1 Preliminaries

1.1. Consider a normal form game with players $I = \{1, 2, \ldots, n\}$. Denote $A_i = \{a_i^1, a_i^2, \ldots, a_i^{m_i}\}$ the set of pure strategies available to player $i \in I$ and let $A = A_1 \times A_2 \times \ldots A_n$. A pure strategy profile is a vector $a \in A$, i. e. $a = (a_1, a_2, \ldots, a_n)$ with $a_i \in A_i$. Payoffs to player $i \in I$ are given by the function $\pi_i : A \to \mathbb{R}$. Thus $\pi_i(a)$ denotes the payoff to player i when the strategy profile is give by a. Define the following concepts:

(a) Mixed strategy for player $i \in I$.
(b) Mixed strategy profile.
(c) Expected payoff for player $i \in I$.
(d) Strictly dominated strategy for player $i \in I$.
(e) Weakly dominated strategy for player $i \in I$.
(f) Nash equilibrium.
(g) Show that all pure-strategies played with positive probability in a Nash equilibrium must give the same payoff.

1.2. Consider the problem of public good provision described in class with two players. Each player must decide whether to (C)ontribute or (N)not and payoffs are given in the following payoff matrix.

		Player 2: C	Player 2: N
Player 1	C	$1-c_1, 1-c_2$	$1-c_1, 1$
	N	$1, 1-c_2$	$0, 0$

Assume that c_1 and c_2 are commonly known. Calculate all the NE in the following cases:

(a) $c_1 = 0.5$ and $c_2 = 1.5$.
(b) $c_1 = 0.5$ and $c_2 = 0.75$.
(c) $c_1 = c_2 = 0$.

1.3. Consider again the problem described in exercise 1. 2. Assume now that each player's cost of contributing is private information, but players have a common prior belief about the probability distribution on the different cost levels.

(a) Suppose $c_1 = 0.5$ and $c_2 \in \{0.75, 1.5\}$; that is, c_2 can take two possible values 0.75 and 1.5. Let $\text{Prob}\{c_2 = 0.75\} = 0.2$. Is there an equilibrium where player 1 does not contribute? Explain.
(b) Suppose now $c_1, c_2 \in \{0.75, 1.5\}$; that is, both players may have two possible cost levels 0.75 or 1.5. Let $p := \text{Prob}\{c_i = 0.75\}$, $i = 1, 2$ with $0 < p < 1$.
   i. Is there an equilibrium where players *always* contribute?
   ii. Is there an equilibrium where players *never* contribute?

1.4. Consider again the problem of public good provision discussed above, but assume now that the good is provided only if *both* players contribute (choose action C).

(a) How does this change the payoff matrix of the game?
(b) Compute the best response of player $i = 1, 2$ to player $j \neq i$, when j contributes with probability $\sigma_j(C) \in [0, 1]$.
(c) Compute all Nash equilibria of the game for the following cases.
   i. $c_1 = c_2 = 0$
   ii. $c_1 = c_2 = 1$
   iii. $0 < c_i < 1,\ i = 1, 2$
   iv. $c_i = 0,\ 0 < c_j < 1,\ i, j = 1, 2\ i \neq j$
   v. $c_i = 1,\ 0 < c_j < 1,\ i, j = 1, 2\ i \neq j$
   vi. $c_i = 0,\ c_j = 1,\ i, j = 1, 2\ i \neq j$