Mas-Colell et al. Ch. 11

Advanced Microeconomics

Externalities and public goods

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Winter Semester 2010/11

Layout

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- Pigouvian taxes
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Introduction

- We concluded that in perfectly competitive markets, market equilibrium is always Pareto efficient.
- However, in the presence of market power, as in a monopoly or oligopoly, the equilibrium allocation is no Pareto efficient.
- In the present chapter, we study further characteristics of markets that imply failure of optimality in equilibrium. Our focus is here on external effects and public goods.
- Depending on the source of market failure, we will see that there is role for government intervention with the introduction of taxes, subsidies, or personalized contributions.
- In general, the mechanisms proposed in this chapter can restore optimality only in the presence of perfect information about preferences and costs.

Externalities

An *externality* is present whenever the well-being of a consumer or the production possibilities of a firm are *directly* affected by the actions of another agent in the economy. not through prices

Examples of externalities

- A firm located on a river that discharges effluent into the river exerts a negative externality for consumers and firms located downstream, who are forced to clean the water before using it.
- Honeybee hives and apple gardens located next to each other exert a mutual positive production externality. The bees act as pollinators and the apple garden is a source of food for them.
- Smoking in a restaurant has a negative impact on other customers' enjoyment of their meals, involving a negative consumption externality.

An example with consumption externalities

Consider the case of two consumers $i = 1, 2$, each with preferences over the consumption of some bundle of goods $x \in \mathbb{R}^n_+$ and both affected by the level of some action $h \in \mathbb{R}_+$ taken by consumer 1.

Let $u_i(x_i, h)$ be the utility of consumer i and assume that $\partial u_2 / \partial h \neq 0$. Let p be the price vector of the goods in x and denote

$$v_i(p, m_i, h) = \max_{x_i} u_i(x_i, h) \text{ s.t. } p \cdot x_i = m_i.$$

Assume that $v_i(p, m_i, h) = \phi_i(p, h) + m_i$ and that p is given and independent of consumers' actions. We suppress p, write $\phi_i(h)$, and assume that $\phi_i''(h) < 0$.

Alternatively, we could have two firms, if we interpret $\phi_i(h)$ as the profits of firm i as a function of some action h, causing the externality (e.g. amount of effluent of a polluting firm).

Drop i

$$u(x,h) = \underset{\text{time devoted}}{h^{1/4}} \cdot \underset{\text{garden tools}}{x_2^{1/2}} + \underset{\text{income for all other goods}}{x_1}$$

$p_1 = 1$

$p_2 = q$

$p = (1, q)$

$$\max_{x} \; u(x,h) = h^{1/4} x_2^{1/2} + x_1$$

s.t. $x_1 + q x_2 = m$

$\hookrightarrow x_1 = m - q x_2 \rightsquigarrow$ subs. in objective fcn.

$$\max_{x_2} \; h^{1/4} x_2^{1/2} + m - q x_2$$

foc. $\frac{1}{2} h^{1/4} x_2^{-1/2} - q = 0$

$$x_2 = \left(\frac{h^{1/4}}{2q}\right)^2 = \frac{h^{1/2}}{4q^2}$$

$$v(p,m,h) = h^{1/4} \cdot \frac{h^{1/4}}{2q} + m - \frac{h^{1/2}}{4q} = $$

$$= \frac{h^{1/2}}{4q} + m \rightarrow \phi(p,h)$$

$q = 1/4 \quad \phi(h) = h^{1/2}$

Nonoptimality of competitive outcome

In a competitive equilibrium, consumer 1 chooses h so as to maximize $v_1 = \phi_1(h) + m_1$. At an optimum h^* for consumer 1 we have

$$\phi_1'(h^*) \leq 0, \qquad \text{with equality if } h^* > 0.$$

Instead the Pareto optimal allocation h° solves the problem

One consumer cannot be further improved w/o harming the other consumer

$$\max_{h,T} \quad \phi_1(h) + m_1 - T = \phi_1(h) + \phi_2(h) + \overbrace{m_1 + m_2 - \bar{u}_2}^{\text{constant}}$$

