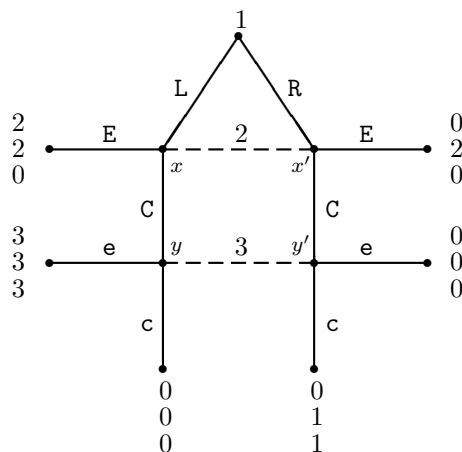


5 Perfect Bayesian equilibrium

5.1. Consider the following two-player game. Player 1 decides whether to give a (wrapped) gift to player 2. Player 2 does not know if player 1 is a true friend or if she is actually an enemy. Player 2 can accept the gift or reject it. If player 1 gives no gift, the payoffs are 0 for both players. If player 1's gift is accepted, her payoff is 1; her payoff is -1 if the gift is rejected (player 1 hates the humiliation of having the gift rejected). If player 2 rejects a gift, her payoff is 0. If player 2 accepts a gift from a friend, her payoff is 1; if the accepted gift comes from an enemy, her payoff is -1 (a friend's gift is something player 2 likes, an enemy's gift is embarrassing or plainly bad). Player 1 is a friend with probability $0 < p < 1$ (exogenous).

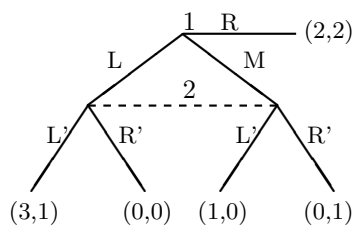
- (a) Draw the extensive-form representation of this game as a game of imperfect information. Please, specify the information sets clearly.
- (b) What is the difference between this and a Bayesian game? And between this and a signaling game?
- (c) Find all the Nash equilibria (NE) in pure strategies. Are they subgame perfect?
- (d) Find all the pure-strategy perfect Bayesian equilibria (PBE). Are there any NE which do not correspond to PBE?
- (e) Change the following payoffs only: the payoff to player 2 if she accepts a gift from an enemy is 0; her payoff if she rejects a gift from an enemy is -1 (the enemy might beat him up). Answer the previous two questions for this new game.

5.2. Consider the following three-player, extensive form game. We denote $x, x', y,$ and y' the decision nodes of players 2 and 3.

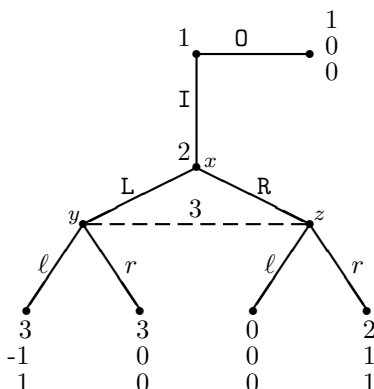


- (a) Find all pure-strategy Nash equilibria.
- (b) Are all Nash equilibria of this game subgame perfect? Explain.
- (c) Find all pure-strategy weak perfect Bayesian equilibria.

5.3. Find all PBE for the following game and discuss their plausibility.

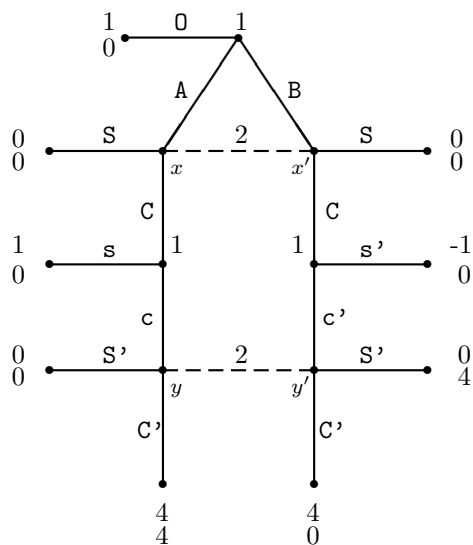


5.4. Consider the following three-player, extensive form game. Each player has only one information set and two actions at each information set. We denote x , y , and z the decision nodes of players 2 and 3.



- (a) Find all pure-strategy Nash equilibria.
- (b) Are all Nash equilibria of this game subgame perfect? Explain.
- (c) Find all pure-strategy weak perfect Bayesian equilibria. Check that there are weak perfect Bayesian equilibria where player 3 plays ℓ . What are the corresponding equilibrium beliefs?
- (d) Are equilibria where player 3 plays ℓ perfect Bayesian? Explain.

5.5. Consider the following two-player, extensive-form game.



- (a) Show that there is a PBE with payoffs $(4, 4)$.
- (b) Show that there is no PBE where player 1 plays B.
- (c) Show that $(0sc', SS')$ is played at a PBE. Find the equilibrium beliefs.
- (d) Try to find all pure-strategy PBE.