debt*

The welfare analysis we conduct below apply to the long-run optimal level of public debt with aggregate fluctuations. Because the aggregate variables are not constant, not even in the limit, we consider for our welfare analysis the values of aggregate variables averaged over long periods of time for a given debt to GDP ratio [9]. We define the optimal level of public debt as the debt over GDP ratio that maximizes the traditional utilitarian welfare criterion μ.

As explained in Lucas (1987) and Mukoyama and Sahin (2006), this criterion measures the amount of consumption that one would have to remove or add in order to make the agent indifferent between the benchmark debt over GDP ratio and some other level of public debt. It verifies:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log((1+\mu)c_t^{bench.})\right] = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t)\right]$$

with $\left\{c_t^{bench.}\right\}_{t=0}^{\infty}$ the consumption stream in the benchmark model when the debt over GDP ratio is equal to 8/3. $\{c_t\}_{t=0}^{\infty}$ is the consumption stream when the debt over GDP ratio is set to some other level than the benchmark level. For logarithmic utility we can show that:

$$\mu = \exp\left(\left[V - V^{bench.}\right]/S\right) - 1,$$

where $V^{bench.} = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t^{bench.})\right]$, $V = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t)\right]$ and $S = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\right]$.

The result of the introduction of public debt on welfare is *a priori* undetermined because of two opposing effects. The first effect is a *crowding out effect*. The crowding out of physical capital clearly reduces consumption and then welfare. Moreover the increase in the income tax rate tends to amplify the negative impact of public debt on welfare. The second effect is a *cost of precautionary saving effect*: the increase in the after-tax interest rate makes it less costly to accumulate precautionary saving in order to smooth consumption as the interest rate gets closer to the time preference rate. This second effect is welfare enhancing [10]. It is difficult to predict which effect overcomes

[9] More details can be found in Appendix A.

[10] Woodford (1990) argues that welfare can be enhanced if the interest rates are kept high enough, that is, closer to time preference rates in a liquidity-constrained economy. We find the same effect here.

the other analytically.

Figure 2 depicts the long-run optimal level of debt in the benchmark economy. In a setting embedding aggregate risk and calibrated on the U.S. economy, the optimal public debt level is 5% of output on an annual basis. Aggregate fluctuations, idiosyncratic risk and credit constraints lead agents to engage in precautionary saving in order to smooth consumption. Without public debt, the cost of precautionary saving is higher because when households accumulate, the interest rate lowers. Any level of public debt, raises the interest rate and reduces the cost of precautionary saving. As a result welfare is enhanced. However any level of public debt also crowds out private capital and increases taxes, and thus reduces welfare.

Here, a lower level of debt than the benchmark level increases welfare out of reducing the crowding out of private capital. At the same time, a lower level of debt than the benchmark level decreases welfare out of increasing the cost of precautionary saving. As long as the first effect dominates the second, a lower level of debt is optimal. At the optimal level, one effect exactly balances the other. For a debt over GDP ratio smaller than 5%, the consumption loss out of increasing the cost of precautionary saving is higher than the consumption gain out of reducing the crowding out of private capital.

As illustrated in Table 2, going from the benchmark level of debt to the optimal level of debt is welfare enhancing. The consumption gain of being at the optimal level of debt instead of the benchmark level is 0.257%. On the contrary, it is welfare decreasing to go to a higher level of public debt than the benchmark level. In an economy where the level of debt is for instance 75%, the consumption loss not to be at the benchmark level (resp. optimal level) would be on average 0.072% (resp. 0.329%). It is noteworthy that the welfare profile we find is not as flat as what Aiyagari and McGrattan (1998) report. In their paper, the costs or gains of going to a broad set of levels of public debt around the optimal level are very low. For instance, they report that choosing a level of public debt of zero percent instead of 67%, which is the optimal level of debt they find, would cost only about 0.08% of consumption.

Our results suggest that the role the interest rate plays in reducing the cost of precautionary saving is central to understand why agents settle for a given optimal level of public debt.

If we block any price movements by considering a price fixed small open economy, it appears that there is no consumption gains or losses out of the *cost of precautionary saving effect*. In the case where all prices are set to their average benchmark level for any level of public debt, there is no consumption loss out of increasing the cost of precautionary saving for a lower level of public debt than the benchmark level. In this case, there are only consumption gains out of reducing the crowding out of private capital.