$$\text{s.t.} \quad \phi_2(h) + m_2 + T \geq \bar{u}_2 \qquad T \underset{\text{= at optimum}}{\geq} \bar{u}_2 - \phi_2(h) - m_2$$

or, equivalently, it maximizes the *joint surplus* of both consumers, solving

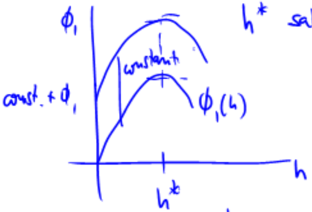
$$\max_h \phi_1(h) + \phi_2(h).$$

This gives us the following condition for h°

$$\phi_1'(h^\circ) \leq -\phi_2'(h^\circ), \qquad \text{with equality if } h^\circ > 0.$$

Given that $\phi_2'(h) \neq 0$, h^* and h° differ at an interior solution.

Max $\phi_1(h)$ same Max $\phi_1(h)$ + constant

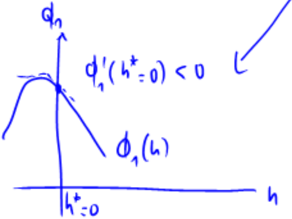


h^* satisfies $\phi_1'(h^*) = 0$

h^* satisfies

$\phi_1'(h^*) \leq 0$, with equality $h^* > 0$

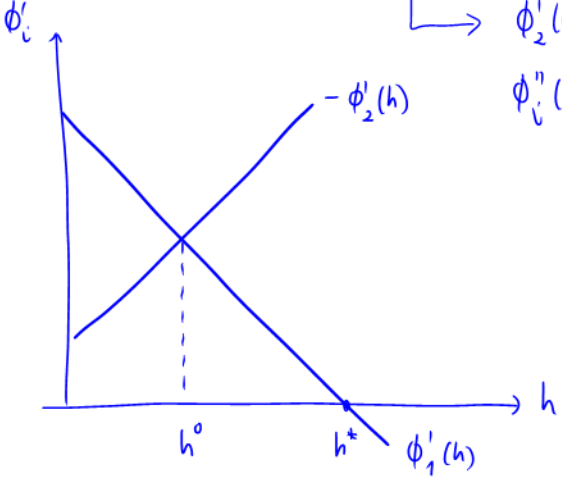
Corner solution



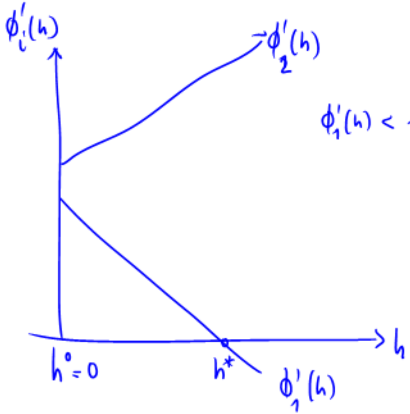
Equilibrium and optimal h with a negative externality

$\phi_2'(h) < 0$

$\phi_i''(h) < 0$



Corner solution for h^o



$\phi_1'(h) < -\phi_2'(h) \ \forall h \Rightarrow h^o = 0$

Pigouvian taxes

Consider the case of a negative externality with $h^\circ < h^*$. Suppose that consumer 1 must pay a tax of t_h per unit of h. Her optimal h must now solve the following problem

$+m_1$

$$\max_h \phi_1(h) - t_h \cdot h,$$

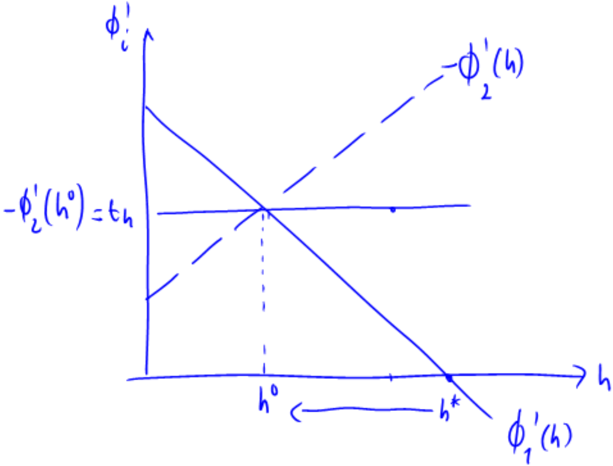
and, thus, it must satisfy the condition

$$\phi_1'(h) \leq t_h, \qquad \text{with equality if } h > 0.$$

A tax of $t_h = -\phi_2'(h^\circ) > 0$ will then implement the optimal level of the externality.

