ECP-2008-EDU-428046

Math-Bridge

Report about culturally different notions, notations and names

Deliverable number	*D-4.1*
Dissemination level	*Confidential*
Delivery date	8^{th} *of January 2010*
Status	*submitted*
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e*Content*plus

This project is funded under the *e*Content*plus* programme[1] ,
a multiannual Community programme to make digital content in Europe more accessible, usable and exploitable.

[1] OJ L 79, 24.3.2005, p. 1.

Project ref.no.	ECP-2008-EDU-428046
Project title	Math-Bridge European Remedial Content for Mathematics

Deliverable dissemination level	Confidential
Contractual date of delivery	December 31^{th}, 2009
Actual date of delivery	January 8^{th}, 2010
Deliverable number	D-4.1
Deliverable title	Report about culturally different notions, notations and names
Type	Deliverable
Status & version	submitted
Number of pages	18
WP contributing to the deliverable	WP4
WP/Task responsible	Franz Embacher/Paul Libbrecht
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EC Project Officer	Marcel Watelet
Keywords	culturally dependent notions, notations and names

Contents

Introduction

This document describes the need for differentiating mathematical notations based on the many cultures of mathematical writing and how these are analyzed, collected, and operationalized. This document forms an introduction to the project of a *notation census* intending to safely collect notations widely used around thee world.

Math-Bridge being a European project aims at serving at least learners in Dutch, English, French, Finnish, German, Hungarian, and Spanish, and thus must *speak the same mathematical language*. The final use of this report and census is to support the rendering of the mathematical formulæ in the Web experience offered by the project using, as much as possible, the classical notation the learner is used to.

Section 1 introduces the cultural diversity of the mathematical languages in the different cultures. Section 2 describes the notation census project which aims at collecting safely the notations around the world. The current state of the census is provided in Section 4.

This deliverable addresses the work planned in task T4.2 (*cultural differences*) in the description of work of the math-bridge project.

1 The cultural dependendence of mathematical notions, notations and names

1.1 Cultural dependence of mathematical notations and names

Mathematics is widely viewed as a universal “methodology of reasoning” and a “language” common to all humans, truly “objective”, and thus independent of subjective beliefs and cultural preformation. In particular, statements written in the mathematical symbol language, i.e. in terms of “formulae”, should thus be understandable in principle by everyone. In a general sense this is indeed true, and it is exactly this fact that makes mathematics a widely applicable tool for (almost) all sorts of problems: The mathematical language provides the possibility to express abstract concepts and logic reasoning in an unambigous way that can be understood independend of the reader’s language and culture. If we say that a mathematical theorem (for example Fermat’s last theorem) has been *proven*, we mean that a certain mathematical truth has been acertained at an objective level: It has been shown to hold *as such*, without reference to a particular language or culture.

However, when *practicing* mathematics (no matter whether the application of pre-established methods or genuine mathematical creativity are concerned, whether easy or difficult), language and culture dependent issues *do* play a role at several levels. Mathematical thought has evolved in history in different cultural contexts (beginning with several hundred years

B.C.), each developing its own methods by which mathematical ideas can be expressed and operated with. Although the development of modern science and a world-wide scientific communication within the last few centuries lead to an extensive unification and "internationalization" of mathematical conventions[2], a lot of variations remained, so that we may talk about "mathematical cultures", which sometimes (but not always) are associated with "cultures" in the sense of countries or languages. In particular, differences exist even between regions as close to each other as the European countries.

To begin with, mathematical statements are usually embedded in phrases of "ordinary language", which may be viewed as "everday language" used in a more or less rigorous way. As an example, the statement

$$\textsf{there exists a natural number } n \textsf{ such that } n^2 = 4 \tag{1.1}$$

contains the verbal elements "there exists", "natural number" and "such that". It is *in principle* possible to completely omit verbal elements of this type. For example, statement (1.1) could be re-expressed as

$$\exists n \in \mathbb{N} : n^2 = 4\,. \tag{1.2}$$

However, in most cases a rigorous elimination of all "words between the symbols" would be tremendously impractical, in particular in texts addressing mathematics students (who are just about to *learn* mathematics) and non-specialists (interested to *apply* mathematical techniques for various purposes). As a consequence, mathematical texts in practice rely on written *words* at least as much as on written *mathematical symbols*, thus suffering from all sorts of variations in meaning and difficulties of translation.