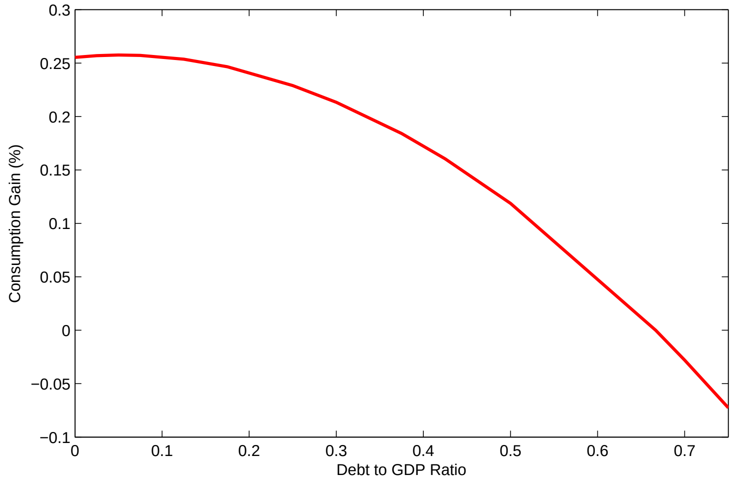

Fig. 2. Welfare gain versus debt/GDP ratio in the benchmark model

2.3 Business cycles and distributional effects

In this section we move on to the effects of public debt along the cycle and across the distribution of the population. To generate statistics along the cycle, we compute separately the average values of our aggregate variables either when the economy is in boom or when it is in recession over the whole sequence of simulation. The last two columns of Table 2 illustrate the consumption gain or loss of being at the optimal level of debt instead of the benchmark level. In recession, the consumption gain of being at the optimal level of debt is higher than on average. On the contrary, in expansion this consumption gain is lower. This is due to a *price fluctuation effect*: in recessions, the cost of postponing consumption to build up a buffer stock of savings is higher because the interest rate is lower as shown in Table 3. In expansion, we have the opposite: interest rates are higher and the cost of precautionary saving is lower.

The fluctuation of prices is not the only relevant effect we need to account for to explain the optimal level of public debt. When we take aggregate risk into consideration, an *employment fluctuation effect* modifies the precautionary saving motive. Along the cycle, the unemployment rate and duration increase in recessions and reduce in booms. In recessions, the precautionary motive becomes stronger: employed agents face a higher risk of loosing their job and unemployed agents find it more difficult to find a job. In recessions agents want to save more for precautionary motives because of the *employment fluctuation effect* but at the same time the *price fluctuation effect* raises the cost of saving. Thus public debt helps to reduce the cost of precautionary saving in recessions. In booms, agents want to save less because of the *employment fluctuation effect*. At the same time the *price fluctuation effect* makes it less costly to save. Thus public debt is less useful in booms.

The overall consumption gains or losses of a change in the level of public debt is shared very differently in the population. To show that we decompose the welfare gains of going from the benchmark level of public debt to the optimal level across the population. The last two rows of Table 2 show this decomposition. For instance, the row *Bottom* 10% shows the welfare gap between the 10% least fortunate people living in an economy with the benchmark level of public debt and those living in an economy with the optimal level of public debt. This decomposition closely matches a decomposition by wealth levels as the lowest (resp. highest) expected utilities refer to people who have experienced the highest unemployment (resp. employment) spells and who end up with the lowest (resp. highest) level of assets.

A change in policy that consist in going from the benchmark level of debt to the optimal debt level leads to an increase in consumption of 0.885% for the poorest 10% of the population. Thus the poorest agents are better off with low levels of public debt. In the meantime, the richest 10% of the population would loose as much as 10.401% of consumption. This is explained by the fact that rich people's income is mainly capital income whereas poor people's income is mostly labor income. Thus when the level of public debt is higher, poor people suffers from the reduction in output caused by higher tax rates, lower wages and crowding out of capital. On the contrary, as interest rates raise with higher level of public debt, rich people are better off. The same type of effects are discussed in Ball and Mankiw (1995) and Floden (2001).

2.4 Optimal level of debt without aggregate fluctuations

In this section we look at setups without aggregate fluctuations and compare them with our benchmark economy. There are several ways of eliminating business cycles[11] to derive a comparable long-run idiosyncratic model. We look at two methods.

2.4.1 Imrohoroglu (1989) method

In the spirit of Lucas (1987) we replace the aggregate stochastic process with its conditional mean. We then follow Imrohoroglu (1989) to derive the labor market process. The transition probabilities of this economy are set so that the average rate of unemployment and the average duration of unemployment are the same between this economy and the benchmark economy. All other calibrated parameters are kept to their benchmark values, especially time preference rates and the risk aversion parameter[12]. This model is similar

[11] For a survey, see Barlevy (2004).
[12] For greater details on the models without aggregate risk and their calibration, see appendix C.

to the benchmark model in many ways: higher levels of debt raise interest rates and crowd out private capital, overall wealth increases with debt and taxes are higher. But as the risk faced by agents is lower, agents save less and interest rates are higher in this model.

