Final exam

1. A single indivisible object is sold to one of two potential buyers. Buyer 1’s value, $X_1$, is uniformly distributed in the interval $[0, 1]$, whereas buyer 2’s value, $X_2$, is uniformly distributed in the interval $[0, 2]$. The two values are independently distributed.

(a) If the seller uses a second-price auction with reserve price $r$, what is the optimal $r$ for a seller who attaches value $x_0 = 0$ to the good?

(b) What is the optimal selling mechanism for this problem?

(c) How would the VCG mechanism allocate the object and what would be the payments in that case?

(d) Which of the selling mechanisms above are efficient? Argue why or give a counterexample.

(e) Rank the three mechanism above in terms of revenues to the seller, without computing expected revenues.

2. A good with common value $V$ is auctioned among $N = 3$ bidders. $V$ is unknown at the time of the auction, but assumed to be uniformly distributed over $[0, 1]$. Bidders get private signals $X = (X_1, X_2, X_3)$ on the value of the good. Given $V = v$, signals are independently and uniformly distributed on $[0, 2v]$.

(a) Derive the joint density of $X$. What is the density of $V$ conditional on $X$?

(b) Compute the symmetric equilibrium bidding strategy for a SPSB auction.

(c) Compute the symmetric equilibrium bidding strategy for a FPSB auction.

(d) As a seller, which one of the last two auctions would you prefer and why?