One might expect now that at least the mathematical *symbols* provide something like a well-defined and culturally independent language (or at least vocabulary). However, mathematical concepts are not only used in the international mathematical research communication – which indeed uses a largely standardized symbolic language and may thus be considered as a "mathematical culture" by its own – but also in local (e.g. technical or economic) contexts or just in every-day life. All these fields of usage of mathematical concepts constitute "cultures" whose customs differ in many respects. The most basic example is how *numbers* are written. Whereas in most countries (as well as in the international communication) the number "twelf and a half" ist written in decimal representation as

$$12.5\,, \tag{1.3}$$

some mathematical cultures would use

$$12,5 \tag{1.4}$$

[2]It should be added that several branches of ancient mathematics that did not fit the development and interests of the western development have simply been forgotten and are now being rediscovered by historians.

instead. In (1.3) the *decimal separator* is indicated by a period or point (we thus say in English language “twelve point five”), whereas in (1.4) a comma is used instead (consequently, in German language one calls this number “zwölf Komma fünf”). Having evolved historically [WKPD01], notational differences of this type have even been standardized in local contexts. For example, the DIN (Deutsch Industrie-Norm – German industrial norm) specifies the comma as used in (1.4) as the “official” decimal separator. Students in German schools are thus acquainted with the notation (1.4) rather than (1.3). Note however that the *meaning* of the period in (1.3) and the comma in (1.4) are precisely the same in both cases! In technical terms, these two differing notations constitute *semantically equivalent concepts*. Consequently, a general definition like

$$\text{the period means the same as the comma}\,, \tag{1.5}$$

spelled out once and for all, would logically be admissible in a mathematical text that uses the period notation but is otherwise written in German (and omits the use of other separators like the thousands separator[3]). However, for learners – and in particular for the target group of Math-Bridge — this would provide unnecessary cognitive load and the danger of misunderstandings. Hence, *automated formula rendering* underlying the generation of educational mathematical texts in different languages (to be used within different cultures) should take these variations into account and always provide the appropriate notations the learners are used to. The notation census contains a page about decimal numbers: `http://wiki.math-bridge.org/display/ntns/Decimal+Numbers`.

Mathematical concepts and operations are often *named* by words or phrases. In some cases, such words may be traced back to an inventor, in other cases their origin has been forgotten (and is possibly recovered by historians). Examples are “number” (German: “Zahl”), “to add numbers” (German: “Zahlen addieren”), “function” (German: “Funktion”), “continuous” (German: “stetig”), and – in order to mention a concept that goes back to the Arab tradition of mathematics – the “sine” (German: “Sinus”, Spanish: “seno”). Just by there use, verbal expressions of this type have become part of the (scientific) language of the speaker or reader. Which word or phrase is used to denote a concept or an operation is to some extent a matter of (official or informal) standardization, but it might as well be a matter of *cultural tradition*. As the above examples show, the particular word or phrase used to denote *one* concept will be different when expressed in different languages. Sometimes these names in different languages are quite accurate translations of each other, in other cases verbal denotations seem to have emerged independently. Now, the key point is that abbreviations of these verbal expressions may become part of the mathematical *symbol language*. As an example, most mathematicians in the world would use the symbol “sin” as an abbreviation of “sine” (“Sinus”), so that we may find a formula like

$$\sin(\pi) = 0 \tag{1.6}$$

[3] In fact, other separators make the situation worse: What does 1.001 mean? Depending on the convention used, it could mean $1 + \frac{1}{1000}$ or $1000 + 1$.

in a mathematical textbook. However, in Spanish language, the “sine” is called “seno”, which has lead to the fact that in the Spanish tradition (or “culture”) of mathematics the abbreviation for this concept is “sen” [WKPD03]. A textbook written in Spanish language will thus rather contain the formula

$$\mathrm{sen}(\pi) = 0 \tag{1.7}$$

instead. Of course Spanish mathematicians are aware of this difference, and when addressing an international audience, they would use “sin” instead of “sen”, but it is a fact that Spanish pupils and students are familiar with “sen” rather than “sin”. If *automated formula rendering* is envisaged, notational differences in naming and abbreviating concepts are at the same footing as the differing conventions to denote numbers, as discussed above. For illustration let us quote two further culturally differing notations:

- Arithmetics – even when addressed at an elementary level such as in secondary school – knows the concept of the “greatest common divisor” of two integer numbers. In many languages the symbol used to denote this concept just consist of the initial letters of the corresponding verbal phrase. In English it is thus written as “gcd”. In the German tradition the same concept is called “größter gemeinsamer Teiler” and abbreviated as “ggT”. In Dutch, the corresponding phase “grootste gemene deler” is abbreviated as “ggd”. In French the corresponding phrase is generally called plus grand commun diviseur and is written“pgcd”. See the notation census page about it: `http://wiki.math-bridge.org/display/ntns/gcd`.
- Sometimes variations in the mathematical notation may be quite drastic. An example is provided by the “binomial coefficients”. What is denoted by

  $$\begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{1.8}$$

  in most mathematical cultures tends to appear in French and Russian textbooks in the form

  $$\mathrm{C}_5^3 \tag{1.9}$$

  instead (see the census page about the binomial coefficient: `http://wiki.math-bridge.org/display/ntns/binomial-coefficient`).

We have stated before that in general *one* mathematical concept may be represented by different cultures in different ways. However, in some cases, a particular name or phrase – that may easily be translated between the languages – denotes *different* concepts. The most prominent example is the notion of "natural numbers". Originally, it meant set of integer numbers greater or equal to 1, i.e.

$$1, 2, 3, 4, 5 \ldots \tag{1.10}$$

and was denoted by the symbol $\mathbb{N}$. However, for practical reasons (whose details are not of interest here), it would be better to include the zero, hence to define the set of natural numbers by

$$0, 1, 2, 3, 4, 5 \ldots \tag{1.11}$$

Meanwhile, several mathematical cultures have followed this idea, while sticking to the original symbol $\mathbb{N}$. In order to denote the set of numbers (1.10), one then usually writes something like $\mathbb{N}^+$ or $\mathbb{N}^*$. In the tradition in which the zero is not included, one may write $\mathbb{N}^0$ or $\mathbb{N}_0$ in order to denote the set (1.11). As a consequence, when the symbol $\mathbb{N}$ (or the name "natural numbers") appears in a mathematical text, it could mean (1.10) or (1.11), depending on the mathematical culture it originates from[4]. When *automated formula rendering* is concerned, ambiguities of this type must be taken into account in order to avoid confusion and let the user recognize easily which variant it is.

It should be added that notational variations of all types discussed so far may even exist within a country or language, e.g. if the scientific language (academic culture) and the notational customs used in classroom teaching diverge.

1.2 Cultural dependence of mathematical ideas and notions

The above examples suggest that – in spite of notational differences – the mathematical concepts to be denoted are the same in all cultures. However, not even this is completely true! The evolution of *mathematics* as well as the evolution of *mathematics teaching* have proceeded in different ways and have thus led to different "atmospheres" in which mathematical ideas and thoughts are embedded and that support different forms of intuition and even feelings. With regards to mathematics education in school – which is usually tightly standardized at national levels – this has for example lead to pedagogical traditions which differ in how much they rely

- on geometrical imagination and reasoning,

[4]In a *good* textbook this point is of course explicitly clarified. The problem is also pointed out in Eric Weissteins *MathWorld* pages: "Regrettably, there seems to be no general agreement about whether to include 0 in the set of natural numbers" [WNN]. *Wikipedia*, the most popular source for scientific content on the web – although not entirely consistent in notation throughout the various fields of mathematics – also reflects this ambiguity in most pages on the subject od natural numbers [WKPD02]. It should be added that the web in general pulls the user out of a single textbook into easy jumps between many content sources, with varying degree of reliability.

- on algebraic reasoning or
- on the idea of sets (set theory in school was a sort of fashion some decades ago).

As an example, whether the derivative of a function at a point is introduced (and thought of) as a *rate-of-change* expressed by a formula or as the *slope* (or *steepness*) of a tangent depends not only on the taste of the lecturer but also on the mathematical culture he or she comes from. Different pedagogical traditions thus affect the way how learners operate with mathematical concepts or how they tackle a given problem. Only at an advanced level of mathematical thought and skill, learners will be able to easily switch between points of view originating from different traditions. In a project providing multi-lingual educational resources, issues like these must be taken into account.

2 The Notation Census

Because mathematical notations are very diverse and their context of occurrence is just as diverse, we propose to establish a **census of mathematical notations**. That census should list all available mathematical notations that are widely spread around the world in a way that enables mathematics readers to see the mathematical notations used in the multiple cultural contexts even though they do not understand the language of the documents.