Figure 3 depicts the welfare profile we find in this idiosyncratic risk

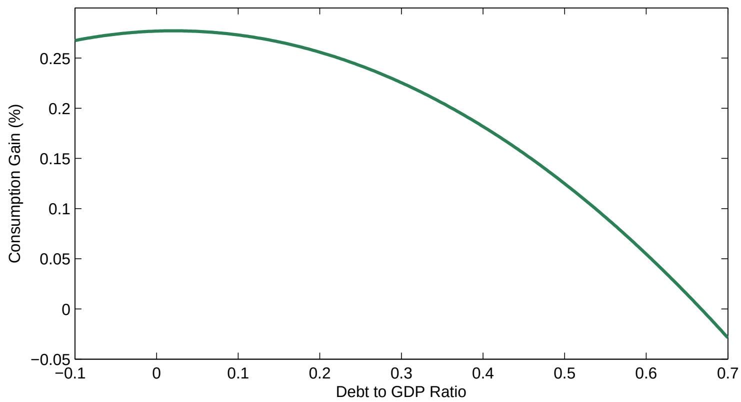

Fig. 3. Welfare gain versus debt/GDP ratio without aggregate fluctuations

model. The optimal level of public debt is 2.5% of output on an annual basis. This level is lower than the level we found in the benchmark economy. This is because there is no *price fluctuation effect* and *employment fluctuation effect* in this economy. In the benchmark economy, as the unemployment rate and the unemployment duration increase during recessions, the precautionary motive is stronger. Moreover, in the benchmark economy the interest rate and the physical capital are smaller in recessions. Therefore, it is more costly to save for precautionary motive. That is why the need for public debt is more important in the benchmark economy. In the idiosyncratic model the risk is lower because agents are subject to a less risky labor market and at the same time they no longer face price fluctuations. The precautionary motive is weaker here than in the benchmark model and agents save less. As a consequence interest rates are at a higher level in this economy.

2.4.2 Brute-force averaging

This method is related to the first one above. We again set the aggregate process to its conditional mean. The transition probabilities on the labor markets are generated as follows. We take the exact same transition process as in the benchmark economy and repeat it over very long periods of time. Thus we create a history of transition on the labor market conditional on the aggregate risk faced by agents. In the end of this simulation, we are left with a sequence of aggregate shocks and corresponding labor market situations for individuals. We then brute-force average the sequence of labor market transitions out. To do so, we count the numbers of transitions from unemployed to unemployed, from unemployed to employed, from employed to employed and from employed to unemployed over the entire simulated sequence and then take the mean. As we simulate over very long periods of time, we respect the law of large numbers. After brute-force averaging over the simulated series, we are left with a transition matrix on the labor market.

The matrix we get by brute-force averaging is very similar to the matrix obtained by the Imrohoroglu method above. The optimal level of public debt is 2.5% of output on an annual basis. We can characterize this optimal level in the same way as we did above.

2.5 Higher unemployment rate and longer unemployment spells

Our benchmark calibration reflects the behavior of the U.S. labor market. In this section we consider an alternative calibration of the labor market as a robustness exercise. This can be seen as a step towards reproducing European labor market features. However as it is not the subject here, we do not take into consideration or reproduce the employment benefit system found in Europe. Obviously, the large employment benefit system found in some European countries is a key element that would change the nature of the results.

We follow the methodology used in Algan and Allais (2004) and the data set of Blanchard and Wolfer (2000). We only modify the labor market features and leave the rest of the calibration unchanged. We fix the unemployment rate to be 13% in recessions and 7% in booms. We also set the duration of an unemployment spell to be 6 quarters in recessions and 4 quarters in booms. We now find the optimal level of debt to be 30% on an annual basis as depicted in Figure 4. Longer unemployment spells and higher unemployment rates tend to raise the optimal level of debt.

This calibration purposely strengthens the *employment fluctuation effect*: unemployed agents have a harder time finding a job in this economy when compared to the benchmark economy and employed agents face a higher risk of loosing their jobs. In recessions, this is amplified. The precautionary motive is stronger here than in the benchmark economy. The harder it is for households to smooth consumption and the higher is the need for public debt. Here a higher level of public debt is needed to help households effectively smooth consumption.

