Spectral Properties of the Dirac Operator with Fixed-Point Action ⋆

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Abstract. Discretization of space and time leads to severe problems, such as cut-off dependence of observables and explicit breaking of chiral symmetry (H. Nielson and M. Ninomiya (1981)). One can cope with these problems by introducing **perfect actions**. The fixed-point action defined by C. B. Lang and T. K. Pany (1998) is called a **classical perfect action**. Its classical predictions agree with those of the continuum (considering the same physical volume), no matter how coarse the lattice. **Quantum perfect actions** agree with all the continuum predictions (if one considers quantum perfect operators or observables).

We studied this fixed-point action for the massless one-flavour Schwinger model and compared the results with the theoretical predictions found by P. Hasenfratz, V. Laliena and F. Niedermayer (1998) and by F. Farchioni and V. Laliena (1998). The numerical results agree nicely with the predicted circular shape of the spectrum. It roughens at low values of β due to the necessary truncation of the couplings and to numerical errors. In order to estimate the scaling behaviour of the deviations of the spectrum from the ideal circular shape, we defined a mean deviation $|\lambda - 1|$ from the unit circle in the region close to $\lambda = 0$, in an angular window of $|\arg(1-\lambda)| < \pi/4$. A behaviour of $\sigma \propto 1/\beta^{2.41} \simeq a^5$ is observed. The parametrized action pA_{FP} is truncated in a finite range (7x7 in our case). Heuristically this implies an error for the eigenvalues, considered as dimension-one gauge-invariant operators of the gauge field, in the form of some operator of higher dimension k. From the observed deviation we estimate an effective value $k \simeq 5$.

We note, that there are configurations with real eigenvalues. We checked the eigenvectors v_i for those and confirm that these modes have definite chirality $\langle v_i \gamma_5 v_i \rangle$. Also, we can clearly distinguish the real values around zero from those around 2 (right-hand part of the spectrum). We may identify these real eigenvalues (around zero) with zero-modes and relate their number n_0 with the geometrically (i.e. from the gauge field configuration) defined topological charge Q_G. We find agreement in the following sense: The ratio of the number of configurations, where these numbers coincide over all configurations approaches unity in the limit $\beta \to \infty$. These results and results on the chiral condensate are presented by F. Farchioni, C.B. Lang and M. Wohlgenannt (1998).

References

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⋆ presented by M. Wohlgenannt

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