The optimality-restoring tax equals the *marginal externality* at the optimal solution; i.e., it corresponds to the amount that consumer 2 would be willing to pay to reduce h slightly from h°.

Optimality-restoring Pigouvian tax

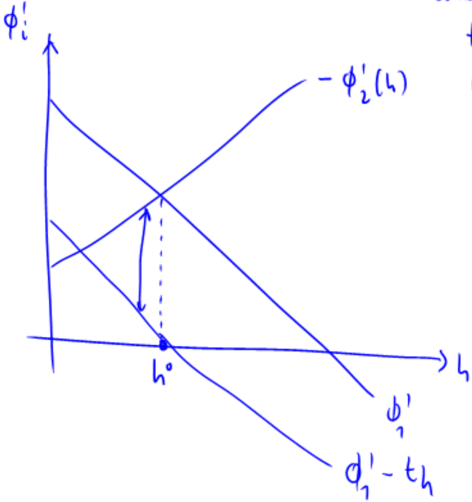


If negotiation between parties is possible :

At h_0 Marginal benefit of C.1 is $\emptyset$ but marginal damage on C.2 is positive

Consumer 2 could pay consumer 1 for further reduction

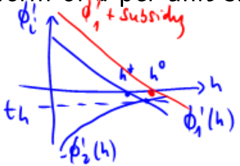
which would not be efficient.



Further comments

- For the case of a positive externality we would have that $h^\circ > h^*$ and $t_h = -\phi_2'(h^\circ) < 0$, so that t_h takes the form of a per-unit *subsidy*.

h° requires $\phi_1'(h^\circ) = -\underbrace{\phi_2'(h^\circ)}_{(+)} < 0$



- For the case of a negative externality, optimality can also be achieved with a subsidy $s_h = -\phi_2'(h^\circ) > 0$ for every unit that consumer 1 reduces h below h^*. In this case the consumer solves

equal to optimal t_h

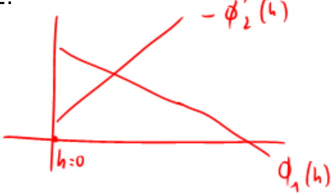
$$\max_h \phi_1(h) + s_h(h^* - h) = \phi_1(h) - t_h \cdot h + \underbrace{t_h \cdot h^*}_{\text{constant}},$$

which exactly replicates the outcome of the tax.

Bargaining

Suppose that we are able to assign enforceable property rights. If consumer 2 is assigned the right to an externality-free environment, then consumer 1 is unable to engage in the externality-producing activity without explicit permission of consumer 2.

Both consumers could profit from bargaining. Note that if the benefit of increasing h above zero for consumer 1 is larger than the damage imposed on consumer 2, then both consumers could agree on some transfer from consumer 1 to consumer 2.



Restoring optimality through bargaining

Suppose consumer 2 makes a take-it-or-leave it offer, demanding a payment T from consumer 1 in return for the permission to generate the externality level h. Consumer 2 will choose her offer to solve

$$\max_{h,T} \quad \phi_2(h) + T$$
$$\text{s.t.} \quad \phi_1(h) - T \geq \phi_1(0) \qquad T \leq \phi_1(h) - \phi_1(0)$$

At the solution $T = \phi_1(h) - \phi_1(0)$. Therefore, consumer 2's optimal offer involves a level h that solves

$$\max_h \phi_2(h) + \phi_1(h) - \underbrace{\phi_1(0)}_{\text{constant}},$$

which coincides with h°. $\quad \phi_1'(h) \leq -\phi_2'(h)$ with eq. for $h > 0$

The optimal payment in this case is $\phi_1(h^\circ) - \phi_1(0) > 0$.