The census should be **visual** because mathematical notations are a graphical artifacts and its rendering in web-browsers should succeed in all situations: this requirement prevents the usage of elaborate display technologies of mathematical formulæ accepting a potentially poorer typography but the certainty of rendering in the notation of that cultural context. The census should be **traceable** so that one can recognize who has written the each part of the census and **commentable** so that the quality of the census can be steadily improved; normal web-authoring practices similarly to those of Wikipedia or others apply here, together with the public visibility of any editing action. The census should be displaying **widely used notations** by relying on extracts of mathematical texts that are widely used themselves; this is fundamental so as to achieve usefulness of the census. Finally, the census should be **verifiable** because the misunderstanding among the cultures can be critically high: any interested reader should be able to find in just a few clicks who are the authors making a claim that a notation is widely used and in which cultural context it is used with the trustability of the source of notation serving as discussable reference point.

Lack of respect of the last point has appeared in the literature: indeed one can find in several texts about mathematical notations the fact that the binomial coefficient in Russian is written as C_k^n whereas the binomial coefficient in French is written as C_n^k, i.e. they are opposite of each other. This is the case of [CIMP01], [Koh06], and [LrAG09]. As the authors could observe and proof in many russian sources, this turned out to be false: the binomial coefficient in Russian is written the same as in French. Tracking down the source of this confusion turned out to be impossible with the inability to verify the origin of such a claim and with, even, easy assertions about the pre-industrial era where such a notation was transported between France and Russia.

2.1 Ingredients

We propose to realize the notation census in a **public wiki** which is made of the following core ingredients:

- **sources** are listed in a bibliography-like approach: on a single page containing all sources and a name of the cultural context they are used in, link to a publisher page and link to possible web-download of the (partial) content are wished. A source reference would be the entry point to judge the *relevance* of the notations with any given cultural context, something a person living in that context can do.
- notations are grouped per page, **one page per semantic**, each grouped in content-dictionaries following the semantic classification of, at least, the official OpenMath content dictionaries
- notations are listed there with a small informal description, links to the content-dictionary entries, and a series of **observations** made of a text containing the observed notation, the name of the symbol in that context, a pointer to the element of the bibliography containing it, and a graphics copy of the relevant bit indicated with a page number or other internal reference allowing a reader to find the used notation fast.

The observations and sources both define a **cultural context** which is fuzzily defined. It is generally accepted to be at least made of a human language but it often goes beyond the sole language. For example the French notation for binomial coefficient, with the big C, is recognized to be widespread at school levels, as can be seen in `http://wiki.math-bridge.org/display/ntns/binomial-coefficient` but several combinatorics researchers agree that the vector notation is preferable even in French [Fra09]. A more widespread example is that of the square root of -1 which is written mostly with the letter i except in electrical engineering where the letter i is too close to that of electrical current hence the square root of unity is written j; the list of different observed notations is presented on `http://wiki.math-bridge.org/display/ntns/nums1_i`. For this reason, the census speaks about the *cultural context* which can, typically, be also defined by major texts.

Conformance is not, yet, strictly enforced for each contribution to the notation census but it is expected that an early result of this deliverable be a check-list before contributing a source and before contributing an observation. We believe the requirements above, except maybe for a web-access to the text's content which is mentioned optional, are minimal.

2.2 Roadmap

The explanations are currently provided in the *notation census manifest* available at `http://wiki.math-bridge.org/display/ntns/Notation-Census-Manifest`. This text intends to be the one stop information source about the census guiding ideas and principles.

We expect to populate the notation census gradually with the following timeline:

- announce the census to the public in December 2009
- cover the MathML CD-group in January 2010
- cover all ActiveMath-available content till the Summer 2010
- cover all of the Math-Bridge available content until its end

3 Notations in Math-Bridge

The reasons of creating a notation census have been presented in the first section but not its exploitation. This is what we turn to in this section which explains how the notation census will be exploited for the purposes of the Math-Bridge project and how the ActiveMath platform can be endowed to honour the cultural contexts by presenting mathematical formulæ in a customizable way.

Math-Bridge intends to offer bridging-courses allowing learners to bridge mathematics knowledge gaps taking them with their pre-existing knowledge, hence by the notations they already know, to lead them to a level ready to start University. It aims, thus, to use the right notations in the cultural contexts it aims at covering, that is late-school-level mathematics for Dutch, English, French, Finnish, German, Hungarian, and Spanish classes. Math-Bridge intends to use the rich formulæ rendering of ActiveMath, coupled with other personalization features of this web-platform. This platform renders mathematical formuæ from their OpenMath semantic source enabling a rich set of services attached to the rendering. These include the transfer to computer algebra systems, exercise inputs, plotter or the searchability and probably will become richer following, e.g., [GLR09].