3 Conclusion

This paper reconsidered the optimal level of public debt in an environment of aggregate fluctuations. Our benchmark model calibrated on the U.S. economy finds that a positive public debt level of 5% of output on an annual

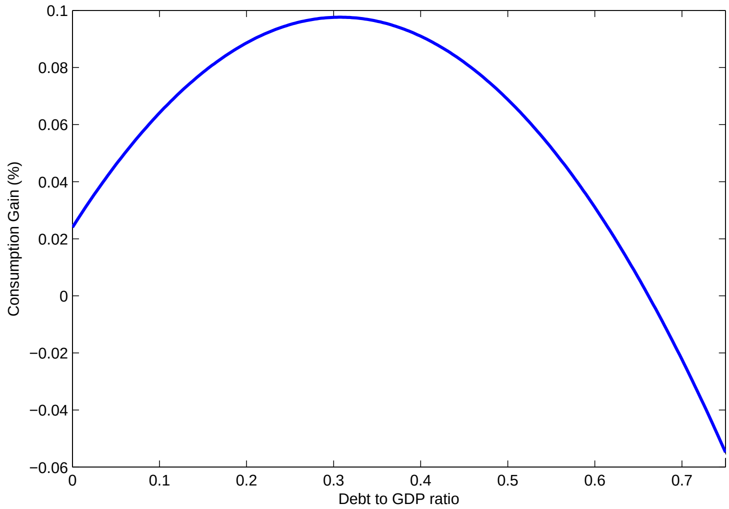

Fig. 4. Welfare gain versus debt/GDP ratio in the model with higher unemployment rate and longer unemployment spells.

basis is optimal. This level is higher than in an economy without aggregate fluctuations. Our benchmark economy shows that in an aggregate fluctuations setting, households are subject to an *employment fluctuation effect* and to a *price fluctuation effect*. These effects make the precautionary saving motive stronger and at the same time the cost of saving higher.

Our results suggest that a higher supply of assets induced by higher levels of public debt tends to raise the interest rate. This helps households to reduce the cost of precautionary saving and make smoothing of consumption easier. As a result there are welfare gains in the economy. However for the poorest agents, higher levels of debt are not optimal. Poor agents rely mainly on labor income and are dependent on higher wages and lower taxes. Public debt decreases wages and increases tax level thus poor agents prefer lower levels of public debt. We also emphasize that longer unemployment spells tend to raise the optimal level of public debt.

This paper focuses on long-run aspects of public debt with aggregate fluctuations. The transitional aspects will be dealt with in a sister paper. Other directions such as the countercyclical aspect of public debt or a time-consistent policy in this type of economies are all subject of upcoming research.

Appendix

A Computational strategy

A.1 Solving the model

We solve the model by using the methodology developed by Den Haan (1996) and Krusell and Smith (1998). They show that agents need only a restrictive set of statistics about the wealth distribution to determine prices. This set includes the mean of the wealth distribution and the aggregate productivity shock. A linear prediction rule based only on the average level of capital provides an accurate prediction. This result comes from the near linearity of the decision rule $a'(a, s, \beta; z, \Gamma)$. As the aggregate capital stock is mainly held by rich people who have approximately the same propensity to save, next period's aggregate capital is accurately predicted by current period's aggregate capital. In our model, we use the following prediction rules:

$$\log(\bar{K}') = a_0 + a_1 \log(\bar{K}), \text{ if } z = z_g$$
$$\log(\bar{K}') = c_0 + c_1 \log(\bar{K}), \text{ if } z = z_b$$

where $\bar{K}'$ and $\bar{K}$ denote respectively the average stock of capital of the next period and of the current period. Thus the strategy is the following:

(1) As the aggregate variables are not constant, even in the limit, in an economy with aggregate fluctuations, we approximate the steady state of this economy by averaging aggregate variables over long periods of time. We first make a guess for the average long-run interest rate [13]. From this we can deduce the long-run GDP and using the debt over GDP ratio and the public expenses ratio, we can derive the level of public debt and public expenses in this economy given our guess of the long-run interest rate. From the long-run budget constraint of the government, we then derive a valid long-run tax rate for the economy. Given the tax rate we just defined, we execute steps 2, 3 and 4 below. When those steps have converged, we update our guess of the long-run interest rate until a fix point is found.

(2) Given a set of parameter values (a_0, a_1, c_0, c_1) for the law of motion, we solve the individual problem. To solve the individual problem, we iterate on the Euler equation:

[13] Further details about this are given in the section A.2 below.

$$U'(c) = E\left[\beta' U'(c')(1 + r'(z', \Gamma')(1 - \tau))/s, \beta; z, \Gamma\right],$$

on a discrete grid until a fix point is found. When the borrowing constraint bind, the solution can be deduced from the budget constraint.

(3) Given the parameter values for individual decision rules, we solve the aggregate problem *i.e.* the coefficients of the law of motion.

(4) If the parameters (a_0, a_1, c_0, c_1) found are close to the parameter values used to solve step (2), the algorithm has converged. Otherwise steps (2), (3) and (4) are repeated until convergence.

When all steps are complete, the long-run average interest rate coincides with the average interest rate in the actual economy with aggregate fluctuations, the government budget constraint is balanced and the lump sum transfer to individuals amounts to zero at the equilibrium.