Importance of property rights

Suppose now that consumer 1 has the right to generate as much of the externality as she wants. In this case, she will agree to externality level h only if $\phi_1(h) - T \geq \phi(h^*)$.

$T = \phi_1(h) - \phi_1(h^*)$

Consumer 2 will offer to set h at the level that solves

$$\max_h \phi_2(h) + \phi_1(h) - \underbrace{\phi_1(h^*)}_{\text{constant}},$$

which again results in h°.

The optimal payment in this case is $\phi_1(h^\circ) - \phi_1(h^*) < 0$. The only effect of the allocation of property rights is on the final wealth of each consumer.

Coase theorem: If trade of the externality is possible, then bargaining leads to an efficient outcome independent of how property rights are allocated.

Externality as a the result of missing markets

Suppose that a competitive market for the right to generate h exists. Consider again the case where consumer 2 has the right to $h = 0$ and let p_h be the price to purchase the right to generate one unit of h.

Consumer 1 would choose h_1 to solve

$$\max_{h_1} \phi_1(h_1) - p_h \cdot h_1,$$

with f.o.c. $\phi_1'(h_1) \leq p_h$, with equality if $h_1 > 0$.

Consumer 2 will decide how many rights to sell by solving

$$\max_{h_2} \phi_2(h_2) + p_h \cdot h_2,$$

with f.o.c. $\phi_2'(h_2) \leq -p_h$, with equality if $h_2 > 0$. $p_h \leq -\phi_2'(h_2)$

In equilibrium $h_1 = h_2 = h^{**}$ and optimality conditions imply that

$$\phi_1'(h^{**}) \leq -\phi_2'(h^{**}), \qquad \text{with equality if } h^{**} > 0,$$

implying that $h^{**} = h^{\circ}$.

Public goods

A commodity is said to be a *public good* if the consumption of a unit does not preclude others from the consumption of the same unit.

Examples of public goods

- The same piece of knowledge can be used simultaneously by any number of individuals.
- National security is enjoyed by everybody in the country.

  Exclusion: Note, however, that although a patent system can exclude someone from the commercial use of a certain piece of knowledge, it is difficult to exclude anyone from the effects of national security.

- Public roads and parks can be used simultaneously by many citizens.

  Congestion: This type of public goods are, however, subject to congestion effects, that difficult the enjoyment of the good when too many try to make simultaneous use.

Conditions for Pareto optimality

Let x denote the amount of public good available for I consumers and denote $\phi_i(x)$ the benefits of x units of the public good to consumer i. Assume $\phi_i'(x) > 0$ and $\phi_i''(x) < 0$ for all i and x.

The cost of supplying x units of the public good is $c(x)$, strictly increasing and strictly convex. $c'' > 0$ $c' > 0$

A Pareto optimal allocation must maximize the total joint surplus

$$\max_x \sum_{i=1}^{I} \phi_i(x) - c(x).$$

The optimal solution x° must satisfy the condition

$$\sum_{i=1}^{I} \phi_i'(x^\circ) \leq c'(x^\circ), \qquad \text{with equality if } x^\circ > 0.$$

Samuelson condition: At an interior Pareto optimum the *sum* of consumers' marginal benefits from the public good must equal its marginal cost of provision.

Private provision of the public good

Suppose that we introduce a market where consumers contribute to the provision of the public good by purchasing slots x_i and then enjoying the total amount $x = \sum x_i$ provided.

Each consumer solves the individual utility-maximization problem

$$\max_{x_i} \phi_i(x_i + \sum_{j \neq i} x_j^*) - p^* x_i,$$

where p^* denotes the equilibrium price. For each consumer the following condition is satisfied in equilibrium

$$\phi_i'(x_i^* + \sum_{j \neq i} x_j^*) \leq p^*, \qquad \text{with equality if } x_i^* > 0.$$

The firm supplying the public good maximizes profit

$$\max_{x} p^* x - c(x)$$

so that at an optimum for the firm

$$p^* \leq c'(x^*), \qquad \text{with equality if } x^* > 0.$$

Inefficiency of private provision

At an equilibrium with private provision of the public good we must have that $x^* = \sum_i x_i^* > 0$ and $p^* = c'(x^*)$.

good is provided

Moreover, for each consumer contributing to the public good, i.e. with $x_i^* > 0$, it must hold that $\phi_i'(x^*) = c'(x^*)$.