ActiveMath presents mathematical formulæ using a few widely supported display technologies on the web: HTML completed with CSS, XHTML with formulæ in MathML, or PDF created by `pdflatex`. The transformation from the OpenMath encoding to the rendering formats is done through several stages that are explained in [ULWM04]. Two aspects are worth mentioning here:

Authorable Notations ActiveMath rendering to HTML, XHTML, or TeX is done through XSLT and Velocity; most of the mathematical formulæ conversion is done through `symbolpresentation` elements which collect `notation` elements each of which is a pair of an OpenMath expression and its associated rendering in MathML, see [MLUM06]. Each `notation` element is annotated with a context of use; typically this context is only a human language; in some cases, one can also indicate the book-collection that would trigger this notation thus supporting any "fuzzy" creation of a cultural context.

Based on the notation census, the content enrichment and translation processes will *encode* the necessary `symbolpresentation` and `notation` elements. This will be done as part of the encoding process and will be integrated in the quality proofing process.

4 Results

The notation census has been announced and presented to the partners of Math-Bridge in their January meeting with satisfaction.

As of the delivery date of this text, the census contains 28 pages with 22 symbols covered containing 33 observations in 10 cultural contexts. Of the math-bridge languages, thus far English, Spanish, French, Dutch, and German are covered.

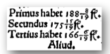
Thus far, at least one external contributor spontaneously contributed.

We reproduce below three pages that show the diversity of the notations as well as the potential of the census. We refer the reader to the census live site to see its current state:

`http://wiki.math-bridge.org/display/ntns`

4.1 A Homogeneous Notation: abs

This symbol is noted almost the same in all contexts, except in keyboard oriented inputs. We reproduce below a copy of `http://wiki.math-bridge.org/display/ntns/abs`:



abs

5 Added by Abdelshafi Bekhit, last edited by Abdelshafi Bekhit on 07 Dec 2009

Absolute Value Notations

The absolute value of a real number x is denoted $|x|$ and defined as the "unsigned" portion of x.

Semantic

- OpenMath's arith1/abs

Observation: Saudi Arabia

In Bibliography we find in page 9 the high-school book shows the absolute value in Arabic. The letter ا (ALEF) is a real number and اذا كان means *if*.

|أ| =
أ إذا كان أ > ٠
٠ إذا كان أ = ٠
-أ إذا كان أ < ٠

Observation: English USA

As found in Bibliography the absolute value of a can be found in the American book from this rule:

$$\begin{array}{ll} \text{if } a \geq 0, & \text{then } |a| = a \\ \text{if } a < 0, & \text{then } |a| = -a \end{array}$$

and in Bibliography we find the book represents absolute value in page number 315 as shown in the image on the right.

$$abs(x+y) \leq abs.x \ + \ abs.y$$

Observation: Germany

In page 24 in the German book the absolute value of x is shown as found in Bibliography.

$$|x| := \begin{cases} x, & \text{falls } x \geqslant 0, \\ -x, & \text{falls } x < 0, \end{cases}$$

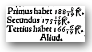
Observation: Spanish

Absolute value of a is represented in the spanish book in page number 13 as found in Bibliography.

$$\begin{array}{ll} |a| = +a & \text{si } a > 0, \\ |a| = -a & \text{si } a < 0. \end{array}$$

4.2 A Highly Varying Notation: gcd

The symbol of the greatest common divisor is bound to the locution hence is arguably made of different letters.



gcd

7 Added by Abdelshafi Bekhit, last edited by Abdelshafi Bekhit on 10 Jan 2010

Greatest Common Divisor Notations

In mathematics, The greatest common divisor of two positive integers a and b, denoted $GCD(a,b)$.

Semantic

- OpenMath's combinat1/gcd
- MathML's gcd element
- MathWorld's Greatest common divisor
- Wikipedia's Greatest common divisor

Observation: Arabic

In Saudi math book for grad VII we find in page number 75 the example shows gcd as 'ق.م.أ' for 63 and 42 in Arabic language.