All results in the model are derived from simulated data. The simulated sample consist of 11000 periods and the first 1000 are discarded. The distribution is approximated by a sample of 30000 households at each period. The algorithm is implemented in the $C++$ language.

A.2 Tax rule

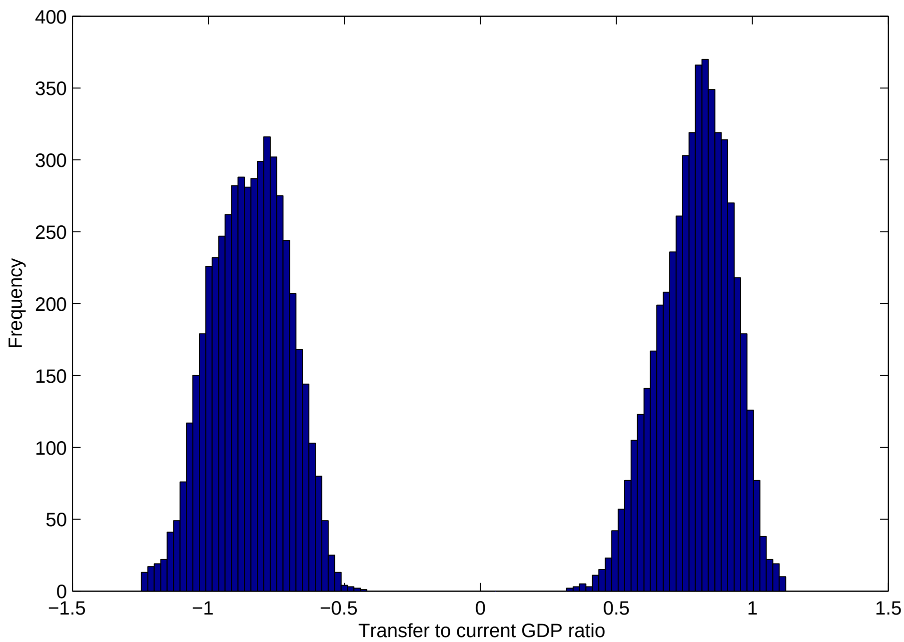

Fig. A.1. Transfer to current GDP ratio

The purpose of the tax rule used here is to avoid having to deal with a short-run government budget constraint. The short-run budget constraint can be written as follows:

$$G_t + r_t B_t = B_{t+1} - B_t + T_t$$

with

$$T_t = \tau(w_t N_t + r_t(K_t + B_t))$$

with G_t the level of public expenses, B_t the level of public debt, T_t tax revenues and r_t the current interest rate. Because next period's level of public debt B_{t+1} is unknown, this expression is cumbersome. Aiyagari and McGrattan (1998) overcome this problem by using the long-run steady state government budget constraint which can be written as follows:

$$G + rB = T$$

We can not directly apply the long-run budget constraint in this model because of the aggregate fluctuations property. Because of aggregate fluctuations, current prices, employment level and aggregate capital fluctuate such that the tax base and the amount of interest paid on the contracted public debt fluctuate. These fluctuations make the government budget constraint unbalanced.

The approach taken here is the following. We guess a value $\bar{r}$ for the interest rate that the fluctuating economy would reach on average in the long run. We derive a constant level of public debt from $\bar{r}$ such that $\bar{B} = b\bar{Y}(\bar{r})$, where $\bar{B}$ is a long-run constant value of public debt, b is the desired debt to GDP ratio and $\bar{Y}$ is the long-run GDP. Similarly, we define $\bar{G} = \gamma\bar{Y}(\bar{r})$ where γ is the fraction of government expenses and $\bar{G}$ a long-run level of public expenses, $\bar{K} = \bar{K}(\bar{r})$ a long-run level of aggregate capital, $\bar{w} = \bar{W}(\bar{r})$ a long-run value for wages and $\bar{N}$ the average employment rate in the economy. We can then define a long-run tax rate in the following manner:

$$\bar{\tau} = \frac{(\bar{G} + \bar{r}\bar{B})}{\bar{w}\bar{N} + \bar{r}(\bar{B} + \bar{K})}$$

At each period in the fluctuating economy, a constant level of public debt $\bar{B}$ is issued by the government along with government expanses $\bar{G}$ and households are taxed at the long-run rate $\bar{\tau}$. Because prices, aggregate capital and employment fluctuate, the government budget might not be balanced at each date along the cycle. Thus we introduce a transfer system that balances the budget at each period. This transfer can be viewed as the error the government makes in closing its budget constraint. We define this transfer Tr in the following manner:

$$Tr_t = \bar{\tau}((K_t + \bar{B})r_t + N_t w_t) - (\bar{G} + r_t \bar{B})$$

Depending on the tax base this transfer can be positive or negative and on average this transfer amounts to zero. Figure $A.1$ depicts the dispersion of the transfer to current GDP ratio over the whole relevant simulation window. The transfer amounts from 0.5% of GDP to 1.25% of GDP (and symmetrically for a negative ratio) with most occurrences around 0.75% of GDP for this simulation scheme. This amount is small compared to usual transfer schemes that usually over 8% of GDP.