Adding up for all consumers we get that for $I > 1$

$$\sum_{i=1}^{I} \phi_i'(x^*) > c'(x^*).$$

Comparing with the condition for Pareto optimality, the assumption on the second derivatives of ϕ_i and c imply that $x^\circ > x^*$; i.e., the level of the public good provided is too low (*free-rider problem*).

For consumers with $x_i^* > 0$ we have $\phi_i^{\prime}(x^*) = c'(x^*)$

$$\sum_{i=1}^{I} \phi_i'(x^*) \geq \sum_{\{i \mid x_i^* > 0\}} \phi_i'(x^*) = \sum_{\{i \mid x_i^* > 0\}} c'(x^*) \geq c'(x^*)$$

$I > 1$

then $$\sum_{i=1}^{I} \phi_i'(x^*) > c'(x^*)$$

if only one consumer contributes the first inequality is strict and the second one is weak (equality)

if more than one consumer contributes then the first inequality is weak and the second is strict.

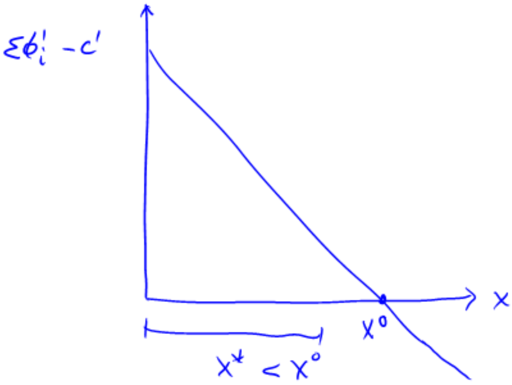
To see that $x^0 > x^*$ recall the condition for

$$x^0 : \sum_{i=1}^{I} \phi_i'(x^0) - c'(x^0) \leq 0$$

$$x^* : \sum_{i=1}^{I} \phi_i'(x^*) - c'(x^*) > 0$$

recall $\phi_i'' < 0$

$c'' > 0$



A market mechanism that achieves optimality

Suppose we could have *a market* for the public good *for each consumer* with a personlized price p_i for consumer i.

Call p_i^{**} the equilibrium personlized price of the public good for consumer i and consider the individual problem of consuming the good as if each consumer could determine the total amount of the good provided.

$$\max_{x_i} \phi_i(x_i) - p_i^{**} x_i.$$

For each consumer, the equilibrium consumption level x_i^{**} must satisfy

$$\phi_i'(x_i^{**}) \leq p_i^{**}, \qquad \text{with equality if } x_i^{**} > 0.$$

Here it is important that the consumer can be excluded from the consumption of the public good if it decides not to contribute.

Lindahl equilibrium

The firm providing the public good produces the amount x, which all consumers can then enjoy simultaneously, but it gets total price $\sum_i p_i^{**}$ per unit provided. Thus, the firm solves

$$\max_x \sum_{i=1}^{I} p_i^{**} x - c(x).$$

Thus, the firm's optimum satisfies

$$\sum_{i=1}^{I} p_i^{**} \leq c'(x^{**}), \qquad \text{with equality if } x^{**} > 0.$$

In a market equilibrium $x_i^{**} = x^{**}$ for all i and

$$\sum_{i=1}^{I} \phi_i'(x^{**}) \leq \sum_{i=1}^{I} p_i^{**} \leq c'(x^{**}),$$ (same as Samuelson condition)

which coincides with the condition for Pareto optimality.

Final comments

- Note that public goods can be considered to be an extreme example of external effect, where the provision by a single agent alone suffices for the enjoyment (or damage, in case of a public bad) of all.
- The introduction of personalized markets reduces the choice of the amount of public good to an individual choice without externalities, which allows to restore efficiency.
- A general problem of the mechanisms proposed in this chapter (including the possibility to bargain) is that they require that the value of the externality or public good to all agents is known. When these are private information, these type of mechanisms will not be able to restore full efficiency.