ق . م . أ = ٢١

Observation: English

In Bibliography we find in page 191 this book shows the greatest common divisor in English.

$$\mathbf{gcd}\{a,b\} = \mathbf{gcd}\{b,r\}.$$

Observation: German

As found in Bibliography the German book shows the greatest common divisor in page 307.

$$\mathrm{ggT}(a,b).$$

Observation: Dutch

In Bibliography the Dutch book shows the greatest common divisor in page 81.
See also another Dutch book, page 9, in Bibliography.

$$\mathrm{GGD}(a,b) = a.$$

Also used is $\mathrm{ggd}(a,b) = a$, as seen in Relaties en Structuren (page 48).

Observation: Spanish

In page 271 in the Spanish book represents the greatest common divisor as found in Bibliography.

$$\mathbf{mcd}(a(x),b(x)) = \mathbf{mcd}(b(x),r_1(x)) = \cdots = \mathbf{mcd}(r_n(x),0) = r_n(x)$$

Observation: French

On page 101 of Intro-math-discrètes, the notation and name of the gcd is described (goto page).

pgcd(n,m) désigne le plus grand div

5 Conclusion

This document introduced the diversity of notations and namings in mathematical texts around the world and the approach to make a census of this diversity within the Math-Bridge project.

The requirement of assessing all notations follows directly from the fact that Math-Bridge intends to take the learners where they are and bring them to a level of mathematics satisfactory for starting university studies.

The instrument proposed to list these notations is called *the notation census*, it is a web-based community oriented set of web-pages which lists *observations* made in widely used sources. We anticipate this community-based-site to attract contributions of many external people who are sensitive to the cause of the preservation of culture. And indeed, a voluntary contribution followed after less than an hour of the public announce.

5.1 Open Questions

In this conclusion, we open the way to unanswered questions raised by this deliverable.

Change the users or change the tools? The diversity of the mathematical notations has not been extensively exploited by most tools doing mathematics. Most pocket calculators found in Spain or France write the sine function as `sin` and the tangent functions as `tan` or write the decimal separator with period. All computer algebra systems known to the authors only use the notations above. It is an accepted practice of a teacher introducing the tools to also introduce the notational differences which learners then tame. One could say that the tools expect to change the users.

The approach taken in Math-Bridge is different: it does all it can to take what is believed to be the learner's notations. This is justified by the fact that it intends to show complete mathematical texts, not only let tools be manipulated. This could also be justified by the web nature of the math-bridge tools: in this nature, there is no *start of the book where the notations are explained* and a first contact may be in the middle of many other things. The web-mediated interaction to the tools is possible in ActiveMath, however, and Math-Bridge provides a uniquely multilingual access to a computer algebra system. As opposed to the widespread *mono-lingual-access* which too often means *english access.*

The efforts of teachers explaining the *different notations* would then become the explanations of *different cultures* enabled by the easy language change a user can do any time in ActiveMath.

Readiness for All Cultures? The web-nature of Math-Bridge of course makes it accessible from any cultural environment in the world; the underlying Unicode and XML infrastructure allows it to translate content in a minimum of time. Will the ActiveMath tools be fully ready for any culture? This remains to be attempted. The directionality of writing texts, which is different at least in the arabic cultures, may appear to be a challenge as has it has been one for MathML [LM06] which expects to be solved by implementations of MathML 3 [CIM08]. Other ingredients as explained by [Mar09] may prove to be crucial.

More Contexts of Notations? The notation census manifest has refused, thus far, to normalize context names. Indeed, what has been observed thus far clearly shows that many liberties for notations are taken by book authors and that new contexts are likely to happen in any contribution.

A few new contexts' differentiations we expect will appear in the notation census:

- **identified schools** of thoughts are likely to define contexts. For example, the Cambridge school of category (represented by many works of the Cambridge University Press) used to write function application after the element (the application of f to x is written xf instead of $f(x)$ as is common elsewhere); also we have been told that some italian mathematics teaching wrote the function composition the other way as the english and french one ($f \circ g$ vs $g \circ f$).
- **old and new**: the naming pgcd, in French, refers to the words plus grand commun diviseur which is obsolete French. Some teachers replace it by *pgdc* but this notation is not yet widespread.

Acknowledgements

The authors wish to thank the very many respondants to all sorts of nerving questions sent around by mail since a few years in order to enable ActiveMath with its level of multilingual notation thus far. This includes Stephen Watt, James Davenport, Abdelshafi Bekhit, George Goguadze, Elena Smirnova, Mohammed Abuhelwa, Christian Gross, Claudio Sacerdoti Cohen, Marc Wagner, Christian Mercat, Josje Lodder, Neil Soiffer, Pascal Fischer, ...

We also wish to thank Abdelshafi Bekhit for much of the contributions to the census contents.

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