3. INDIRECT UTILITY & EXPENDITURE

1. Indirect utility
2. Properties of indirect utility
3. Hicksian demand
4. Expenditure function
5. Properties of expenditure function
6. Duality: relation between indirect utility and expenditure

INDIRECT UTILITY

Utility evaluated at the maximum

$$v(p,m) = u(x^*) \text{ for any } x^* \in x(p,m)$$

Marshallian demand maximizes utility subject to consumer's budget. It is a function of prices and income.

Substituting Marshallian demand in the utility function we obtain indirect utility as a function of prices and income.

PROPERTIES OF INDIRECT UTILITY

1. Homogeneity of degree zero
2. Strictly increasing in m and decreasing in p_i $\forall i$
3. $v(p, m)$ is quasi-convex
4. Continuous in p and m
5. Roy's identity

$$x_i(p, m) = -\frac{\partial v(p, m)/\partial p_i}{\partial v(p, m)/\partial m}$$

HICKSIAN DEMAND

Consider the dual to the consumer's problem

$$\min_{x \geq 0} \quad p \cdot x$$
$$\text{s. t.} \quad u(x) \geq u$$

Hicksian demand (also called **compensated demand**) is the solution to this cost-minimization problem, $x^h(p, u)$.

Notice the parameters of the cost-minimization problem are prices p and *target utility* u.

EXPENDITURE FUNCTION

Expenditure evaluated at its minimum

$$e(p,u) = p \cdot \widetilde{x} \text{ for any } \widetilde{x} \in x^h(p,u)$$

Hicksian demand solves the cost-minimization problem. It is a function of prices p and target utility u.

Substituting Hicksian demand in the expenditure objective we obtain expenditure as a function of p and u.

PROPERTIES OF EXPENDITURE

1. Homogeneous of degree one in p
2. Strictly increasing in u and increasing in p_i $\forall i$
3. Concave in p
4. Continuous in p and u
5. If $u(\cdot)$ is strictly quasi-concave and $e(p, u)$ is differentiable we have Shephard's lemma

$$\frac{\partial e(p^0, u^0)}{\partial p_i} = x_i^h(p^0, u^0) \qquad i = 1, \ldots, n$$

DUALITY

Let $p >> 0,\ m > 0\ u > u(0)$

1. $e(p, v(p, m)) = m$
2. $v(p, e(p, u)) = u$

Duality between Marshallian and Hicksian demand

1. $x(p, m) = x^h(p, v(p, m))$
2. $x^h(p, u) = x(p, e(p, u))$