On the computational side, as long as the average interest rate in the simulated economy is far from the guessed long-run rate $\bar{r}$, the transfer might be large. But when the fix point on the average interest rate has been found (this is the case in Figure $A.1$), the transfers are small and average out to zero so that the government budget constraint is balanced over the cycle.

B Calibration

We now show how we derived the transition matrices for aggregate state changes (η), for joint transition between aggregate states and labor market statuses (Π) and for discount factor changes (Υ).

B.1 Aggregate state change transition matrix

To deduce the aggregate state change transition matrix η, we solve the following system:

$$\left.\begin{aligned} \eta_{gg} &= \eta_{bb} \\ \eta_{bg} &= \eta_{gb} \\ \eta_{gg} + \eta_{gb} &= 1 \\ \eta_{bb} + \eta_{bg} &= 1 \\ \eta_{bg} &= \frac{1}{8} \end{aligned}\right\} \Longrightarrow \begin{pmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{pmatrix}$$

As we assumed that the duration of a boom or a recession is the same, we deduce the two first equations. Moreover, the duration of a cycle is set to 8 quarters, it follows that $\eta_{bg} = Pr(z_{t+1} = g/z_t = b) = \frac{1}{8}$ and $\eta_{gb} = Pr(z_{t+1} =$

$b/z_t = g) = \frac{1}{8}$.

B.2 Matrix for joint transition between aggregate states and labor market statuses

The determination of the matrix Π that describes the transition between unemployment and employment requires the identification of the aggregate shock (whether we are in a recession or in a boom). The transition matrix Π is built thanks to the matrix η and to the transition matrixes Π^{gg}, Π^{bb} ,Π^{gb} and Π^{bg}. Π verifies:

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix}$$

We assumed that in recessions the duration of the unemployment, that we note *durub*, amounts to 2.5 quarters and the unemployment rate u_b is set to 10%. In booms, the duration of unemployment, *durug*, is equal to 1.5 quarters and the unemployment rate, u_g, is set to 4%. From this information, we can deduce the matrices Π^{gg} and Π^{bb}.

The transition matrix Π^{gg} corresponds to the case $(z, z') = (g, g)$. It verifies:

$$\Pi^{gg} = \begin{pmatrix} \Pi^{gg}_{uu} & \Pi^{gg}_{ue} \\ \Pi^{gg}_{eu} & \Pi^{gg}_{ee} \end{pmatrix}$$

Solving the system below gives the values of Π^{gg}_{ee}, Π^{gg}_{eu}, Π^{gg}_{ue} and Π^{gg}_{uu}:

$$\begin{cases} \Pi^{gg}_{ee} + \Pi^{gg}_{eu} = 1 \\ \Pi^{gg}_{ue} + \Pi^{gg}_{uu} = 1 \\ \Pi^{gg}_{ue} = \dfrac{1}{durug} \\ \Pi^{gg}_{ee} = 1 - \dfrac{u_g \Psi^{gg}_{ue}}{1 - u_g} \end{cases} \Longrightarrow \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

The transition matrix Π^{bb} corresponds to the case $(z, z') = (b, b)$. It verifies:

$$\Pi^{bb} = \begin{pmatrix} \Pi^{bb}_{uu} & \Pi^{bb}_{ue} \\ \Pi^{bb}_{eu} & \Pi^{bb}_{ee} \end{pmatrix}$$

Solving the system below gives the values of Π^{bb}_{ee}, Π^{bb}_{eu}, Π^{bb}_{ue} and Π^{bb}_{uu}:

$$\begin{cases} \Pi^{bb}_{ee} + \Pi^{bb}_{eu} = 1 \\ \Pi^{bb}_{ue} + \Pi^{bb}_{uu} = 1 \\ \Pi^{bb}_{ue} = \dfrac{1}{durub} \\ \Pi^{bb}_{ee} = 1 - \dfrac{u_b \Pi^{bb}_{ue}}{1 - u_b} \end{cases} \Longrightarrow \Pi^{bb} = \begin{pmatrix} 0.6 & 0.4 \\ 0.0445 & 0.9555 \end{pmatrix}$$

When the cycle changes the unemployment rate changes. The transitions between unemployment and employment get modified. We make the same assumptions as Krusell and Smith (1998):

$$\begin{cases} \Pi^{bg}_{uu} = \Pr(s_{t+1} = u^g / s_t = u^b) = 0.75 \Pi^{gg}_{uu} \\ \Pi^{gb}_{uu} = \Pr(s_{t+1} = u^b / s_t = u^g) = 1.25 \Pi^{bb}_{uu} \end{cases}$$

The probability to remain unemployed when the next period is a recession (resp. boom), increases (resp. decreases) since by assumption the unemployment rate is higher in recession than in boom.

The transition matrix Π^{bg} corresponds to the case $(z, z') = (b, g)$. It verifies:

$$\Pi^{bg} = \begin{pmatrix} \Pi^{bg}_{uu} & \Pi^{bg}_{ue} \\ \Pi^{bg}_{eu} & \Pi^{bg}_{ee} \end{pmatrix}$$

The system below gives us Π^{bg}_{ee}, Π^{bg}_{eu}, Π^{bg}_{ue} and Π^{bg}_{uu}:

$$\begin{cases} \Pi_{ee}^{bg} + \Pi_{eu}^{bg} = 1 \\ \Pi_{ue}^{bg} + \Pi_{uu}^{bg} = 1 \\ \Pi_{uu}^{bg} = 0.75\Pi_{uu}^{gg} \\ \Pi_{ee}^{bg} = \dfrac{((1-u_g) - u_b\Pi_{ue}^{bg})}{1-u_b} \end{cases} \Longrightarrow \Pi^{bg} = \begin{pmatrix} 0.25 & 0.75 \\ 0.0167 & 0.9833 \end{pmatrix}$$

The transition matrix Π^{gb} corresponds to the case $(z, z') = (g, b)$.

$$\mathbf{\Pi}^{gb} = \begin{pmatrix} \Pi_{ee}^{gb} & \Pi_{eu}^{gb} \\ \Pi_{ue}^{gb} & \Pi_{uu}^{gb} \end{pmatrix}$$

The system below gives us Π_{ee}^{gb}, Π_{eu}^{gb}, Π_{ue}^{gb} and Π_{uu}^{gb}:

$$\begin{cases} \Pi_{ee}^{gb} + \Pi_{eu}^{gb} = 1 \\ \Pi_{ue}^{gb} + \Pi_{uu}^{gb} = 1 \\ \Pi_{uu}^{gb} = 1.25\Pi_{uu}^{bb} \\ \Pi_{ee}^{gb} = \dfrac{((1-u_b) - u_g\Pi_{ue}^{gb})}{1-u_g} \end{cases} \Longrightarrow \Pi^{gb} = \begin{pmatrix} 0.75 & 0.25 \\ 0.0729 & 0.9271 \end{pmatrix}$$

B.3 Matrix for discount factor changes

We assumed that the discount factors follow a three-states first-order Markov process. Therefore, the matrix describing the transition from the discount factor β_i to the discount factor β_j is the following:

$$\Upsilon = \begin{pmatrix} \Upsilon_{ll} & \Upsilon_{lm} & \Upsilon_{lh} \\ \Upsilon_{ml} & \Upsilon_{mm} & \Upsilon_{mh} \\ \Upsilon_{hl} & \Upsilon_{hm} & \Upsilon_{hh} \end{pmatrix}$$

As we assumed that there is no immediate transition between β_l and β_h as in Krusell and Smith (1998), it involves that $\Upsilon_{lh} = \Upsilon_{hl} = 0$. Moreover, as we set the duration of the extreme states (β_l and β_h) to 50 years namely 200 quarters, we have $\Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm}$. Solving the following system gives us the transition matrix Υ:

$$\left.\begin{array}{c} \Upsilon_{ll} + \Upsilon_{lm} + \Upsilon_{lh} = 1 \\ \Upsilon_{ml} + \Upsilon_{mm} + \Upsilon_{mh} = 1 \\ \Upsilon_{hl} + \Upsilon_{hm} + \Upsilon_{hh} = 1 \\ \Upsilon_{lh} = \Upsilon_{hl} = 0 \\ \Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm} \\ \Upsilon_{ml} = \frac{\Pr(\beta_t=\beta_l)\Upsilon_{lm}}{\Pr(\beta_t=\beta_m)} \\ \Upsilon_{mh} = \frac{\Pr(\beta_t=\beta_h)\Upsilon_{hm}}{\Pr(\beta_t=\beta_m)} \end{array}\right\} \Longrightarrow \Upsilon = \begin{pmatrix} 0.995 & 0.005 & 0 \\ 0.0007 & 0.9979 & 0.0014 \\ 0 & 0.005 & 0.995 \end{pmatrix}$$

C Model without aggregate risk

We now briefly detail the model without aggregate risk and its calibration. Most of this model is similar to the benchmark model, thus we underline only differences.

Model

As this model serves comparison purposes most of the benchmark assumptions remain unchanged. The assumptions about the representative firm are similar with the exception of the technical progress z. In the absence of aggregate risk, z is fixed to its average value. In the absence of aggregate risk, the budget constraint of the government is:

$$G_t + r_t B_t = B_{t+1} - B_t + T_t$$

Household preferences are unchanged. In the absence of aggregate risk, the matrix that describes the transition on the labor market becomes:

$$\pi = \begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix}$$

with $\pi_{uu} = \Pr(s_{t+1} = u | s_t = u)$. When there is no aggregate risk, it is no longer necessary to distinguish the nature of the cycle. In the absence of aggregate risk, the state variables are summarized by the vector (a, s, β). The program the household solves is:

$$v(a, s, \beta) = \max_{c,\, a'}\{u(c) + \beta E\left[v(a', s', \beta')|(s, \beta)\right]\} \tag{C.1}$$

subject to:

$$c + a' = (1 + r(1 - \tau))a + w\chi(s) \tag{C.2}$$
$$c \geq 0 \tag{C.3}$$
$$a' \geq 0 \tag{C.4}$$

with

$$\chi\left(s\right) = \begin{cases} \theta \text{ if } s = u, \\ (1 - \tau) \text{ if } s = e \end{cases}$$

Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s,), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital K, factor prices $\{r, w\}$, tax rate τ satisfying these conditions:

(1) Given the prices $\{r, w\}$, the decision rules $\{c(a, s, \beta), a'(a, s, \beta)\}$ solve the dynamic programming problem (C.1) subject to the constraints (C.2), (C.3) and (C.4)
(2) Market price arrangements are:

$$r = \alpha z K^{\alpha-1} N^{1-\alpha} - \delta$$
$$w = (1 - \alpha) z K^{\alpha} N^{-\alpha}$$

(3) Government budget constraint is balanced.
(4) Capital Market clears when:

$$K + B = \int a'(a, s, \beta) d\Gamma(a, s, \beta)$$

with $\Gamma(a, s, \beta)$ the distribution of agents over asset holdings, employment status and preferences.

Calibration

We present here the calibration strategy in an economy without aggregate risk. The calibration of the preferences, the discount factors and the behavior of the government are unmodified. The calibration of z and the characteristics of the labor market differ in the absence of aggregate risk. In an economy without aggregate risk, z is constant and set to the average value of the aggregate shock, namely the unit value. As we consider two alternative calibrations of the idiosyncratic labor process, we detail them sequentially.

In the Imrohoroglu method, the unemployment rate u and the unemployment spells $duru$ are respectively set to 7% and 2 quarters (we average over good and bad periods in the benchmark model). These two assumptions define the transition matrix π:

$$\begin{cases} \pi_{uu} + \pi_{ue} = 1 \\ \pi_{eu} + \pi_{ee} = 1 \\ \pi_{ue} = \dfrac{1}{duru} \\ \pi_{ee} = 1 - \dfrac{u\pi_{ue}}{1-u} \end{cases} \Longrightarrow \begin{pmatrix} 0.5 & 0.5 \\ 0.0376 & 0.9624 \end{pmatrix}$$

In the brute-force averaging method, we first simulate the transition process and find the transition probabilities. This yields the following transition matrix:

$$\begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix} = \begin{pmatrix} 0.50749 & 0.49251 \\ 0.03708 & 0.96292 \end{pmatrix}$$

With this matrix, we derive an unemployment rate of 7%.

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Table 1
Distribution of wealth: Benchmark model and data

	Held by Top				Gini
Source	1%	5%	10%	20%	
Benchmark Model	22	52	71	89	.82
Data	30	51	64	79	.79
	Held by Bottom				
Source	20%	40%	60%	80%	
Benchmark Model	1	2	4	11	
Data	0	1	6	19	

Table 2
Consumption gain (%) of going to the optimal level of debt in the benchmark economy

	Business Cycle		
Population	Average	Recessions	Booms
All	0.257	0.267	0.247
Bottom 10%	0.885	0.901	0.868
Top 10%	-10.401	-10.535	-10.264

Table 3
Macro variables along the cycle in the benchmark economy

	Level of debt (% of output)					
	66			5		
Statistics	Booms	Average	Recessions	Booms	Average	Recessions
Agg. Capital	12.23	12.17	12.10	12.58	12.52	12.46
Output	1.18	1.14	1.11	1.20	1.16	1.12
Before Tax Interest Rate (%)	0.99	0.90	0.80	0.93	0.83	0.74
Before Tax Wage	0.7902	0.7900	0.7899	0.7982	0.7981	0.7980